

My Theory. Voevodsky. ^{12/22}. Huzaruro. M. Novl.

$\mathcal{D}(\mathbb{R})$ $DMgm(\mathbb{R})$
 motivic coh. \cong *

$DMgm(\mathbb{R}) = \left(K^b(SmCor(\mathbb{R})) / \mathbb{I} \right)$ pseudo-Abel \mathbb{R}
 + Tate obj. invertible $[\mathbb{Z}(1)^{-1}]$

$\mathcal{D}(\mathbb{R})$ の定義:

$\widetilde{Symb}(\mathbb{R})$ DG-category.

Object: $\bigoplus_{\alpha \in I} (X_\alpha, r_\alpha)$ $r_\alpha \in \mathbb{Z}$ X_α : proj. smooth. \mathbb{R} .
 index set: finite

Morph: $Hom((X, r), (Y, s))$

$\cong \sum^{dim X+s-r} (X \times Y, -\bullet)$ cycle cpx of Bloch.

$X \times \square^n$ $\square^n = (\mathbb{A}^1, \{0, 1\})^n$

$Hom((X, r), (Y, s)) \times Hom((Y, s), (Z, t))$

$\longrightarrow Hom((X, r), (Z, t))$

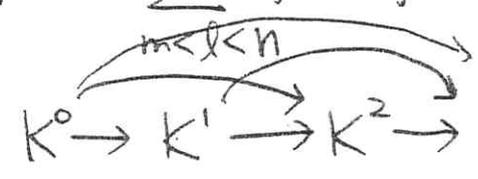
$\sum^a (X \times Y, \bullet) \otimes \sum^b (Y \times Z, \bullet) \longrightarrow \sum^{a+b-d} (X \times Z, \bullet)$

$\Delta Symb(\mathbb{R})$: DG-category

Object: C-diagram in $\widetilde{Symb}(\mathbb{R})$

(K^m, f_{K^m, K^n}) $f_{m,n} \in Hom(K^m, K^n)^{-(n-m)}$
 symbol.

$(-1)^n \partial f^{mn} + \sum_{m < l < n} f^{ln} \circ f^{ml} = 0$



$$\text{Hom}(K, L) = \bigoplus_{m \leq n} \text{Hom}(K^m, L^n)$$

$$D = \sum \omega + \sum (-1)^i \cdot f_{K^i} + \sum f_{L^i} \quad (\rightarrow) = 0.$$

compos def'd \Rightarrow DG-category.

$\text{Ho}(\widetilde{\text{Symb}}(\mathbb{k}))$: object $\mathbb{Z}[1]$, Morph $\text{Hom}(K, L)$
 $\Delta \widetilde{\text{Symb}}(\mathbb{k})$ $\left\{ \begin{array}{l} \mathcal{D}(\mathbb{k}) \end{array} \right.$

$\mathcal{D}_{\text{finite}}(\mathbb{k})$

$H^0(\text{Hom}(K, L))$

\uparrow
triangulated

$$\mathcal{D}(\mathbb{k}) = (\mathcal{D}_{\text{finite}}(\mathbb{k}))^{\text{b}} \quad \text{b: ps-Abel } \mathbb{k}$$

• $\mathcal{D}(\mathbb{k})$ is triangulated category tensor.

• $\mathbb{Z}(1) := (\text{pt}, 1)[-2]$

• $h: (\text{SmProj}/\mathbb{k})^{\text{opp}} \rightarrow \mathcal{D}(\mathbb{k})$

$h(\mathbb{I}) \otimes \mathbb{Z}(k)[2r]$

$X \rightarrow (X, 0)[0]$

$= (Y, k)[0]$

$\text{Hom}_{\mathcal{D}(\mathbb{k})}(h(X), h(Y) \otimes \mathbb{Z}(k)[2r-n])$

$\simeq \text{CH}^r(X \times \mathbb{I}, n)$

• $(\text{q-proj}/\mathbb{k})^{\text{opp}} \xrightarrow{h} \mathcal{D}(\mathbb{k})$ の存在. (char $\mathbb{k} = 0$ とき)

• $H_{\text{Bett}}^* : \mathcal{D}(\mathbb{k}) \rightarrow \underline{\text{Vec}}_{\mathbb{Q}}$

$[00]$; My papers

Invention Math

[95]: Res. Math letters

[04]: Invention. Math

Thm: $(\text{ch}_k=0) \exists$ equiv. of triangulated category tensor.

$$DM_{gm}(k) \xrightarrow{\cong} \mathcal{D}(k)$$

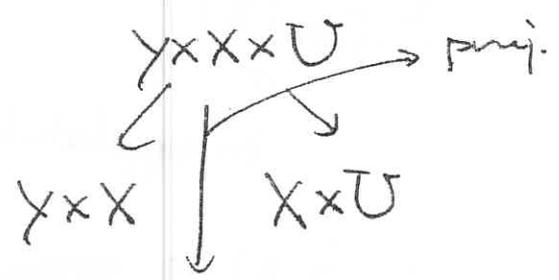
X : smooth proj. / $k \cup$ smooth / k .

$$H((X, r), U)_{\alpha}^{\bullet} = \sum^{\dim X \pm} (X \times U, \dots)$$

I : smooth proj. $\text{Hom}((Y, s), (X, r))_{\alpha}^{\bullet}$ acts from right;

$$\alpha \circ V \in H((I, s), U)^{\bullet}$$

U, U' : smooth quasi-proj. / k



$$\sum C U \times U' \quad \text{Corr}(U, U')$$

finite surj. to U' a component.

acts from left $\cdot Y \times U$.

$$U \circ \alpha \in H((X, r), U)^{\bullet}$$

$$(\alpha \circ V) \circ V' = \alpha \circ (V \circ V')$$

$\text{SmCor}(k)$:

$$u' \circ (u \circ \alpha) = (u' \circ u) \circ \alpha$$

Object: U : sm-q+proj / k

$$u \circ (\alpha \circ v) = (u \circ \alpha) \circ v$$

$$\text{Hom}(U, U')$$

$$= \text{Corr}(U, U')$$

$$f^{i+1} \circ f^i = 0$$

$$C^b(\text{SmCor}(k))$$

$$U^{\bullet} = [\rightarrow U^0 \xrightarrow{f^0} U^1 \xrightarrow{f^1} U^2 \rightarrow \dots]$$

$$K^b(\text{SmCor}(k))$$

$$T [U] \xrightarrow{\cong} [U \times \mathbb{A}^1]$$

$$0 \rightarrow [U \cup V] \rightarrow [U \sqcup V] \rightarrow [U \cap V] \rightarrow 0$$

\parallel
 W : smooth.

$$(K^b(\text{SmCor}(k)) / T)^{\#} \cdot [2(-)^{\#}]$$

$$\mathcal{Z}(H) = h^1(\mathbb{P}^1)$$

Def: [00] $U^\bullet \in K^b(\text{SmCor}(\mathbb{k}))$ A left resolution of U^\bullet
 is an object $L \in \mathcal{D}_{\text{finite}}(\mathbb{k})$

+ $\alpha \in H^0(H(L, U^\bullet))$ satisfying

$$K \in \mathcal{D}_{\text{finite}}(\mathbb{k}) \quad U^\bullet \in K^b \text{SmCor}(\mathbb{k})$$

$$H(K, U^\bullet)^\circ = \bigoplus_{m \leq n} H(K^m, U^m)^\circ \quad D = \partial + \text{of}_K + f_{U^\bullet}$$

(*): $\text{Hom}_{\mathcal{A}(\mathbb{k})}(K, U) \xrightarrow{\alpha \circ (-)} H^0(H(K, U^\bullet))$
 \uparrow
 $H^0(\text{Hom}(K, U))$

Thm: (1) $\forall U^\bullet \in K^b(\text{SmCor}(\mathbb{k}))$ has a left resolution
 (char $\mathbb{k} = 0$) $\exists ! L, L(U^\bullet) \in \mathcal{D}_{\text{finite}}(\mathbb{k}) \cong \mathbb{C}_0$

(2) $\exists !$ functor $L: \mathcal{U} \rightarrow \mathcal{L}(U^\bullet)$ L : triangulated tensor cat of functor
 s.t. (*) $U^\bullet \mapsto L(U^\bullet)$ functorial.

\Rightarrow $L: K^b(\text{SmCor}(\mathbb{k})) \xrightarrow{\mathcal{E}} \mathcal{D}_{\text{finite}}(\mathbb{k})$ \times def. $\exists ! \mathcal{E}$
 $L(A) \xrightarrow{\sim} (A \otimes \mathcal{Z}(H))$ equivalence in $\mathcal{D}(\mathbb{k})$
 $L(T) = 0$
 $(K^b(\text{SmCor}(\mathbb{k})) / \mathcal{T}) \xrightarrow{h} \mathcal{D}(\mathbb{k})$
 $[\mathcal{Z}(H)^\bullet]$

generated by $h(X) \otimes \mathcal{Z}(K) [s]$ as triangulated category

• $\text{hom}(h(X), h(Y)(r)[2r+n])$

$= CH^{duX+r}(X \times Y, n)$

$\therefore DM_{gm}(k) \xrightarrow{\sim} \mathcal{D}(k)$: category equivalence

sk
(pt)

U : sum of q -proj. $L(U)$

$U \cap X \cdot Y = X - U = [\dots \rightarrow (Y_{(1)}, -1) \xrightarrow{\text{inclusion}} (X, 0)]$

$= \bigcup_i Y_i \rightarrow (Y_{(2)}, -2)$

$Y_{(j)} = \bigoplus_i Y_{i, j} \perp \dots \perp Y_{i, 1}$

\perp normal crossing divisor