IUT III

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In short, in IUT II,

we performed "Galois evaluation"

theta fct $\longmapsto$ theta values

"env" labels "gau" labels

($\mathcal{MF}^{\nabla}$-objects $\longmapsto$ Galois rep'ns
(filtered φ-modules))

Two Problems

1. Unlike "theta fcts",

"theta values" DO NOT admit

a multiradial alg'm

in a NAIVE way.

2. We need

ADDITIVE str.

for (log-) height fcts.

$\mu^{\log}$

On 1.

$$\text{theta fcts} \xmapsto{\text{Gal. eval.}} \text{theta values}$$

(admit multirad. alg'm
cf. [IUT II, Prop 3.4 (i)])

$$\text{theta values} \supseteq \prod_{\mathbb{F}_l^*} (\text{constant monoids})$$

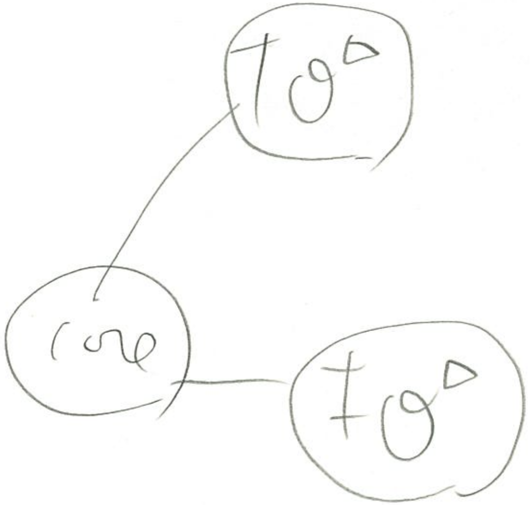
requires cycl. rig via

LCFT

(cf. [IUT II, Prop 3.4 (ii), Cor 3.7 (ii), Rem 4.10.2 (ii)])

Recall cycl. rig. via LCFT

uses $\Theta^{\triangleright} = (\text{unit portion}) \times (\text{value gp portion})$



We DO NOT share it in both sides of Θ-link!

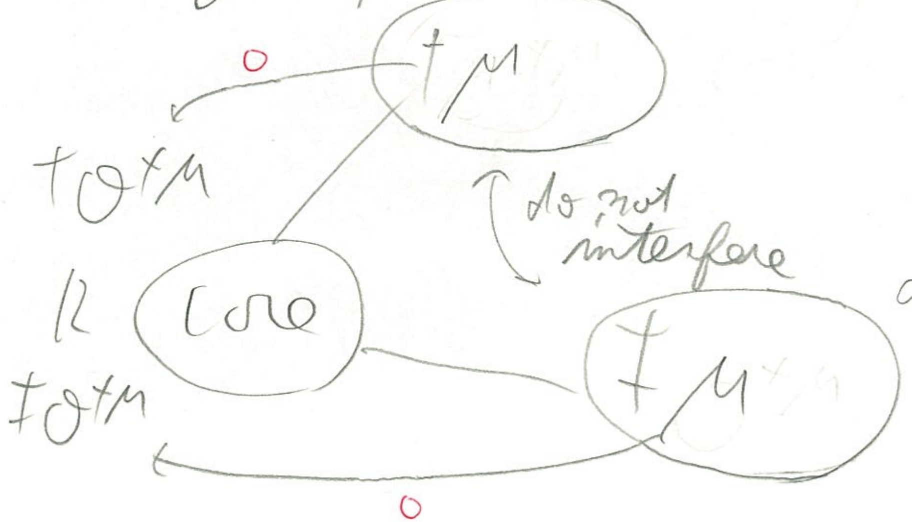
"$\{\underline{\underline{q}}^{j^2}\}_j \longmapsto \underline{\underline{q}}$"

theta values DO NOT admit a multirad. alg'm in a NAIVE way.

cf.

- cycl. rig. via mono-theta env.
- cycl. rig. via $\hat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$

use only "μ-portion" → admit a multirad. alg'm



⑧

To overcome these problems,

→ use log link!

(& allowing <u>mild</u> indet's

non-interference

etc.

(later))

want to see alien ring str.

eye

$$
\begin{tikzcd}
\boxed{+} & & \bullet \arrow[r, "\Theta\text{-link}"] & \bullet \\
\boxed{\times} \arrow[u] & \mathcal{O}^{\times\mu} & \bullet \arrow[u, "\text{log-link}"'] &
\end{tikzcd}
$$

<u>Note</u> $\mathbb{F}_l^{\rtimes\pm}$-symm. isom's are compatible w/ log-links

$\rightsquigarrow$ can pull-back Ψ_{gau} via log-link

However,

$$\begin{tikzcd}
\boxed{+} & \bullet \arrow[r, "\circlearrowleft"] \arrow[r, bend left=40, "\circlearrowleft"] & \bullet \\
\boxed{\times} \arrow[u] & \bullet \arrow[u, "\log"'] \arrow[r, "\circlearrowleft"'] & \bullet \arrow[u, "\log"']
\end{tikzcd}$$

$\theta^{\times\mu}$

log is highly

<u>non-</u>commutative

(i.e. $\log(a^N) \neq (\log a)^N$)

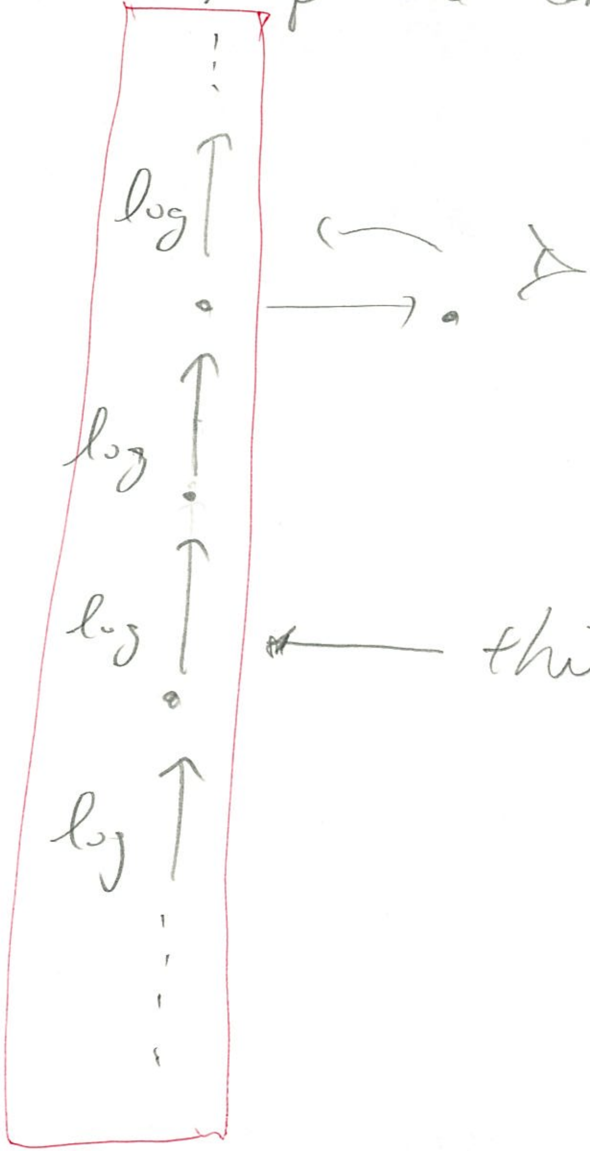
cannot see

from the right

$$\begin{tikzcd}
\bullet \\
\bullet \arrow[u, "\log"']
\end{tikzcd}$$

We consider

the infinite chain of log-links



this is invariant

by one shift!

Important Fact

$k/\mathbb{Q}_p$ fin. <u>log shell</u>

$$\log \mathcal{O}_k^{\times} \subset \frac{1}{2p} \log \mathcal{O}_k^{\times} =: \mathcal{I}_k$$

$$\begin{array}{c} \uparrow \log \\ \mathcal{O}_k^{\times} \end{array}$$

$\leftarrow$ the domain & codomain of log are contained in the log-shell

upper semi-compatibility

(Note also: log-shells are <u>rigid</u>)

Besides theta values, we need
<u>another thing</u>:

we need <u>NF</u> (:= number field)

to convert $\boxtimes$-line bdles

into $\boxplus$-line bdles

and vice versa.

$\boxtimes$ - line bdles

— def'd in terms of <u>torsors</u>

$\boxplus$ - line bdles

— def'd in terms of <u>fractional ideals</u>

$\exists$ natural cat. equiv. in a scheme theory

$\boxtimes$-line bdles

$\Leftarrow$ def'd only in terms of $\boxtimes$-str's

$\rightsquigarrow$ admits precise log-Kummer corr.

But, difficult to compute log-volumes

$\boxplus$-line bdles

$\Leftarrow$ def'd by both of $\boxtimes$ & $\boxplus$-str's

$\rightsquigarrow$ only admits upper semi-compatible log-Kummer corr.

But, suited to <u>explicit estimates</u>

We also include NFs
as data

$$\left(\text{an NF}\right)_j \subset \prod_{\underline{v}_{\mathbb{Q}}} \log(\Theta^{\times})$$

theta values
NFs

← story goes *in a parallel way* in some sense
(of course ∃ essential difference
cf. [IUT III, Rem 2.3.2, 2.3.3])

To obtain the final multirad. alg'm:

Frob.-like • data assoc. to $\mathcal{F}$-prime-strips

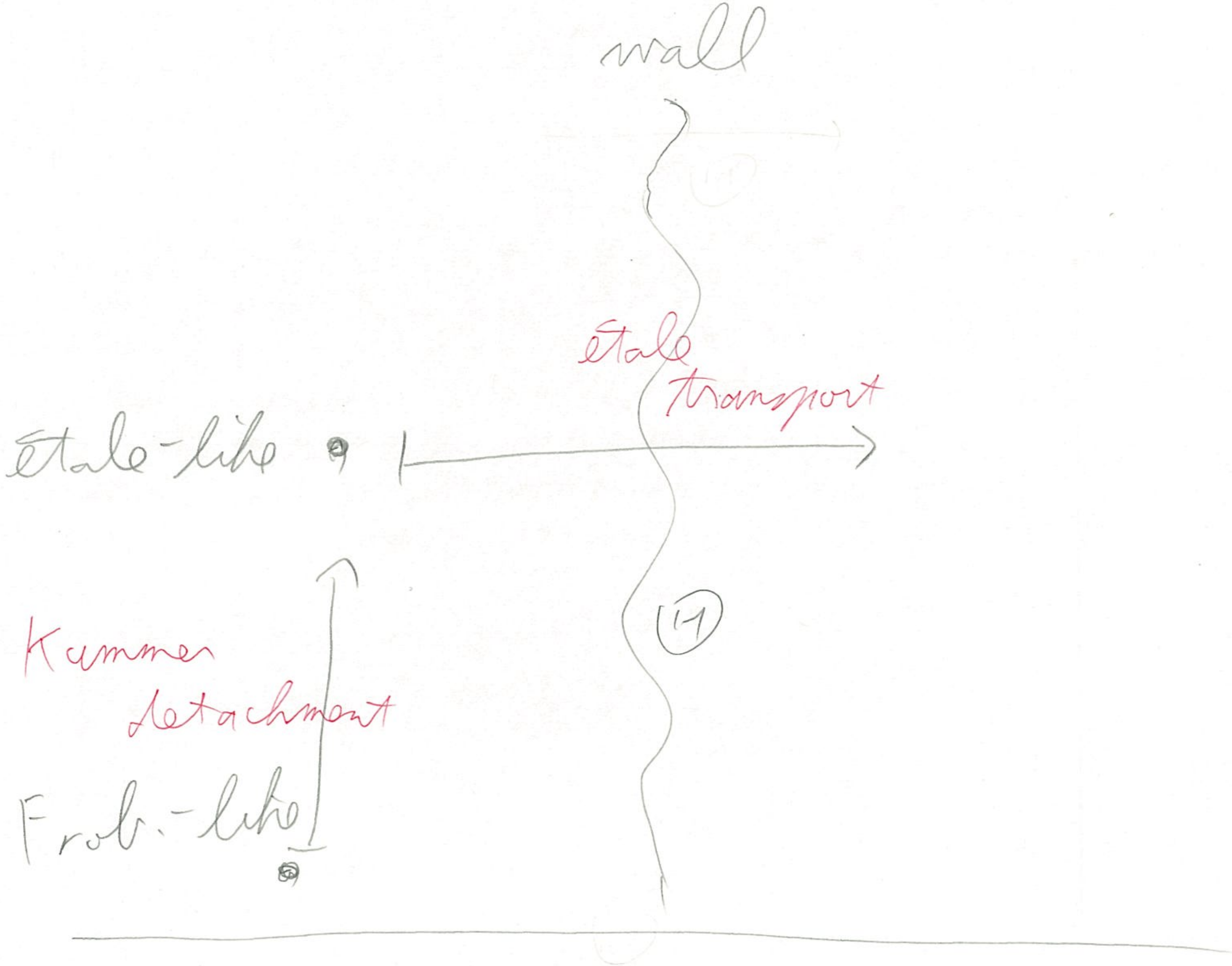
$\downarrow$ $\quad$ } Kummer theory

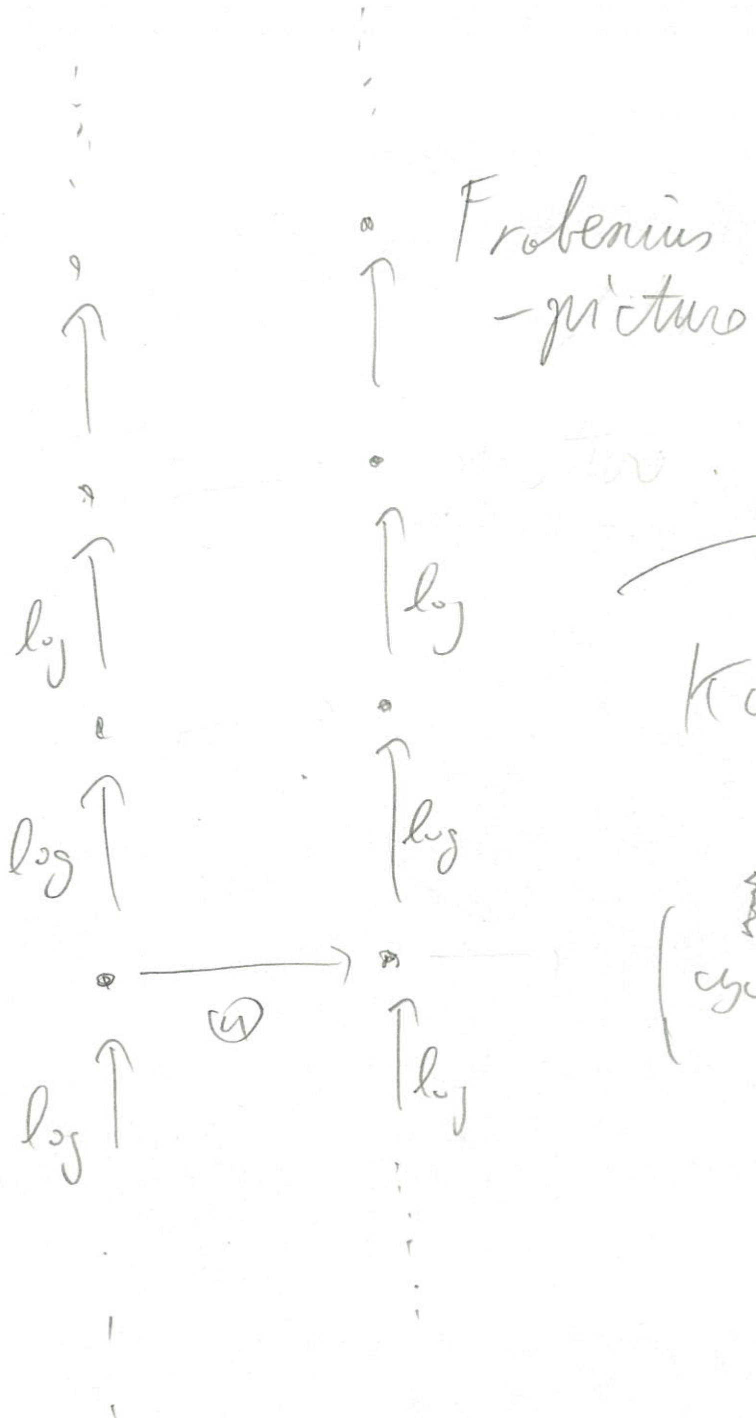
étale-like • data assoc. to $\mathcal{D}$-prime-strips

arith.-hol.

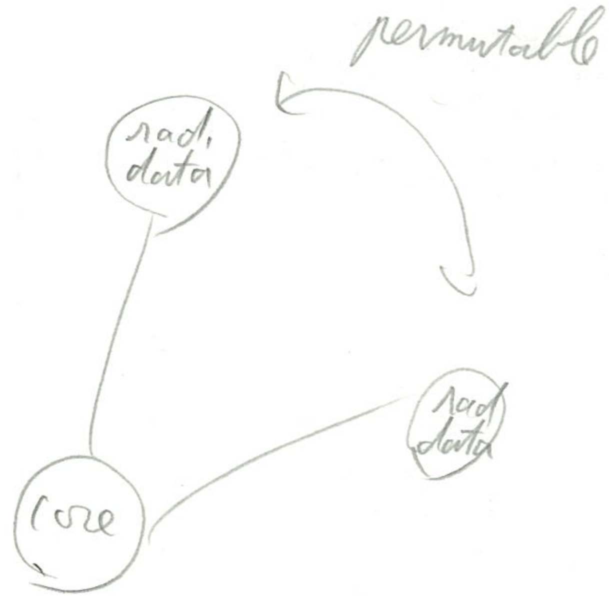
$\downarrow$ $\quad$ } forget arith. hol. str.

mono-an. • data assoc. to $\mathcal{D}^{\vdash}$-prime-strips


wall
étale transport
étale-like
Kummer detachment
Frob.-like
(1)



Kummer theory
(cycl. rig.'s)



3 portions of Θ-link

local

- unit: $\quad {}^{\dagger}\mathcal{O}^{\times\mu} \xrightarrow{\sim} {}^{\ddagger}\mathcal{O}^{\times\mu}$, acted on by ${}^{\dagger}G_v \xrightarrow{\sim} {}^{\ddagger}G_v$ ← share (∃ lnt fct)

- value gp: $\quad {}^{\dagger}\underline{\underline{q}}^{(1^2, \dots, (l^*)^2)\mathbb{N}} \xrightarrow{\sim} {}^{\ddagger}\underline{\underline{q}}^{\mathbb{N}}$ ← drastically changed

- global realified: $\quad (\mathbb{R}_{\geq 0})_{\underline{v}}(\dots, j^2, \dots) \xrightarrow{\sim} (\mathbb{R}_{\geq 0})_{\underline{v}} \log \underline{\underline{q}}$ (∃ lnt fct)

$\underline{\mathbb{V}} \ni \underline{v}$

Kummer theory

unit portion

$${}^{\dagger}G_v \curvearrowright {}^{\dagger}\mathcal{O}^{\times\mu} := {}^{\dagger}\mathcal{O}^{\times}_{\bar{k}}/\mu \quad \mathbb{Q}_p\text{-module}$$

$$+ \text{ integral str. i.e. } \operatorname{Im}(\mathcal{O}^{\times}_{\bar{k}^H}) \subseteq (\mathcal{O}^{\times\mu}_{\bar{k}})^H$$

fin. gen. $\mathbb{Z}_p$-mod. $\quad \forall H \subset G_v$ open

log-shell

($\exists$ computable log-vol.)

Θ-wall --- | --- non-ring theoretic

$${}^{\ddagger}G_v \curvearrowright {}^{\ddagger}\mathcal{O}^{\times\mu}$$

22

$$\left({}^{\dagger}G_v \curvearrowright \mathcal{O}^{\times}_{\bar{k}} \right) \xrightarrow[\text{Kummer}]{\sim} \left({}^{\dagger}G_v \curvearrowright \mathcal{O}^{\times}_{\bar{k}}({}^{\dagger}G_v) \right)$$

unlike the case of $\mathcal{O}^{\triangleright}_{\bar{k}}$,

(← now, we cannot use $\mathcal{O}^{\triangleright}_{\bar{k}}$. use only $\mathcal{O}^{\times}_{\bar{k}}$)

$\hat{\mathbb{Z}}^{\times}$-indet. occurs

(log-vol. is invariant under this $\hat{\mathbb{Z}}^{\times}$-indet. OK)

cycl. rig. $\mu(G_v) \xrightarrow[?]{\sim} \mu(\mathcal{O}^{\times}_{\bar{k}})$

via LCFT

does not hold.

We want to protect

$\left\{\begin{array}{l}\text{value gp portion} \\ \text{global real'd portion}\end{array}\right.$

from this $\widehat{\mathbb{Z}}^{\times}$-indet.!

(sharing ${}^{\dagger}\mathcal{O}^{\times\mu} \xrightarrow{\sim} {}^{\ddagger}\mathcal{O}^{\times\mu}$ w/ int. str.s

$\rightsquigarrow$ (Ind 2))

$\bullet \xrightarrow[\text{(n)}]{} \bullet$ horizontal indet.

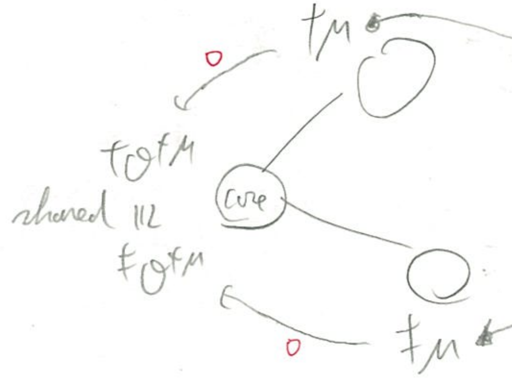
value gp portion

$\mathcal{O}^{\triangleright}$

mono-theta cycl. rig

(unlike LCFT cycl. rig.)

only $\mathcal{M}$ is involved



do not obstruct each others

multirad.

(on the function level)

NF

$\hat{\mathbb{Z}}^{\times} \cap \mathbb{Q}_{>0} = \{1\} \rightsquigarrow$ cycl. rig.

Note also

mono-theta cycl. rig.

is compat w/ prof. top.

$\rightsquigarrow$ $\mathbb{F}_l^{\rtimes\pm}$-sym. (conj. synchro.)

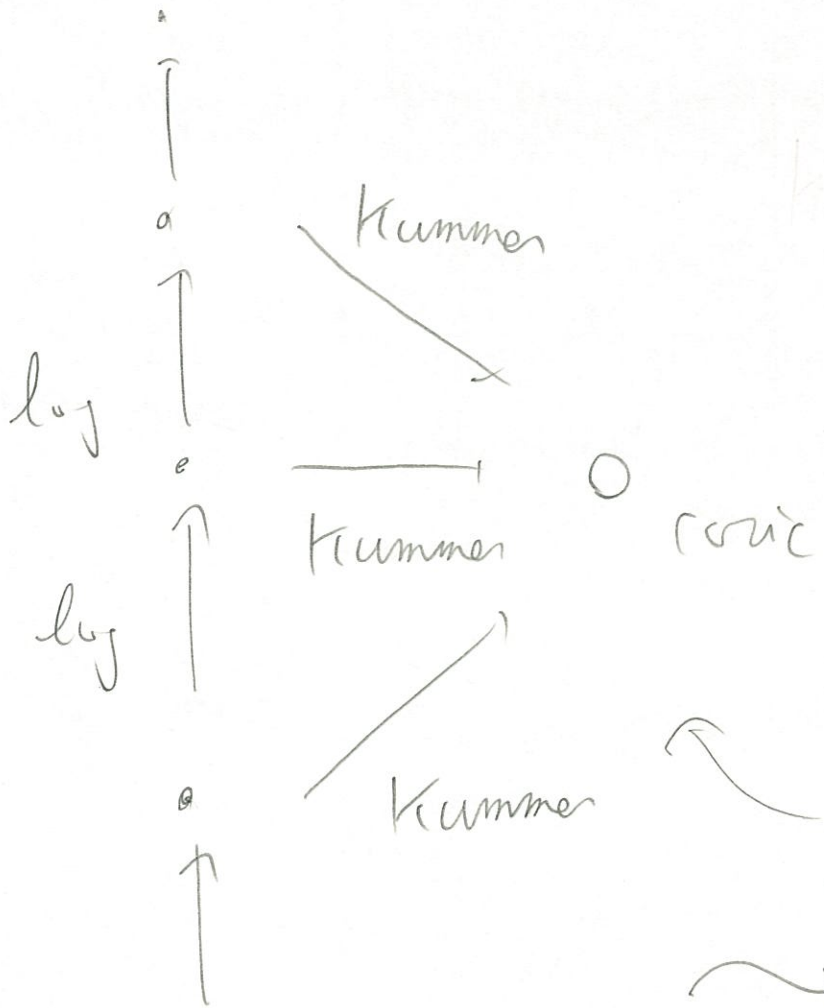
$$\begin{tikzcd} \boxplus \\ \boxtimes \arrow[u, "\log"] \end{tikzcd} \quad \mathbb{F}_l^{\rtimes\pm}\text{-sym.}$$

is compat. w/ log-links

$\rightsquigarrow$ can <u>pull-back</u> coric (diagonal) obj via log-links

later

$\rightsquigarrow$ LGP monoid (logarithmic Gaussian Procession)



After

$(\text{Kummer}) \circ (\log)^n \quad (n \geq 0),$

take the action of "$\underline{q}^{j^2}$"

on $\mathcal{I} \otimes \mathbb{Q}$

log-Kummer correspondence

not compat.

rigid!

$\rightsquigarrow$ consider at a common upper bound given by log-shell

(Ind3)

$\uparrow \log$ vertical indet.

const. multiple rig.

$\log_\mu \uparrow$ $\exists$ splitting modulo μ of

label 0

hor. core

$$0 \to \mathcal{O}^\times \to \mathcal{O}^\times \cdot q^{j^2} \to \mathcal{O}^\times \cdot q^{j^2} / \mathcal{O}^\times \to 0$$

&

$\log_\mu |\mu| = 0$

$\longrightarrow$ No new action appears by the iterations of log's

No interference

Note also

$$\mu^{\log}(\log_{\mu}(A)) = \mu^{\log}(A)$$

if $\quad A \xrightarrow[\text{bij}]{\sim} \log_{\mu}(A)$

(compatibility of log-volumes w/ log-links)

$\rightsquigarrow$ do not need to care about <u>how many times</u> log's are applied.

In the Archimedean case,

we use a system (cf. [IUT II, Rem 4.8.2 (v)])

$$\{ \cdots \to\!\!\!\to \mathcal{O}^\times/\mu_N \to\!\!\!\to \mathcal{O}^\times/\mu_{N'} \to\!\!\!\to \cdots \}$$

& μ_N is killed in $\mathcal{O}^\times/\mu_N$

& constructions start from $\mathcal{O}^\times/\mu_N$'s, not $\mathcal{O}^\times$
(of log-links, ...) (cf. [IUT III, Def 1.1 (ii)])

& we put "weight N" on $\mathcal{O}^\times/\mu_N$
for the log-volumes
(cf. [IUT III, Rem 1.2.1 (ii)])

<u>NF</u> as well, consider the actions of $(F^{\times}_{\mathrm{mod}})_j$

after $(\text{Kummer}) \circ (\log)^n$ $(n \geq 0)$

By $F_{\mathrm{mod}} \cap \prod_v \mathcal{O}_v = \mu$

$\rightsquigarrow$ No new action appears in the iteration of log's

No interference

け	multirad. geom. container	contained in a mono-analytic container
val gp	theta fct $\xrightarrow{\text{eval}}$ (depends on labels & hol. str.)	theta values $\underline{q^{j^2}}$
NF	$\infty\kappa$-coric fcts $\xrightarrow{\text{eval}}$ (indep. of labels dep. on hol. str.) Belyi cusp'tion	NF $F_{mod}^{\times}$ (up to $\{\pm 1\}$)

	cycl. rig.	log-Kummer
theta	mono-theta cycl. rig.	no interference by const. mult. rig.
NF	$\hat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$ cycl. rig	no interference by $F_{mod} \cap \prod_v \mathcal{O}_v = \mu$

vicious cycles

$\dagger\mu_{Fr}$ $\xrightarrow{\text{Kummer}}$ ${}^{\circ}\mu^{\Theta}_{et}$

$\uparrow \log$

$\ddagger\mu_{Fr}$ $\xrightarrow{\text{Kummer}}$

$\uparrow \log$

μ^{Θ}_{et}

$\wr$

indet $=: \mathbb{I}^{ord}$

$\cap$

$\mathbb{N}_{\geq 1} \times \{\pm 1\}$

theta: $\mathbb{I}^{ord} = \{1\}$

by zero of order $= 1$ at each cusps

NF: $\mathbb{I}^{ord} \twoheadrightarrow \mathrm{Im} \subset \mathbb{N}_{\geq 1}$

$\parallel$

$\{1\}$

by $\mathbb{Q}_{>0} \cap \hat{\mathbb{Z}}^{\times} = \{1\}$

we have $(F^{\times}_{mod})^{\circ} \curvearrowright \{\pm 1\}$-indet.

($\{t, \underline{q}^{j^2}\}$'s are not inv under $\{\pm 1\}$)

OK. $\leftarrow$ However the totality of $F^{\times}_{mod}$ is invariant under $\{\pm 1\}$

cf. [IUT IV, Fig 2.7]

$\mathbb{F}_\ell^{\rtimes\pm}$-sym.

0 is also permutated

theta

local & transcendental

$q = e^{2\pi i z}$

compat. w/ prof. top.

theta fct

zero of order $= 1$ at each cusp

"only one valuation"

cycl. rig.

Note theta fcts / theta values does not have $\mathbb{F}_\ell^{\rtimes\pm}$-sym.

But, the cycl. rig DOES use $[\,,\,]$

NF

global & algebraic

rat. fcts.

$\mathbb{F}_\ell^{*}$-sym.

0 is isolated

incompat. w/ prof. top.

Never for alg. rat. fcts

$\hat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$

"many valuations" ← global

sacrifice the compat. w/ prof. top.

Note also Gal. eval. or use hol. ith.

labels

<u>theta</u> Gal eval & Kummer

← compat. w/ labels

<u>NF</u> ← the output F_{mod}^{*} does not depend on labels

global val'd monoids are mono-analytic nature (← units are killed)

→ do not depend on hol. str.

unit $\quad {}^{\dagger}\mathcal{O}^{\times\mu} \cong {}^{\ddagger}\mathcal{O}^{\times\mu}$ (Ind 2) $\rightarrow$

val. gp $\quad \{\underline{\underline{q}}^{j^2}\}_{j} \curvearrowleft_{I \otimes \mathbb{Q}}$ w/ (Ind 3) $\uparrow$

MF $\quad \|\overset{(-)}{M}_{mod} \curvearrowleft$

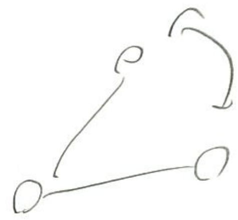
Kummer detachment

étale-like objects

étale transport

$$ {}^{\dagger}G \xrightarrow[\sim]{\text{full poly}} {}^{\ddagger}G $$

(Ind 1)



permutation indet.

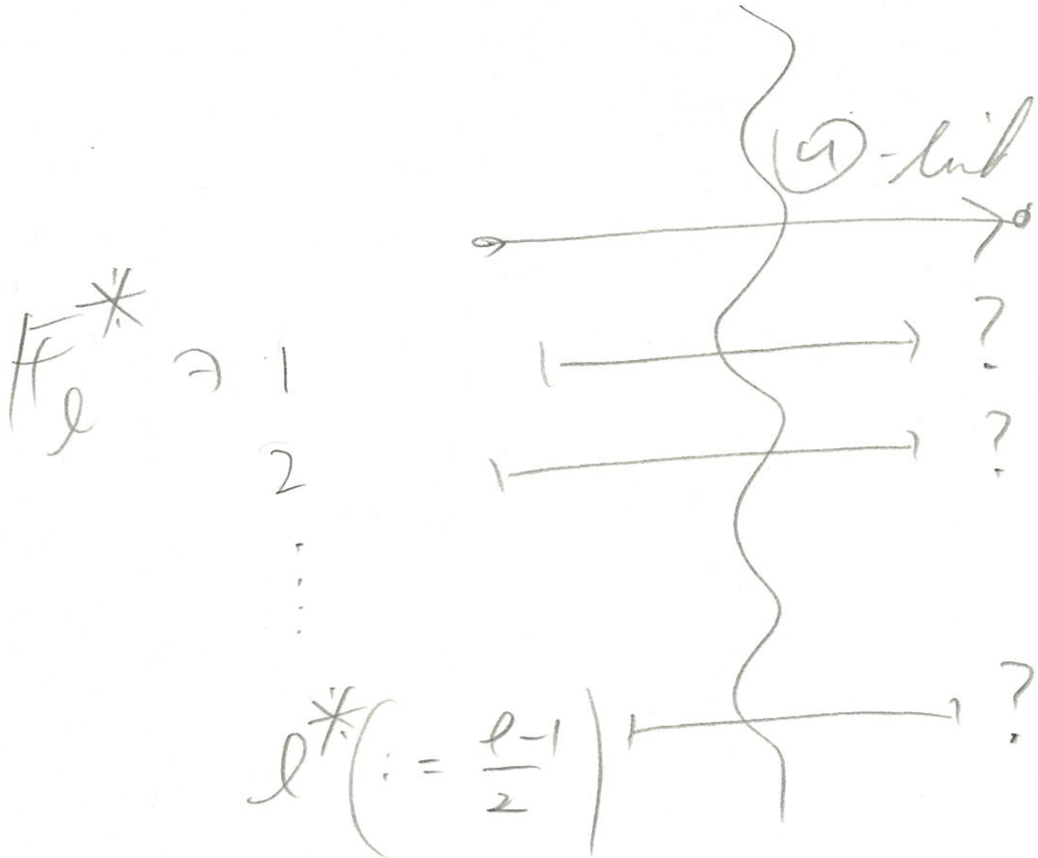
$\rightsquigarrow$ we can transport the data over the Θ-wall

Another thing

$$\Psi_{gau} \subset \prod_{j \in \mathbb{F}_\ell^{*}} \text{(const. monoids)}$$

labels come from arith. hol. str.

cannot transport the labels for ⓪-link



$0 \longmapsto ?$ $l^{\pm}$-possibilities

$1 \longmapsto ?$

$\vdots$

$l^{*} \longmapsto ?$

in total

$(l^{\pm})^{l^{\pm}}$-possibilities

$\left(l^{\pm} := l^{*} + 1 = \frac{l+1}{2} \right)$

the final inequality is weak.

use processions

$$
\begin{array}{ccccccccc}
\{0\} & \subset & \{0,1\} & \subset & \{0,1,2\} & \subset & \cdots & \subset & \{0,\ldots,\ell^*\} \\
\downarrow & & \downarrow & & \downarrow & & & & \\
\{?\} & \subset & \{?,?\} & \subset & \{?,?,?\} & \subset & \cdots & \subset & \{?,\ldots,?\}
\end{array}
$$

$\rightsquigarrow$ then, in total $(\ell^{\pm})!$-possibilities

gives more strict inequality than the former case

A rough picture of the final multirad. rep'n:

$$\begin{array}{ccccccc} & & (F^{\times}_{\mathrm{mod}})_1 & & \dots & & (F^{\times}_{\mathrm{mod}})_{l^*} \\ & & \wr & & & & \wr \\ \{\mathcal{I}^{\mathbb{Q}}_0\} & \subset & \{\mathcal{I}^{\mathbb{Q}}_0, \underset{\sim}{\mathcal{I}^{\mathbb{Q}}_1}\} & \subset & \dots & \subset & \{\mathcal{I}^{\mathbb{Q}}_0, \dots, \underset{\sim}{\mathcal{I}^{\mathbb{Q}}_{l^*}}\} \end{array}$$

$\underset{=}{q}^{1^2}$ $\underset{=}{q}^{2^2}$ acts $\underset{=}{q}^{(l^*)^2}$

$\underline{\Psi}^{\perp}_{LGP}$

const. mult. rig.

value gp portion via canonical splitting modulo μ

By this multirad rep's

& the compatibility w/ ⊛-link:

$$\left\{\underline{\underline{q}}^{j^2}\right\}_{1\le j\le \ell^*} \qquad\qquad \underline{\underline{q}}$$

w/ indet's $\qquad$ w/ indet's

deg << 0 $\qquad$ deg ≈ 0

we cannot distinguish them!!

(Ind 1) permutation indet.  ${}^\dagger\mathcal{G} \simeq {}^\ddagger\mathcal{G}$

in the étale transport

(Ind 2) horizontal indet. $\bullet \xrightarrow{\circlearrowleft} \bullet$ ${}^\dagger\mathcal{O}^{\times\mu} \simeq {}^\ddagger\mathcal{O}^{\times\mu}$

in the Kummer detach. w/ int. str.

(Ind 3) vertical indet. $\bullet \xrightarrow{\log} \bullet$ (vertical) $\quad \log(\mathcal{O}^\times) \subset \frac{1}{2p}\log(\mathcal{O}^\times)$, $\uparrow \log$, $\mathcal{O}^\times$

in the Kummer detach.

can be considered as

a kind of "descent data from

$\mathbb{Z}$ to $\mathbb{F}_1$"

$\mathbb{Z} \otimes \mathbb{Z}$

$\mathbb{F}_1$

hol. hull

(Ind 1)

(Ind 2)

(Ind 3)

mono-analytic

container

||

log-shell

$\mathfrak{I}$

possible image of "{q}"

somewhere, it contains q

Recall $\{\underline{q}^{j^2}\}_j \longmapsto \underline{\underline{q}}$

$\rightsquigarrow \; 0 \lesssim -(\text{ht}) + \underbrace{(\text{indet})}_{(\boxed{1}+\varepsilon)\left(\text{log-diff} \; (+\text{log-cond})\right)}$

$\rightsquigarrow \; (\text{ht}) \lesssim (1+\varepsilon)(\text{log-diff} + \text{log-cond})$

calculation in Hodge-Arakelov

miracle equality

$$\left(\left(\begin{aligned} \sum_j j^2 [\,\cdot\,] &\approx \frac{l^2}{24}[\,\cdot\,] \\ \sum_j j [\,\cdot\,] &\approx \frac{l^2}{24}[\,\cdot\,] \end{aligned}\right)\right)$$

<u>cf.</u> Hodge Arakelov

<u>IF</u> a global mult. subspace existed

$$\Longrightarrow \quad \begin{array}{ccc} \underline{\underline{q}}\,\vartheta & \hookrightarrow & \underline{\underline{q}}^{j^2}\,\vartheta \\ \uparrow & & \uparrow \\ \deg \doteq 0 & & \deg \ll 0 \end{array}$$

$$\Longrightarrow \quad -(\text{large}) \gtrsim 0$$

[IUT III, Th3.11] In summary,

Tempered conj. vs prof. conj. (semi graphs of anab.) $\mathbb{F}_l^{\rtimes\pm}$-conj. synchro ($\rightsquigarrow$ diagonal) ($\rightsquigarrow$ hor. conj.)

	(i) (objects)	(ii) (log-Kummer)	(iii) (compat w/ $\boxplus^{\times\mu}_{LGP}$-link)
$\mathbb{F}_l^{\rtimes\pm}$-symm. $\boxplus$	$\mathcal{I}^{\mathbb{Q}}$ $\rightsquigarrow$ unit	invariant after admitting (Ind3) $\uparrow$	invariant after admitting (Ind2) $\rightarrow$ $\hat{\mathbb{Z}}^\times$-indet.
	$\overline{\mathcal{I}}^{\perp}_{LGP}$ val gp ← compat. of log-link w/ $\mathbb{F}_l^{\rtimes\pm}$-symm.	no interference by const. mult. rig. ell. cusp'tion ← pro-p anal. + hidden endom.	(only μ is involved $\rightsquigarrow$ multirad) protected from $\hat{\mathbb{Z}}^\times$-indet. by mono-theta cycl. rig. [quadratic str. of Heisenberg gp
$\mathbb{F}_l^{*}$-symm. $\boxtimes$	$\overset{(-)}{\mathbb{M}}_{mod}$ NF ← Belyi cuspition ↑ pro-p anal. + hidden endom.	no interference by $F_{mod} \cap \prod_v \mathcal{O}_v = \mu$	protected from $\hat{\mathbb{Z}}^\times$-indet. by $\hat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$
others	(compat. of log-volumes w/ log-links)	(arch. theory: Aut-hol. space ell. cusp'tion) (auton. of processions are included)	(étale picture: permutable after admitting (Ind1) $\circlearrowleft$)

some questions

How about the following variants of ④-link?

i) $\{\underline{\underline{q}}\partial^2\}_j \longmapsto \underline{\underline{q}}^N$ $(N>1)$

ii) $\{(\underline{\underline{q}}\partial^2)^N\}_j \longmapsto \underline{\underline{q}}$ $(N>1)$

i) $\{ \underline{q}^{j^2} \}_j \longmapsto \underline{q}^N$

$\uparrow$

$\deg \doteq 0$ $\left(\begin{array}{c} \ell \approx ht \\ \& \\ \deg \ll \ell \end{array} \right)$

<u>it works</u>

$\rightsquigarrow$ $N \cdot 0 \lesssim -(ht) + (\text{indet})$

(as for $N \ll \ell$)

(When $N > \ell \Rightarrow$ the inequality is weak)

ii) $\left\{ \left(\underline{q}\, 0^2 \right)^N \right\}_0 \longmapsto \underline{\underline{q}}$

A **DOES NOT** work!

Because

1. $\Theta \overset{\text{replace}}{\rightsquigarrow} \Theta^N \Rightarrow$ ~~mono-theta cycl. rig.~~

mono-theta cycl. rig. comes from the quadraticity of $[\ ,\ ]$

cf. [EtTh, Rem 2.19.2]

$\rightsquigarrow \Theta^N$ $(N>1)$ $\rightsquigarrow$ $\nexists$ Kummer compat.

2 mono-theta

constant multiple rig.

as well

ext'n str. of

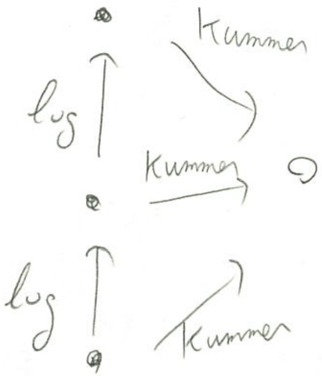
$$0 \to \mathcal{O}^\times(-) \to \mathrm{Aut}_{\mathcal{E}}((-)) \to \mathrm{Aut}_{\mathcal{D}}((-)^{\mathrm{bs}}) \to 1$$

c.f. [EtTh, Rem 5.12.5]

3 vicious cycles

Θ^N zero of order $= N > 1$ at cusps

various Frob-like $\mu \overset{\text{Kummer theory}}{\cong}$ étale-like $\mu \rightsquigarrow$ cusp



cf. [IUT III, Rem. 2.3.3 (vi)]

loop $\rightsquigarrow$ one loop gives once N-power

IF ~~it~~ WORKED

$\rightsquigarrow 0 \lesssim -N(\mathrm{ht}) + (\mathrm{ind\,et})$

$\rightsquigarrow (\mathrm{ht}) \lesssim \frac{1}{N}(1+\varepsilon)(\text{log-diff.} + \text{log-cond.})$

$\rightsquigarrow$ contradiction

to a lower bound
given by analytic number theory
(Masser, Stewart-Tijdeman)