$H^{m,n}(X;\mathbb{Z}/p) = H^m_{Zar}(X;\mathbb{Z}/p(n))$

$ch(k)=0 \qquad \zeta_p \in k \qquad X$: smooth

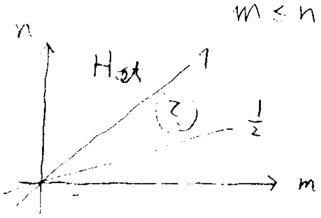
$H^{m,n}(X;\mathbb{Z}/p) = 0$ if $m \geq 2n$

$H^{2n,n}(X,\mathbb{Z}/p) = CH^n(X)/p$

(Bloch-Kato)

$H^{m,n}(X;\mathbb{Z}/p) = H^m_{et}(X;\mathbb{Z}/p)$

$m \leq n$



$Q_i : H^{*,*'}(X;\mathbb{Z}/p) \longrightarrow H^{*+2p^i-1,\, *'+(p^i-1)}(X;\mathbb{Z}/p)$

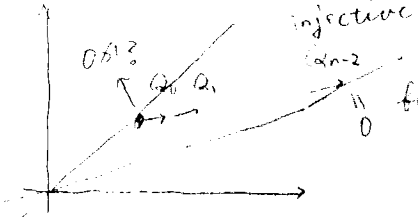
if $p \geq 3 \qquad Q_i = [Q_{i-1}, P^{p^{i-1}}]$

$\{-1\} \in K_1^M(k)/2 = H^1(\mathrm{Spec}\,k, \mathbb{Z}/2)$

$p=2 \qquad Q_i =$ 〃 mod ρ 〃

P^i 傾き $\frac{1}{2}$ ($CH^*(X)/p$)

Q_i 〃 $\frac{1}{2}$ 少し小



by Rost

$x \in H^{*,*}(X(\mathbb{C}),\mathbb{Z}/p)$

$Q_i(x) \neq 0$

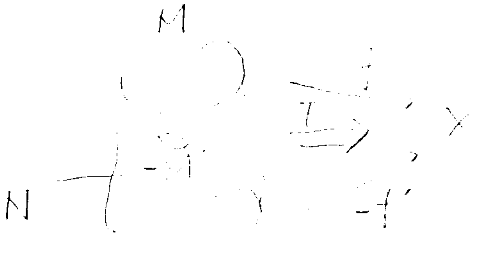
$\Rightarrow x$ は cl

2. Cobordism theory

complex cobordism: $\quad [X, MU] \quad MU = Th_{BU}(EU)$

$$MU^*(X) = [f : M \to X] / (\text{cobordism relation})$$

M: weak complex mfd $\quad$ $T_M \oplus \varepsilon$ complex bundle
(U-mfd) $\quad$ ε trivial



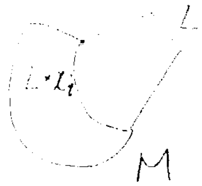
$[M, f] \sim [M', f']$
$\Leftrightarrow \exists$ U-mfd N $\quad F : N \to X$
$\partial N = M \cup (-M')$
$F|_M = f \quad F|_{M'} = -f'$

$MU^*(pt) = \mathbb{Z}[x_1, \cdots] \quad |x_i| = 2i$
(Milnor, Novikov)

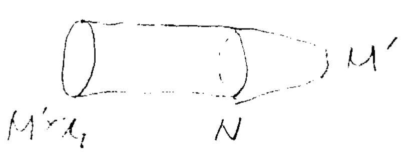
bordism theory of singularity of type x_i

$MU(x_i)^*(X)$

by $\hat{M} = M \cup L \times \text{cone}\, x_i$
$\partial M \simeq L \times x_i$



$M = M' \times x_i$



$M' \times x_i = 0$ in $MU(x_i)^*(X)$

Th (Sullivan)

$$\begin{tikzcd} MU^*(X) \arrow[rr, "\times x_i"] & & MU^*(X) \arrow[dl, "\rho"] \\ & MU(x_i)^*(X) \arrow[ul, "\delta"] & \end{tikzcd}$$

$MU(x_{i_1}, x_{i_2}, \ \ )^*(X)$ も考えられる

$i \neq p^i - 1 \qquad MU(x_i \mid i \neq p^i-1)^*(X)_{(p)} = MU^*(X)/(x_i \mid i \neq p^i-1)$

$\underset{\mathrm{def}}{=} BP^*(X)$ (Brown-Peterson theory)

$(p, x_{p-1}, x_{p^2-1}, \ \ )$ (V_n, l)-mfd

$\quad \ \ \| \qquad \|$
$\quad \ v_1 \quad \ v_2$

$BP^*(pt) = \mathbb{Z}_{(p)}[v_1, v_2, \ \ ]$

norm variety

$MU(p, x_1, x_2, \ \ )^*(X) = BP(p, v_1, \ \ )^*(X)$

$MU(p, x_1, x_2, \cdots)^*(pt) = MU^*/(p, x_1, \ \ ) = \mathbb{Z}/p$

Th (Sullivan)

$MU(p, x_1, \ \ )^*(X) = H^*(X; \mathbb{Z}/p)$

$MU(x_1, x_2, \cdots)^*(X) = H^*(X; \mathbb{Z})$

Cor (Yagita)

$x \in H^*(X; \mathbb{Z}/p) = BP(p, v_1, \cdots)^*(X)$

$\|$

$\hat{M} = M \cup \hat{M}_0 \times \mathrm{cone}(p) \cup \hat{M}_1 \times \mathrm{cone}(v_1) \cup \cdots$

$Q_{v_i}(\hat{M}) = \hat{M}_i \Rightarrow$ cohomology operation $Q_{v_i} = Q_i =$

Morava K-theory

$$BP(p, \ldots, \overset{\text{omit}}{\hat{v}_n}, \ldots)^*(X) \overset{\text{おく}}{=:} k(n)^*(X)$$ corrected Morava K-theory

$$k(n)^*(pt) = \mathbb{Z}/p[v_n]$$

$$[v_n^{-1}]\, k(n)^*(X) = K(n)^*(X)$$

$$\begin{tikzcd} k(n)^*(X) \arrow[rr, "v_n"] & & k(n)^*(X) \arrow[dl, "\rho"] \\ & H^*(X;\mathbb{Z}/p) \arrow[ul, "\delta"] & \end{tikzcd}$$

<u>Cor</u> $k(n)^*(X)$: v_n-tor. $\Rightarrow$

$$H^*(X;\mathbb{Z}/p) \simeq \mathbb{Z}/p\{1, Q_n\} \otimes k(n)^*(X)/p$$

$$\Rightarrow MH(H^*(X;\mathbb{Z}/p); Q_n) = 0$$

$$\operatorname{Ker} Q_n / \operatorname{Im} Q_n$$

$MU^*(X)$ $\qquad MGL^{*,*}(X) = AMU^{*,*}(X)$

$$k(n)^*(X) \Rightarrow Ak(n)^{2*,*}(X)$$

<u>Thm</u> (Voevodsky) $, v_n$

$$Ak(n)^*(\tilde{C}(V_a)) \text{ is } v_j\text{-torsion}$$

$0 \neq a \in K_n^M(k)/2$ $\quad V_a$ norm variety

( ) $\tilde{C}(X) \longrightarrow C(X) \longrightarrow \operatorname{Spec}(k)$

$$C(X) \simeq X \times C(X)$$

$$Ak(n)^{2*,*}(\tilde{C}(X)) \xrightarrow[p^*]{} \underset{\substack{\| \\ 0}}{Ak(n)^{2*,*}(\tilde{C}(X)\times X)} \xrightarrow[p_*]{} Ak(n)^{2*,*}(\tilde{C}(X))$$

$$p_* p^*(x) = \underset{= 0}{v_n \times x}$$