IUT III

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11/Dec/2015 at Oxford

The author expresses his sincere gratitude to RIMS secretariat for typesetting his hand-written manuscript.

In short, in IUT II,
we performed "Galois evaluation"

theta fct	$\longmapsto$	theta values
"env" labels		"gau" labels

$$\left(\begin{array}{l} \mathcal{MF}^{\nabla}\text{-objects} \\ \text{(filtered } \varphi\text{-modules)} \end{array} \quad \longmapsto \quad \text{Galois rep'ns} \right)$$

Two Problems

1. Unlike “theta fcts”, “theta values” DO NOT admit a multiradial alg’m in a NAIVE way.

2. We need ADDITIVE str. for (log-) height fcts. $\mu^{\log}$

On 1.

theta fcts $\overset{\text{Gal. eval.}}{\longmapsto}$ theta values

$\uparrow$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\cap$

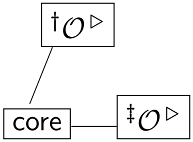
(admit multirad. alg'm *cf.* [IUT II, Prop 3.4(i)]) $\qquad\qquad$ $\prod_{\mathbb{F}_\ell^{\circledast}}$ (constant monoids)

$\uparrow$

requires cycl. rig. via LCFT
(*cf.* [IUT II, Prop 3.4(ii), Cor 3.7(ii), Rem 4.10.2(ii)])

Recall cycl. rig. via LCFT uses

$$\mathcal{O}^{\triangleright} = (\text{unit portion}) \times (\text{value gp portion})$$



theta values

We DO NOT share it

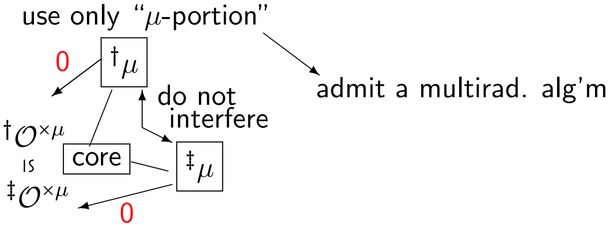
in both sides of Θ-link!

$$\text{“}\{\underline{\underline{q}}^{j^2}\}_j \longmapsto \underline{\underline{q}}\text{”}$$

DO NOT admit a multirad. alg'm in a NAIVE way.

cf.

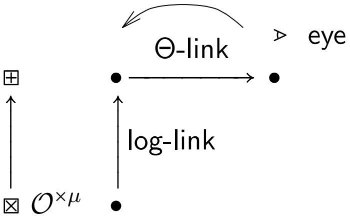
$$\begin{cases} \text{cycl. rig. via mono-theta env.} \\ \text{cycl. rig. via } \widehat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\} \end{cases}$$



To overcome these problems,
$\longrightarrow$ use log link!

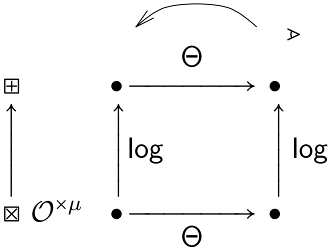
$$\left(\begin{array}{c} \text{\& allowing } \underset{\uparrow}{\text{mild}} \text{ indet's} \\ \\ \text{non-interference etc. (later)} \end{array} \right)$$

want to see alien ring str.



(Note $\mathbb{F}_{\ell}^{\rtimes\pm}$-symm. isom's
are compatible w/ log-links
$\rightsquigarrow$ can pull-back Ψ_{gau} via log-link)

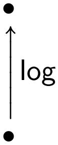
However,



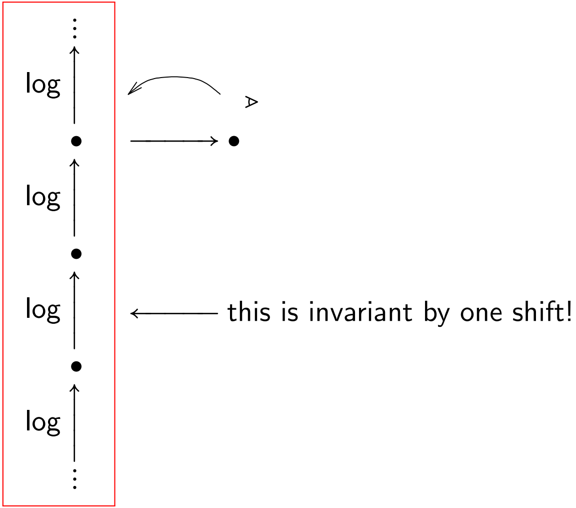
is highly <u>non</u>-commutative

(*cf.* $\log(a^N) \neq (\log a)^N$)

⇝ cannot see from the right



We consider the infinite chain of log-links



Important Fact

$k/\mathbb{Q}_p$ fin.

$$\begin{array}{ccll} \log O_k^\times & \subset & \frac{1}{2p}\log O_k^\times & = \underset{}{\overset{\text{log shell}}{\mathcal{I}_k}} \\ \uparrow \log & \circlearrowleft & \text{the domain \& codomain} & \\ & & \text{of log are} & \\ O_k^\times & & \text{contained in the log-shell} & \end{array}$$

upper semi-compatibility

(Note also: log-shells are rigid)

Besides theta values, we need
another thing :

we need NF (:= number field)
to convert ⊠-line bdles
into ⊞-line bdles
and vice versa.

$\boxtimes$ -line bdles

$\leftarrow$ def'd in terms of torsors

$\boxplus$ -line bdles

$\leftarrow$ def'd in terms of fractional ideals

$\nearrow$

$\exists$ natural

cat. equiv. in a scheme theory

⊠ -line bdles

← def'd only in terms of ⊠ -str's

→ admits precise log-Kummer corr.

But, difficult to compute log-volumes

⊞ -line bdles

← def'd by both of ⊠ & ⊞-str's

→ only admits upper semi-compatible log-Kummer corr.

But, suited to explicit estimates

We also include NFs as data

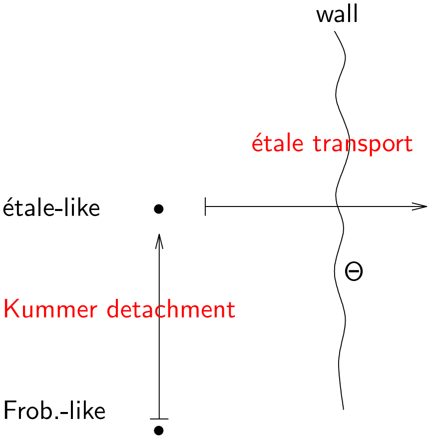
$$(\text{an NF})_j \subset \prod_{v_{\mathbb{Q}}} \log(\mathcal{O}^{\times})$$

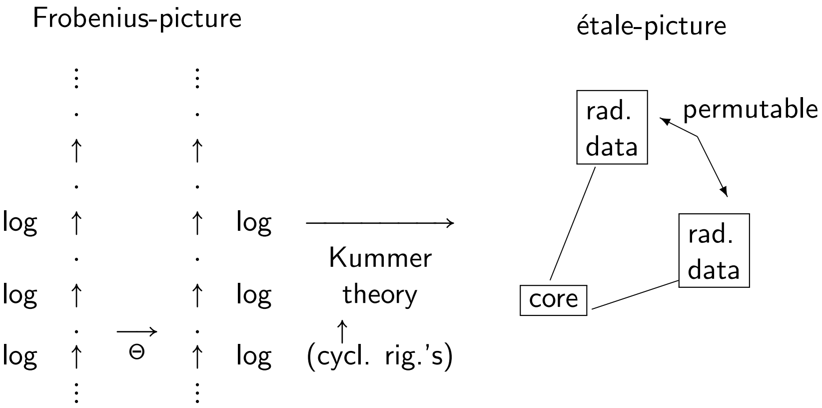
$$\left.\begin{array}{l}\text{theta values}\\ \text{NFs}\end{array}\right\} \longleftarrow \quad \text{story goes } \underline{\text{in a parallel way}} \text{ in some sense}$$

(of course ∃ essential difference
cf. [IUT III, Rem 2.3.2, 2.3.3])

To obtain the final multirad. alg'm:

Frob.-like	$\rightarrow$	• data	assoc. to $\mathcal{F}$-prime-strips
$\downarrow$		$\rightsquigarrow$	Kummer theory
étale-like	$\searrow$		
		• data	assoc. to $\mathcal{D}$-prime-strips
arith.-hol.	$\nearrow$		
$\downarrow$		$\rightsquigarrow$	forget arith. hol. str.
mono-an.	$\rightarrow$	• data	assoc. to $\mathcal{D}^{\vdash}$-prime-strips


wall
étale transport
étale-like
⊖
Kummer detachment
Frob.-like


Frobenius-picture
étale-picture
log
log
log
log
log
log
Θ
Kummer
theory
(cycl. rig.'s)
rad.
data
permutable
rad.
data
core

3 portions of Θ-link

local

- unit $\quad {}^{\dagger}G_{\underline{v}} \xrightarrow{\sim} {}^{\ddagger}G_{\underline{v}}$ ↻ ↻ ← share (⇝ ht + fct) $\quad {}^{\dagger}\mathcal{O}^{\times\mu} \xrightarrow{\sim} {}^{\ddagger}\mathcal{O}^{\times\mu}$
- value gp $\quad {}^{\dagger}\underline{\underline{q}}^{\begin{pmatrix}1^2\\ \vdots \\ j^2\end{pmatrix}\mathbb{N}} \xrightarrow{\sim} {}^{\ddagger}\underline{\underline{q}}^{\mathbb{N}}$ ← drastically changed
- global realified ($\underline{\mathbb{V}} \ni \underline{v}$) $\quad (\mathbb{R}_{\geq 0})_{\underline{v}}(\cdots, j^2, \cdots) \xrightarrow{\sim} (\mathbb{R}_{\geq 0})_{\underline{v}} \log \underline{\underline{q}}$

  ⇝ (ht fct)

Kummer theory

unit portions

${}^{\dagger}G_{\underline{v}} \curvearrowright {}^{\dagger}\mathcal{O}^{\times\mu} := {}^{\dagger}\mathcal{O}^{\times}_{\bar{k}}/\mu$ $\mathbb{Q}_p$-module

$+$ integral str. *i.e.* $\mathrm{Im}(\mathcal{O}^{\times}_{\bar{k}^H}) \subseteq (\mathcal{O}^{\times\mu}_{\bar{k}})^H$ $\forall H \subset G_{\underline{v}}$ open

$\uparrow$ fin. gen. $\mathbb{Z}_p$−mod.

Θ wall ↓ ≀ ← non-ring theoretic ⇝ log-shell

(⇝ computable log-vol.)

${}^{\ddagger}G_{\underline{v}} \curvearrowright {}^{\ddagger}\mathcal{O}^{\times\mu}$

$$({}^{\dagger}G_{\underline{v}} \curvearrowright {}^{\dagger}\mathcal{O}_{\bar{k}}^{\times}) \underset{\text{Kummer}}{\overset{\sim}{\longrightarrow}} ({}^{\dagger}G_{\underline{v}} \curvearrowright \mathcal{O}_{\bar{k}}^{\times}({}^{\dagger}G_{\underline{v}}))$$

↑

unlike the case of $\mathcal{O}_{\bar{k}}^{\triangleright}$,

$\left(\leftarrow \text{now, we cannot use } \mathcal{O}_{\bar{k}}^{\triangleright}. \text{ use only } \mathcal{O}_{\bar{k}}^{\times} \right)$

$\widehat{\mathbb{Z}}^{\times}$- indet. occurs

↑ (↘ container is invariant under this $\widehat{\mathbb{Z}}^{\times}$-indet.)

OK

cycl. rig. $\mu(G_{\underline{v}}) \tilde{\to} \mu(\mathcal{O}_{\bar{k}}^{\times})$

via LCFT ?

does not hold.

We want to protect

$$\begin{cases} \text{value gp portion} \\ \text{global real'd portion} \end{cases}$$

from this $\widehat{\mathbb{Z}}^{\times}$- indet!

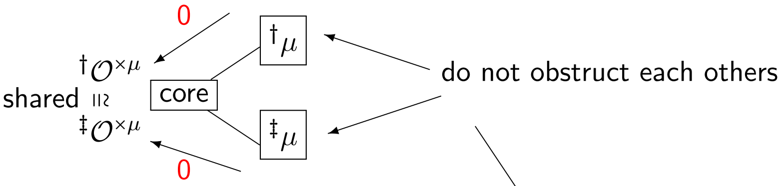
$$\left(\begin{array}{l} \text{sharing } {}^{\dagger}\mathcal{O}^{\times\mu} \overset{\sim}{\to} {}^{\ddagger}\mathcal{O}^{\times\mu} \quad \text{w/ int. str.} \\ \qquad\qquad\qquad \rightsquigarrow \textcolor{red}{\text{(Ind 2)}} \\ \qquad\qquad\qquad \bullet \underset{\ominus}{\longrightarrow} \bullet \quad \text{horizontal indet.} \end{array} \right)$$

value gp portion

$\mathcal{O}^{\triangleright}$

$\downarrow$

mono-theta cycl. rig. $\nwarrow$ only μ is involved (unlike LCFT cycl. rig.)



NF portion

$\widehat{\mathbb{Z}}^{\times} \cap \mathbb{Q}_{>0} = \{1\} \rightsquigarrow$ cycl. rig.

multirad.

(on the function level)

Note also

mono-theta cycl. rig.

is compat. w/ prof. top.

$\rightsquigarrow$ $\mathbb{F}_\ell^{\rtimes\pm}$- sym. (conj. synchro.)

• $\boxplus$

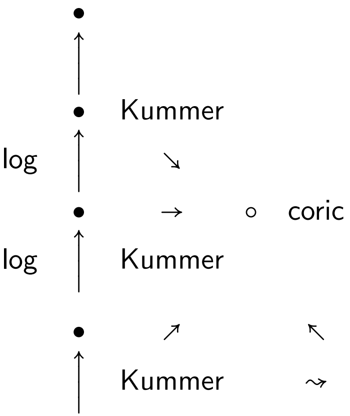
log↑ $\mathbb{F}_\ell^{\rtimes\pm}$- sym. is compat. w/ log-links

• $\boxtimes$

$\rightsquigarrow$ can pull-back coric (diagonal) obj. via log-links

later ↙

$\rightsquigarrow$ LGP monoid (Logarithmic Gaussian Procession)



value gp portion

After

$(\text{Kummer}) \circ (\log)^n \quad (n \geqq 0)$,

take the action of "$\underline{\underline{q}}^{j^2}$" on $\mathcal{I} \otimes \mathbb{Q}$

log-Kummer correspondence

unit portion

not compat.

consider of a common rigid $\downarrow$

upper bound given by log-shell

(Ind 3)

$\bullet$

$\uparrow$ log vertical indet.

$\bullet$

value gp portion

- const. multiple rig.

log $\uparrow$ $\xrightarrow[\text{label } 0]{} \rightsquigarrow$ $\exists$ splitting modulo μ of

↗ hor. core

- $0 \to \mathcal{O}^{\times} \overset{\curvearrowleft}{\to} \mathcal{O}^{\times} \cdot \underline{\underline{q}}^{j^2} \to \mathcal{O}^{\times} \cdot \underline{\underline{q}}^{j^2} / \mathcal{O}^{\times} \to 0$

&

$\log_p(\mu) = 0$

$\rightsquigarrow$ No new action appears

by the iteraions of log.'s

No interference

Note also

$$\mu^{\log}(\log_p(A)) = \mu^{\log}(A)$$

$$\text{if } A \underset{\text{bij}}{\tilde{\to}} \log_p(A)$$

$$\begin{pmatrix} \text{compatibility of log-volumes} \\ \text{w/ log-links} \end{pmatrix}$$

$\rightsquigarrow$ do not need to care about
<u>how many times</u> log.'s are applied.

In the Archimedean case,

we use a system (*cf.* [IUT III, Rem 4.8.2(v)])

$$\{\cdots \twoheadrightarrow \mathcal{O}^{\times}/\mu_N \twoheadrightarrow \mathcal{O}^{\times}/\mu_{N'} \twoheadrightarrow \cdots\}$$

& μ_N is killed in $\mathcal{O}^{\times}/\mu_N$

& constructions (of log-links, $\cdots$)

start from $\mathcal{O}^{\times}/\mu_N$'s, not $\mathcal{O}^{\times}$ (*cf.* [IUT III, Def 1.1(iii)])

& we put "weight N" on $\mathcal{O}^{\times}/\mu_N$

for the log-volumes (*cf.* [IUT III, Rem.1.2.1(i)])

NF portion

as well, consider the actions of $(F^{\times}_{\text{mod}})_j$
after (Kummer) $\circ(\log)^n \quad (n \geqq 0)$

By $F^{\times}_{\text{mod}} \cap \prod_v \mathcal{O}_v = \mu$

$\rightsquigarrow$ No new action appears

in the interation of log.'s

No interference

$\underline{cf}$	multirad. geom. container		contained in a mono-analytic container
$\underline{\text{val gp}}$	theta fct	eval $\rightsquigarrow$ $\left(\begin{array}{c}\text{depends on labels}\\ \text{\& hol. str.}\end{array}\right)$	theta values $\underline{\underline{q}}^{j^2}$
$\underline{\text{NF}}$	$_{(\infty)}\kappa$-coric fcts	eval $\rightsquigarrow$ $\left(\begin{array}{c}\text{indep. of labels}\\ \text{dep. on hol. str.}\end{array}\right)$ Belyĭ cusp'tion	NF $F^{\times}_{\text{mod}}$ (up to $\{\pm 1\}$)

	cycl. rig	log-Kummer
<u>theta</u>	mono-theta cycl. rig.	no interference by const. mult. rig.
<u>NF</u>	$\widehat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$ cycl. rig	no interference by $F^\times_{\mathrm{mod}} \cap \prod_v \mathcal{O}_v = \mu$

vicious cycles

$\bullet^{\dagger}\mu_{Fr}$ $\searrow$ Kummer

$\uparrow$ log $\quad \xrightarrow{\text{Kummer}}$ $\circ\mu_{\acute{e}t}^{\forall}$

$\bullet^{\ddagger}\mu_{Fr}$ $\quad$ <u>theta</u>

$\uparrow$ log $\quad \nearrow$ $\quad$ <u>NF</u>

$\mu_{\acute{e}t}^{\forall}$

$\downarrow$

indet. $=: \mathbb{I}^{\text{ord}}$

$\cap$

$\mathbb{N}_{\geq 1} \times \{\pm 1\}$

$\mathbb{I}^{\text{ord}} = \{1\}$
by zero of order $= 1$
at each cups

$\mathbb{I}^{\text{ord}} \twoheadrightarrow \text{Im} \subset \mathbb{N}_{\geq 1}$

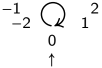
$\|$

$\{1\}$

by $\widehat{\mathbb{Z}}^{\times} \cap \mathbb{Q}_{>0} = \{1\}$

(*cf.* $\underline{\underline{q}}^{j^2}$'s are not inv. under $\{\pm 1\}$) $\searrow$

OK. $\leftarrow$ However the totally of $F_{\text{mod}}^{\times}$ is invariant under $\{\pm 1\}$ $\longleftarrow$ we have $(F_{\text{mod}}^{\times}) \curvearrowleft \{\pm 1\}$-indet.



0 is also permuted

cf. [IUT III, Fig 2.7]

$\mathbb{F}_\ell^{\rtimes\pm}$- sym.

theta

local & transcendental

$q = e^{2\pi iz}$

compat. w/ prof. top.

⟵ theta fct

zero of order = 1 at each cusp

"only one valuation"

⇝ cycl. rig.

(Note theta fcts/ theta values do not have $\mathbb{F}_\ell^{\rtimes\pm}$- sym. But, the cycl. rig. DOES.
↑
use [,])

NF

global & algebraic rat. fcts. Never for alg. rat. fcts

incompat. w/ prof. top. ↙

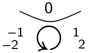
$\widehat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$ sacrifice the compat. w/ prof. top.

"many valuations" ← global

$\mathbb{F}_\ell^{\circledast}$-sym.

0 is isolated

↓



Note also Gal. eval. ← use hol. str.

labels ↙

<u>theta</u> Gal. eval. & Kummer

← compat. w/ labels

<u>NF</u> ← { the output $F^{\times}_{\mathrm{mod}}$ does not depend on labels.

global real'd monoids are

mono-analytic nature (←units are killed)

⇝ do not depend on hol. str. }

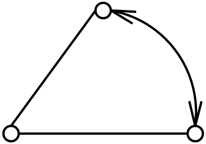
unit	${}^{\dagger}\mathcal{O}^{\times\mu} \cong {}^{\ddagger}\mathcal{O}^{\times\mu}$	(Ind 2) $\rightarrow$	
val. gp	$\{\underline{\underline{q}}^{j^2}\}$	w/ (Ind 3) $\uparrow$	Kummer detachement
	$\curvearrowright \mathcal{I} \otimes \mathbb{Q}$		
NF	$\overset{(-)}{\mathbb{M}}_{\mathrm{mod}} \curvearrowright$		

$\rightsquigarrow$ étale-like objects

étale transport

full poly

$^{\dagger}G_{\underline{v}} \xrightarrow{\sim} {}^{\ddagger}G_{\underline{v}}$ (Ind 1)



permutative indet.

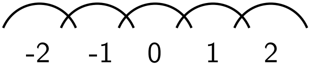
$\rightsquigarrow$ we can transport the data over the Θ-wall

Another thing

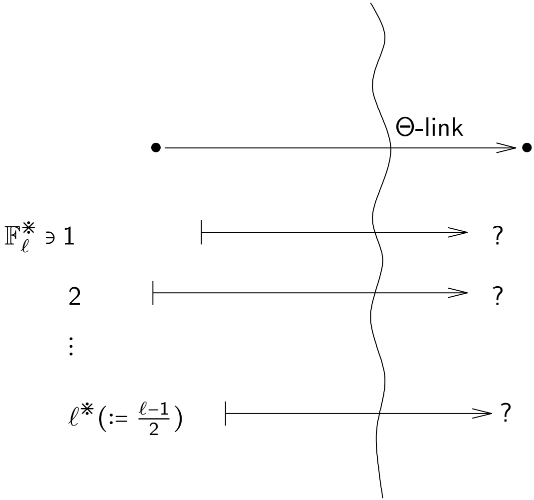
$$\Psi_{\text{gau}} \subset \prod_{t \in \mathbb{F}_\ell^{\circledast}} \quad \text{(const. monoids)}$$

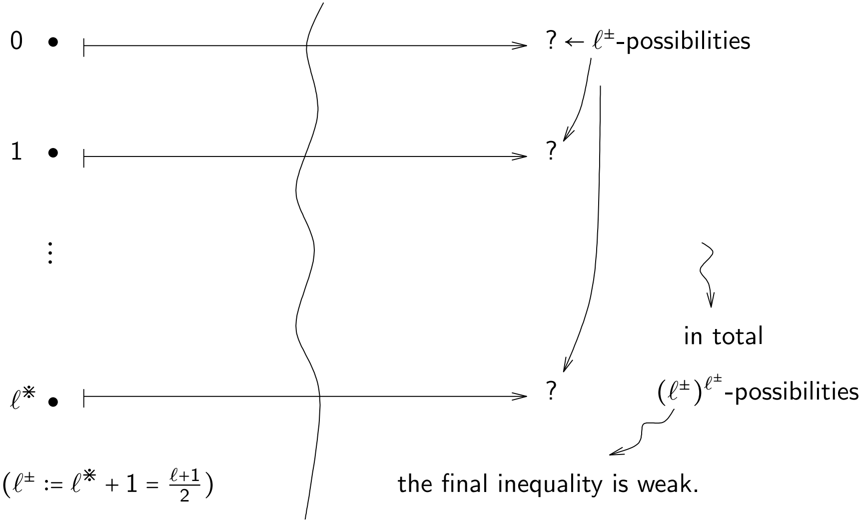
$\uparrow$

labels come from
arith. hol. str.



cannot transport the labels for Θ-link




0
1
ℓ^{*}
? ← $\ell^{\pm}$-possibilities
?
?
in total
$(\ell^{\pm})^{\ell^{\pm}}$-possibilities
$(\ell^{\pm} := \ell^{*} + 1 = \frac{\ell+1}{2})$
the final inequality is weak.

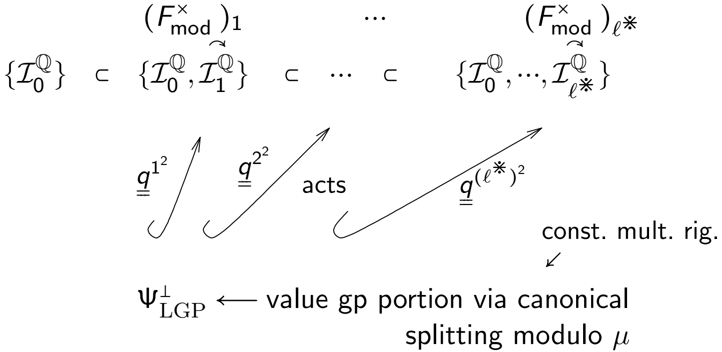
use <u>processions</u>

$$\begin{array}{ccccccccc} \{0\} & \subset & \{0,1\} & \subset & \{0,1,2\} & \subset \cdots \subset & \{0,\cdots,\ell^*\} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \{?\} & \subset & \{?,?\} & \subset & \{?,?,?\} & \subset \cdots \subset & \{?,\cdots,?\} \end{array}$$

$\longrightarrow$ then, in total $(\ell^{\pm})!$-possibilities

$\nearrow$ gives more strict inequality than the former case

A rough picture of the final multirad. rep'n:



By this multirad. rep's & the compatibility w/ Θ-link :

$$\{\underline{\underline{q}}^{j^2}\}_{1\leq j\leq \ell^*} \longleftrightarrow \underline{\underline{q}}$$

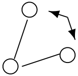
w/ indet.'s

we cannot

distinguish them!!

w/ indet.'s

deg $\ll 0$ $\qquad$ deg ≈ 0

(Ind1) permutative indet.  $^{\dagger}G_{\underline{v}} \cong {}^{\ddagger}G_{\underline{v}}$

in the étale transport

(Ind2) horizontal indet. $\bullet \xrightarrow{\Theta} \bullet$ $^{\dagger}\mathcal{O}^{\times\mu} \cong {}^{\ddagger}\mathcal{O}^{\times\mu}$

in the Kummer detach. w/ int. str.

(Ind3) vertical indet. $\bullet \overset{\log}{\uparrow} \bullet$

$$\begin{array}{ccc} \log(\mathcal{O}^{\times}) & \subset & \\ \uparrow \log & & \frac{1}{2p}\log(\mathcal{O}^{\times}) \\ \mathcal{O}^{\times} & \subset & \end{array}$$

in the Kummer detach.

can be considered as a kind of

"descent data from $\mathbb{Z}$ to $\mathbb{F}_1$"

$$\mathbb{Z} \quad \otimes_{\mathbb{F}_1} \quad \mathbb{Z}$$

$$\searrow \qquad\qquad \nearrow$$

(Ind1) hol. hull

(Ind2)

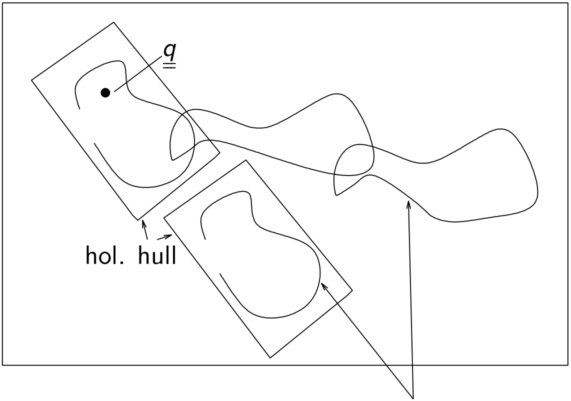
(Ind3)

mono-analytic container

$\parallel$

log-shell

$\mathcal{I}^{\mathbb{Q}}$



possible images of "$\{\underline{\underline{q}}^{j^2}\}_j$"
somewhere, it contains a region
with the same log-volume as $\underline{\underline{q}}$

Recall $\{\underline{\underline{q}}^{j^2}\}_j \longmapsto \underline{\underline{q}}$

$$\longrightarrow \quad 0 \lesssim -(\mathrm{ht}) + \underbrace{(\text{indet})}_{(\boxed{1}+\varepsilon)\left(\begin{array}{c}\text{log-diff}\\ (+\text{log-cond})\end{array}\right)}$$

$$\longrightarrow \quad (\mathrm{ht}) \lesssim (1+\varepsilon)(\text{log-diff} + \text{log-cond})$$

calculation in Hodge-Arakelov

miracle equality $\left(\begin{array}{c}\frac{1}{\ell^2}\sum_j j^2[\ \log q\ ] \approx \frac{\ell^2}{24}[\ \log q\ ]\\ \frac{1}{\ell}\sum j[\ \omega_E\ ] \approx \frac{\ell^2}{24}[\ \log q\ ]\end{array}\right)$

cf. Hodge-Arakelov

IF a global mult. subspace existed

$$\Longrightarrow \quad \underline{\underline{q}}\mathcal{O} \quad \hookrightarrow \quad \underline{\underline{q}}^{j^2}\mathcal{O}$$

$$\uparrow \qquad\qquad \uparrow$$

$$\deg \doteqdot 0 \qquad\qquad \deg \ll 0$$

$$\Longrightarrow \quad -(\text{large}) \geq 0$$

[IUT III, Th 3.11] In summary,

tempered conj. vs prof. conj. (semi-graphs of anbd.) ↘ $\mathbb{F}_\ell^{\rtimes\pm}$-conj. synchro $\left(\begin{matrix}\rightsquigarrow \text{diagonal} \\ \rightsquigarrow \text{hor. core}\end{matrix}\right)$

(i)(objects)		(ii)(log-Kummer)	(iii) (compat. w/ $\Theta^{\times\mu}_{\mathrm{LGP}}$-link)
$\mathbb{F}_\ell^{\rtimes\pm}$-symm. ⊞	$\mathcal{I} \hookleftarrow$ unit	invariant after admitting (Ind3) ↑	invariant after admitting (Ind2) → ↑ $\widehat{\mathbb{Z}}^\times$-indet.
	$\Psi^\perp_{\mathrm{LGP}}$ val gp compat. of log-link w/ $\mathbb{F}_\ell^{\rtimes\pm}$-symm.	no interference by const. mult. rig. ell. cusp'tion←pro-p anab. + hidden endom.	only μ is involved ⇝ multirad. protected from $\widehat{\mathbb{Z}}^\times$-indet. by mono-theta cycl. rig. quadratic str. of Heisenberg gp
$\mathbb{F}_\ell^{\circledast}$-symm. ⊠	$\overset{(-)}{\mathbb{M}}_{\mathrm{mod}}$ NF Belyĭ cusp'tion ↑ pro-p anab. + hidden endom.	no interference by $F_{\mathrm{mod}}^\times \cap \prod_v \mathcal{O}_v = \mu$	protected from $\widehat{\mathbb{Z}}^\times$-indet. by $\widehat{\mathbb{Z}}^\times \cap \mathbb{Q}_{>0} = \{1\}$
others	(compat. of log-volumes w/ log-links)	(arch. theory: Aut-hol. space ell. cusp'tion)	(étale picture: permutable after admitting (Ind1)) ↑ (autom. of processes are included)

Some questions

How about the following variants of Θ -link ?

i) $\{\underline{\underline{q}}^{j^2}\}_j \longmapsto \underline{\underline{q}}^{N} \qquad (N > 1)$

ii) $\{(\underline{\underline{q}}^{j^2})^{N}\}_j \longmapsto \underline{\underline{q}} \qquad (N > 1)$

i) $\{\underline{\underline{q}}^{j^2}\}_j \longmapsto \underline{\underline{q}}^N$

$$\uparrow$$

$$\deg \doteqdot 0 \begin{pmatrix} \ell \approx \mathrm{ht} \\ \& \\ \leftarrow \deg \ll \ell \end{pmatrix}$$

<u>it works</u>

$$\longrightarrow N \cdot 0 \precsim -(\mathrm{ht}) + (\mathrm{indet.})$$

(as for $N \ll \ell$)

(When $N > \ell \Rightarrow$ the inequality is weak)

ii) $\{(\underline{\underline{q}}^{j^2})^N\}_j \longmapsto \underline{\underline{q}}$

it DOES NOT work !

Because

① $\Theta \underset{\text{replace}}{\rightsquigarrow} \Theta^N \Rightarrow$ ~~mono-theta cycl. rig.~~

mono-theta cycl. rig. comes from the quadraticity of [,]

cf. [EtTh, Rem2.19.2]

$\longrightarrow \Theta^N \ (N > 1) \longrightarrow \nexists$ Kummer compat.

② mono-theta

constant multiple rig. as well

↑

ext'n str. of

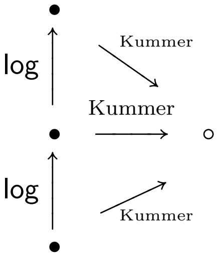
$0 \to \mathcal{O}^{\times}(-) \to \mathrm{Aut}_{\mathcal{C}}((-)) \to \mathrm{Aut}_{\mathcal{D}}((-))^{\mathrm{bs}}) \to 1$

cf. [EtTh, Rem5.12.5]

③ vicious cycles

$$\Theta^N \quad \text{zero of order} = N > 1 \quad \text{at cusps}$$

$$\text{various Frob-like } \mu \overset{\text{Kummer theory}}{\simeq} \text{étale-like } \mu \leftarrow \text{cusp}$$



cf. [IUT III, Rem.2.3.3(vi)]

↻ loop → one loop gives once N-power

IF it WORKED

$$\longrightarrow \quad 0 \leq -N(\mathrm{ht}) + (\mathrm{indet.})$$

$$\longrightarrow \quad (\mathrm{ht}) \leq \frac{1}{N}(1+\varepsilon)(\text{log-diff.} + \text{log-cond.})$$

$\longrightarrow$ contradition to a lower bound

given by analytic number theory

(Masser, Stewart-Tijdeman)