My Theory. Voevodsky. 12/22. Hanamura. M No.1.

$\mathcal{D}(k)$ $\quad$ $DM_{gm}(k)$

motivic coh. $\cong$ *.

$$DM_{gm}(k) = \left(K^b(SmCor(k)) \big/ T \right)^{\text{pseudo-Abel 化}} + \text{Tate obj. invertible } [\mathbb{Z}(1)^{-1}]$$

$\mathcal{D}(k)$の定義:

$\widetilde{Symb}(k)$ DG-category.

Object: $\bigoplus_{\alpha \in I} (X_\alpha, r_\alpha)$ $\quad$ $r_\alpha \in \mathbb{Z}$ $\quad$ X_α: proj. smooth /k.

I: index set: finite.

Morph: $\mathrm{Hom}((X,r),(Y,s))^\bullet$

$$\stackrel{dfn}{:=} \quad Z^{\dim X + s - r}(X \times Y, -\bullet) \text{ cycle cpx of Bloch.}$$

$X \times \square^n$ $\quad$ $\square^n = (\mathbb{A}^1, \{0,1\})^n$

$$\mathrm{Hom}((X,r),(Y,s))^\bullet \times \mathrm{Hom}((Y,s),(Z,t))^\bullet$$
$$\cdots\longrightarrow \mathrm{Hom}((X,r),(Z,t))^\bullet$$

$$Z^a(X\times Y,\cdot) \otimes Z^b(Y\times Z,\cdot) \cdots\longrightarrow Z^{a+b-d_Y}(X\times Z,\cdot)$$

$\Delta \widetilde{Symb}(k)$: DG-category

Object; C-diagram in $\widetilde{Symb}(k)$

$(K^m, f^{m,n})_{K}$ $\quad$ $\underset{m<n}{f^{m,n}} \in \mathrm{Hom}(K^m, K^n)^{-(n-m-1)}$

K^m: symbol.

$$(-1)^n \partial f^{mn} + \sum_{m<l<n} f^{ln} \circ f^{ml} = 0$$

$$K^0 \to K^1 \to K^2 \to$$

$$\mathrm{Hom}(K,L)^{\cdot} = \bigoplus_{m\le n} \mathrm{Hom}(K^m, L^n)^{\cdot}$$

$$D = \sum \partial + \sum (\cancel{\pm}) \cdot f_K^{\cdots} + \sum f_L^{\cdots} (-) = 0.$$

compos def'd $\Rightarrow$ DG-category.

$Ho(\widetilde{Symb}(k))$; object 同じ. Morph. $\mathrm{Hom}(K,L)$

def // $\Delta\widetilde{Symb}(k)$ $\mathcal{D}(k)$

$\mathcal{D}_{finite}(k)$ $H^0(\mathrm{Hom}(K,L)^{\cdot})$

↑
triangulated

$\mathcal{D}(k) = (\mathcal{D}_{finite}(k))^{q}$ q : ps-Abel 化

- $\mathcal{D}(k)$ is triangulated category tensor.
- $\mathrm{Hom}_{\mathcal{D}(k)}(h(X), h(Y))$
- $\mathbb{Z}(1) \overset{dfn}{:=} (pt, 1)[-2]$
- $h : \left(SmProj/k\right)^{opp} \longrightarrow \mathcal{D}(k)$ $h(Y) \otimes \mathbb{Z}(r)[2r]$

$X \longmapsto (X,0)[0]$ $. = (Y, r)[0]$

$$\mathrm{Hom}_{\mathcal{D}(k)}\left(h(X), h(Y) \otimes \mathbb{Z}(r)[2r-n]\right) \simeq CH^r(X \times Y, n)$$

- $\left(q\text{-}proj/k\right)^{opp} \xrightarrow{h} \mathcal{D}(k)$ の存在. (char $k = 0$ なら)
- $H^*_{Bett} : \mathcal{D}(k) \longrightarrow \underline{Vec}_{\mathbb{Q}}$

[00]; My papers Inventions Math

[95]: Res. Math letters

[04]: Invention. Math

<u>Thm</u>: ($\mathrm{ch}\,k = 0$) $\exists$ equiv. of triangulated category tensor.

$$DM_{gm}(k) \xrightarrow{\cong} \mathcal{D}(k)$$

X : smooth proj. $/k$. U smooth$/k$.

$$H((X,r),U)^{\bullet}_{\mathcal{D}\alpha} = Z^{\dim X - r}(X \times U, -\bullet)$$

Y : smooth proj. $\mathrm{Hom}((Y,s),(X,r))^{\bullet}_{\mathcal{D}}$ V. acts from right;

$$\alpha \circ V \in H((Y,s),U)^{\bullet}$$

$Y \times X \times U \to$ proj. $Y \times X$, $X \times U$, $Y \times U$

U, U' : smooth quasi-proj $/k$

$Z \subset U \times U'$. $\mathrm{Corr}(U,U')$

$Z \to U$, $Z \to U'$ finite surj. to a component.

acts from left. $Y \times U$.

$$U \circ \alpha \in H((X,r),U')^{\bullet}$$

$(\alpha \circ V) \circ V' = \alpha \circ (V \circ V')$

$u' \circ (u \circ \alpha) = (u' \circ u) \circ \alpha$.

$u \circ (\alpha \circ v) = (u \circ \alpha) \circ v$.

$\mathrm{SmCor}(k)$:

<u>Object</u>: U : sm-q-proj$/k$.

$\mathrm{Hom}(U,U') = \mathrm{Corr}(U,U')$ $\quad f^{i+1} \circ f^{i} = 0$

$C^b(\mathrm{SmCor}(k))$ $\quad U^{\bullet} = [\to U^0 \xrightarrow{f^0} U^1 \xrightarrow{f^1} U^2 \to \cdots]$

$K^b(\mathrm{SmCor}(k))$

T $\quad [U] \longmapsto [U \times \mathbb{A}^1]$

$0 \to [U \cup V] \to [U \amalg V] \to [U \cap V] \to 0$

$\|$

W : smooth. $\quad \left(K^b(\mathrm{SmCor}(k))/T\right)^{\natural}[\mathbb{Z}(1)^{-1}]$

$\mathbb{Z}(-1) = h^1(\mathbb{P}^1)$

Def: [OO] $U^\cdot \in K^b(\mathrm{SmCor}(k))$ A left resolution of $U^\cdot$ is an object $L \in \mathcal{D}_{\mathrm{finite}}(k)$ + $\alpha \in H^0(H(L, U^\cdot)^\cdot)$ satisfying

$K \in \mathcal{D}_{\mathrm{finite}}(k)$ $U^\cdot \in K^b\mathrm{SmCor}(k)$

$$H(K, U^\cdot)^\circ = \bigoplus_{m \le n} H(K^m, U^n)^\circ \quad D = \partial + f_K^\cdot + f_{U^\cdot}^\cdot$$

(*): $\mathrm{Hom}_{\mathcal{D}(k)}(K, L) \xrightarrow[\alpha \circ (-)]{\simeq} H^0(H(K, U^\cdot))$

$\in$

$H^0(\mathrm{Hom}(K, L)^\cdot)$

Thm: (char $k=0$) (1): $^\forall U^\cdot \in K^b(\mathrm{SmCor}(k))$ has a left resolution

これを、$L(U^\cdot) \in \mathcal{D}_{\mathrm{finite}}(k)$ とかく。

(2): $\exists!$ functor. $L : U^\cdot \longmapsto L(U^\cdot)$ L: triangulated tensor cat の functor

s.t. (*) $U^\cdot$ について functorial.

これで、$L : K^b(\mathrm{SmCor}(k)) \longrightarrow \mathcal{D}_{\mathrm{finite}}(k)$ が def. できる.

$L(A) \mapsto L(A \otimes \mathbb{Z}(-1))$ equivalence in $\mathcal{D}(k)$

$L(T) = 0$ $\left(K^b(\mathrm{SmCor}(k))/T\right)[\mathbb{Z}(-1)^{-1}] \longrightarrow \mathcal{D}(k)$

T: generated by $h(X) \otimes \mathbb{Z}(1)[s]$ as triangulated category

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- $\hom(h(X), h(Y)(r)[2r-n])$

$$= CH^{\dim X + r}(X \times Y, n)$$

$\therefore$ $DM_{gm}(k) \xrightarrow{\sim} \mathcal{D}(k)$: category equivalence

st. (prf)

U : sm of q-proj. $\quad L(U)$

$$\begin{array}{l} U \\ \cap| \\ X \end{array} \quad Y = X - U = \bigcup_i Y_i \text{ : normal crossing divisor}$$

$$= [\cdots \to (Y_{(2)}, -2) \to (Y_{(1)}, -1) \xrightarrow{\text{inclusion}} (X, 0)]$$

$$Y_{(1)} = \coprod Y_{i_1} \amalg \cdots \amalg Y_{i_p}$$