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F: any field $\qquad p(X) \in F[X]$

$p^{(n)} = p \circ \cdots \circ p$ $\Leftarrow$ n-th iterate
n-times

$x \in F$ a pre periodic point for p if the set

$\{ p^{(n)}(x) \in F \mid n \in \mathbb{N} \}$ is finite

Assume now that $F = \mathbb{F}_2(T)$, $p(X) = TX + gX^2 + \Delta X^4$
$g, \Delta \in \mathbb{F}_2(T)$

Thm(A.P): The set of preperiod points p in F has cardinality at most 4

Crutial property of p = additivity

$p(X+Y) = p(X) + p(Y)$

Let $\mathcal{B}$ be an $\mathbb{F}_q$-alg $\quad \mathrm{End}_{\mathbb{F}_q}(\mathbb{G}_a)/\mathcal{B} = \mathcal{B}\{\tau\}$

$\mathcal{B}\{\tau\} = \{ \sum_{i=0}^{n} a_i \tau^i \mid a_i \in \mathcal{B} \}$ $\quad \tau b = a^q \tau$, $b \in \mathcal{B}$

$x \mapsto x^q$ has τ

If $p = T + \sum a_i \tau^i$, $a_n \in \mathbb{F}_q(T)^*$

then $\varphi : \underset{= A}{\mathbb{F}_q[T]} \longrightarrow \mathbb{F}_q(T)\{\tau\}$ s.t. $\varphi(T) = p$

1) $\partial_0(\varphi(p)) = p \quad \forall p \in A \qquad \partial_0 : \mathcal{B}\{\tau\} \longrightarrow \mathcal{B}$ constant

2) $\deg(\varphi(p)) = \deg(p) \cdot \deg(p) \quad \forall p \in A$

$\Leftarrow$ def. of Drinfeld module of rank $\deg(p)$

$\Leftarrow$ sort of a motivic over $\mathcal{B}$ with coef in $\mathbb{F}_q[T]$

$\varphi : A \to F\{\tau\}$ of rk. d, $m \triangleleft A$

$(F = \mathbb{F}_q(T))$

$\varphi[m] = \{ x \in \bar{F} \mid \varphi(P)(x) = 0 \ \forall P \in m \}$

Prop As an A-mod. $\varphi[m] = (A/m)^{\oplus d}$

Tate module of φ $\quad T(\varphi) = \varprojlim_{m \triangleleft A} \varphi[m] \circlearrowleft Gal(\bar{F}/F)$

Def. : $\lambda : \varphi \to \psi$, if $\lambda \in F\{\tau\}$ s.t.

$\lambda \circ \varphi = \psi \circ \lambda$ (isogeny) $\qquad$ isomorphism $\Leftrightarrow \lambda \in F^{\times}$

Rem Tate's conjecture on isogeny holds

(due to A Tamagawa, Taguchi)

$\varphi(T) = T + gT + \Delta\tau^2$ $\quad$ its F-rational torsion

$\varphi_{tor}(F) = \{ x \in F \mid \exists\, 0 \neq P \in A \ \ \varphi(P)(x) = 0 \}$

$|\varphi_{tor}(F)| \leq 4 \qquad (q = 2)$

j-invariant of φ : $\quad j(\varphi) = \dfrac{g^{q+1}}{\Delta} \in F \qquad \Delta \overset{F}{\underset{\in}{\neq}} 0$

$j(\varphi)$ depends only on the isomorphism class of φ

$\Rightarrow$ "j-line"

$Y_1(P)$ = Drinfeld modular curve parametrizing Drinfeld modules $\varphi : A \to L\{\tau\}$ $\quad (A \underset{\text{injection}}{\hookrightarrow} L)$

with a level str. $\quad i : A/p \to \varphi[p](L)$

$(p \triangleleft A$: prime ideal$)$

Thm $Y_1(p)(F) = \phi$ when $\deg(p) \geq 3$

$\nexists\ \varphi : A \longrightarrow F\{\tau\}$ of rk. two with non-trivial F-rational p-torsion.

counter example $\Longrightarrow$

$$0 \to A/p \to \varphi[p] \to \varphi_0[p] \to 0$$

$\varphi_0(T) = T - \Delta\tau$ (unique Drinfeld module with this property).

Hamahata ($\otimes$-cat. of t-motives)

$$\det(T(\varphi)) = T(\varphi_0)$$

$$\det(\varphi) = \varphi_0$$

K = field of def. of $\varphi_0[p] \subset \bar{F}$

$\varphi_0[p] \underset{Gal(\bar{K}/K)}{\simeq} \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$ $\quad * : Gal(\bar{K}/K) \longrightarrow (A/p)^\times$

Prop A: $*$ above is unramified for $\forall$ place v of K for $\forall v$ which is above ∞, the place of F corresponding to the valuation deg

Prop B: this class group of p-torsion homomorphism is trivial when $q=2$ $\quad \varphi(T) = T + \tau$

(1) sketch of the proof of the second half of Prop A

F_∞ = completion of F w.r.t. ∞

def. An A-submod $\Lambda \subset \bar{F}_\infty$ is called an F-lattice if

(1) fin. gen and free A-mod

(2) Λ is $Gal(\bar{F}_\infty/F_\infty)$-invariant

(3) $|\Lambda \cap \{ z \in \bar{F}_\infty \mid |z| < c \}| < \infty$

<u>Rem</u> $\mathrm{Gal}(\bar{F}_\infty / F_\infty) \circlearrowright \Lambda$ is through a finite quotient

<u>Def.</u> two F-lattice Λ, Λ' are isom if $c \in F^\times$ $\Lambda = c\Lambda'$

<u>Thm</u> $\exists$ equivalence of categories

$$\{F\text{-lattices}\} \xrightarrow{\ c\ } \{\varphi : A \to F_\infty\{\tau\}\}$$

s.t. $T(c(\Lambda)) = \Lambda \otimes_A \hat{A}$ ← profinite completion of A

<u>Lem</u> $G < GL_2(\Lambda)$ finite subgroup $\Rightarrow$

G is conjugate to subgroup of $GL_2(\mathbb{F}_2)$ or

" of $U(A) = \{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \mid a \in A \}$

<u>proof</u>: $GL_2(\Lambda) \curvearrowright$ Bruhat-Tits tree

<u>Fact</u> finite $G \circlearrowright \mathcal{T} \Rightarrow G$ has a fixed vertex

$\Lambda = A^2$, $c(\Lambda)[p]$ has a non-trivial $\mathrm{Gal}(\bar{F}_\infty / F_\infty)$-inv subgrp. $\Rightarrow$ $\Lambda \simeq \langle 1, \tau \rangle_A$ $\quad \tau \in \bar{F}_\infty - F_\infty$

$$\begin{array}{ccc} \Lambda \simeq \langle 1, \tau \rangle_A & & \tau \in \bar{F}_\infty - F_\infty \\ \cup & & \cap \\ A \cdot 1 & & U(A) \backslash \Omega \\ & & \uparrow \\ & & U(A) \backslash \hat{\Omega} \end{array}$$

Drinfeld's upper half plane