Talks [1]:

(1) "Review of Taylor-Wiles system."
We will give a review of the method of Taylor-Wiles system in [TW], and [D1]. We also explain how the method of Taylor-Wiles system developed until now.

(2) "Galois representations associated to Hilbert modular forms via congruence after Taylor."
We explain the construction of Galois representations associated to Hilbert modular forms in the case of $2 \mid [F : \mathbb{Q}]$ via congruences after Taylor [T1].

(3) "Global-local compatibility after Carayol."
We explain the global-local compatibility of Langlands correspondence for Hilbert modular forms in $\ell \neq p$ after Carayol [Ca1].

(4) "Modularity lifting for potentially Barsotti-Tate deformations after Kisin I."
We explain axiomatically Kisin's technique of $R^{\mathrm{red}} = T$ in [K1]. We study global deformation rings over local ones, and a moduli of finite flat group schemes to get informations about local deformation rings in [K1]. We can use this technique in the non-minimal cases too.

(5) "Base change argument of Skinner-Wiles."
We explain Skinner-Wiles level lowering technique allowing solvable field extensions in Kisin's paper [K1].

(6) "Integral p-adic Hodge theory after Breuil and Kisin."
We prepare the tools of integral p-adic Hodge theory used in [K1]. We can consider them as variants of Berger's theory.

(7) "Modularity lifting for potentially Barsotti-Tate deformations after Kisin II."
The sequel to the previous talk.

(8) "Modularity lifting for crystalline deformations of intermediate weights after Kisin."
We show Kisin's modularity lifting theorem for crystalline deformations of intermediate weights [K3]. We use results of Berger-Li-Zhu [BLZ] and Berger-Breuil [BB1] about mod p reduction of crystalline representations of intermediate weights.

(9) "p-adic local Langlands correspondence and mod p reduction of crystalline representations after Berger, Breuil, and Colmez."
We explain results of Berger-Li-Zhu and Berger-Breuil about mod p reduction of crystalline representations of intermediate weights [BLZ], [BB1]. We use p-adic local Langlands ([C1], [C2], [BB2]) in the latter case.

(10) "Modularity lifting of residually reducible case after Skinner-Wiles."
We explain Skinner-Wiles' modularity lifting theorem for residually reducible representations [SW1].

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(11) "Potential modularity after Taylor."
We explain Taylor's potential modularity [T2], [T3]. This is a variant of Wiles' $(3, 5)$-trick replaced by Hilbert-Blumenthal abelian varieties.

(12) "Taylor-Wiles system for unitary groups after Clozel-Harris-Taylor I."
We explain Clozel-Harris-Taylor's Taylor-Wiles system for unitary groups [CHT], and Taylor's improvement for non-minimal case by using Kisin's arguments [T4].

(13) "Taylor-Wiles system for unitary groups after Clozel-Harris-Taylor II."
The sequel to the previous talk.

(14) "Proof of Sato-Tate conjecture after Taylor et al."
We show Sato-Tate conjecture after Taylor et al. under mild conditions. We use a variant of $(3, 5)$-trick replaced by a family of Calabi-Yau varieties [HSBT].

(15) "First step of the induction of the proof of Serre's conjecture after Tate, Serre, and Schoof."
We show the first step of the proof of Serre's conjecture, that is, $p = 2$ [Ta2], $p = 3$ [Se2], and $p = 5$ [Sc]. We use Odlyzko's discriminant bound, and Fontaine's discriminant bound.

(16) "Proof of Serre's conjecture of level one case after Khare."
We explain Khare-Wintenberger's constuction of compatible systems by using Taylor's potential modularity [T2], [T3] and Böckle's technique of lower bound of the dimension of global deformation rings [Bo]. We show Serre's conjecture of level one case after Khare [Kh1].

(17) "Proof of Serre's conjecture after Khare-Wintenberger."
We prove Serre's conjecture after Khare-Wintenberger [KW2], [KW3].

(18) "Breuil-Mézard conjecture and modularity lifting for potentially semistable deformations after Kisin."
We explain Breuil-Mézard conjecture, and Kisin's approach of modularity lifting theorem for potentially semistable deformations via Breuil-Mézard conjecture [K6].

References

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[Se1]: Serre's conjecture. [Ta1]: Sato-Tate conjecture. [FM]: Fontaine-Mazur conjecture.

[W1] Wiles, A. *Modular elliptic curves and Fermat's last theorem.* Ann. of Math. (2) **141**(3) (1995), 443–551.

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[DDT] Darmon, H., Diamond, F., Taylor, R. *Fermat's last theorem.* Elliptic Curves, Modular Forms, and Fermat's last Theorem (Hong Kong 1993), Internat. Press, Cambridge, MA, 1995, 1–154.
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[S2] Saito, T. *Fermat conjecture II.* Iwanami publisher, 2008.
[D1] Diamond, F. *The Taylor-Wiles construction and multiplicity one.* Invent. Math. **128** (1997) no. 2, 379–391.
[D2] Diamond, F. *On deformation rings and Hecke rings.* Ann. of Math. **144** (1996), 137–166.
[CDT] Conrad, B., Diamond, F., Taylor, R. *Modularity of certain potentially Barsotti-Tate Galois representations.* J. Amer. Math. Soc. **12**(2) (1999), 521–567.
[BCDT] Breuil, C., Conrad, B., Diamond, F., Taylor, R. *On the modularity of elliptic curves over* $\mathbb{Q}$*: wild 3-adic exercises.* J. Amer. Math. Soc. **14**(4) (2001), 843–939.
[F] Fujiwara, K. *Deformation rings and Hecke algebras for totally real fields.* preprint.

[W1]: Fermat's last theorem. [TW]: Taylor-Wiles system. [DDT]: Survey of the proof of Fermat's last theorem. [S1], [S2]: Books about Fermat's last theorem. [D1]: Axiomization and improvement of Taylor-Wiles system. The freeness of Hecke modules became the output from the input. [D2]: Shimura-Taniyama conjecture for elliptic curves, which are semistable at 3 and 5. [CDT]: Shimura-Taniyama conjecture for elliptic curves, whose conductor is not divisible by 27. [BCDT]: Shimura-Taniyama conjecture in full generality. [F]: $R = T$ in totally real case.

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[W2]: Construction of Galois representations associated to Hilbert modular forms in the $2 \mid [F : \mathbb{Q}]$ and nearly ordinary case (including parallel weight 1) by using Hida theory. [T1]: Construction of Galois representations associated to Hilbert modular forms in the $2 \mid [F : \mathbb{Q}]$ case by the congruences. [H]: GL_2 Hida theory for totally real case.

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[S4] Saito, T. *Hilbert modular forms and and p-adic Hodge theory.* preprint.

[Ca1]: Global-local compatibility for $\ell \neq p$ for totally real case. [S3]: Global-local compatibility for $\ell = p$ for $\mathbb{Q}$. [S4]: Global-local compatibility for $\ell = p$ for totally real case.

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[SW3] Skinner, C., Wiles, A. *Base change and a problem of Serre.* Duke Math. J. **107**(1) (2001), 15–25.

[SW1]: Modularity lifting in the residulally reducible case. Taylor-Wiles arguments in the Hida theoretic situations. [SW2]: Modularity lifting for the nearly ordinary deformations in the residually irreducible case by the method of [SW1]. Minor remark: we do not need to assume that $\overline{\rho}|_{\mathrm{Gal}(\overline{F}/F(\zeta_p))}$ is irreducible. [SW3]: Level lowering technique allowing solvable field extensions.

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[PR] Pappas, G., Rapoport, M. *Local models in the ramified case. I. The EL-case.* J. Algebraic Geom. **12** (2003), 107–145.
[G] Gee, T., *A modularity lifting theorem for weight two Hilbert modular forms.* Math. Res. Lett. **13** (2006), no. 5, 805–811.
[I] Imai, N., *On the connected components of moduli spaces of finite flat models.* preprint.
[B1] Breuil, C., *Integral p-adic Hodge theory.* Algebraic Geometry 2000, Azumino, Adv. Studies in Pure Math. **36** (2002), 51–80.
[K2] Kisin, M., *Crystalline representations and F-crystals.* Algebraic geometry and number theory, Progr. Math. **253**, Volume in honor of Drinfeld's 50th birthday, Birkhäuser, Boston (2006), 459–496.
[K3] Kisin, M., *Modularity for some geometric Galois representations.* preprint.
[BLZ] Berger, L., Li, H., Zhu, H. J. *Construction of some families of* 2*-dimensional crystalline representations.* Math. Ann. **329**(2) (2004), 365–377.
[BB1] Berger, L., Breuil, C., *Sur la réduction des représentations cristallines de dimension* 2 *en poid moyens.* preprint.
[K4] Kisin, M., *Potentially semi-stable deformation rings.* preprint.
[K5] Kisin, M., *Modularity of* 2*-adic Barsotti-Tate representations.* preprint.
[K6] Kisin, M., *The Fontaine-Mazur conjecture for* GL_2*.* preprint.
[K7] Kisin, M., *Modularity of potentially Barsotti-Tate Galois representations.* preprint.

[K1]: Furthur improvement of $R = T$ for potentially Barsotti-Tate representations studying global deformation rings over local ones. We study a moduli of finite flat group schemes to get informations of local deformation rings. We can also use this technique in non-minimal case. [PR]: Used in [K1] to get informations of a moduli of finite flat group schemes. [G]: Connectedness of the moduli of finite flat models considered in [K1] in the case where the residue field is not $\mathbb{F}_p$ and the residual representation is trivial. [I]: Connectedness of the moduli of finite flat models considered in [K1] in the case where the residue field is not $\mathbb{F}_p$ and the residual representation is not trivial. [B1]: Used in [K1] to study a moduli of finite flat group schemes in terms of linear algebra. [K2]: Generalization of [B1], which is a variant of Berger's theory too. [K3]: Modularity lifting for crystalline representations of intermediate weights by the method of [K1]. [BLZ]: Explicite construction of a family of Wach modules. The determination of the mod p reduction of crystalline representations of intermediate weights is used in [K3], and [KW1]. [BB1]: By using p-adic local Langlands ([C1], [C2], and [BB2]), we determine the mod p reduction of crystalline representations of intermediate weights, which are not treated in [BLZ]. This is used in [K3]. [K4]: Construction of potentially semistable deformation rings. [K5]: $p = 2$ version of [K1]. Used in [KW2] and [KW3]. [K6]: Proof of many cases of Breuil-Mézard conjecture by using p-adic local Langlands ([C1], [C2], and [BB2]), and deduce a modularity lifting theorem in a high generality from this. [K7]: Survey of [K1], [T2], [T3], and [KW1].

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[T4] Taylor, R. *Automorphy for some ℓ-adic lifts of automorphic mod ℓ Galois representations II.* preprint.
[HSBT] Harris, M., Shepherd-Barron, N., Taylor, R. *A family of Calabi-Yau varieties and potential automorphy.* preprint.

[T2]: Potential modularity in the ordinary case. Variant of $(3,5)$-trick replaced by Hilbert-Blumenthal abelian variety. [T3]: Potential modularity in the crystalline of lower weights case. [BR]: Motive of Hilbert modular forms. Used in [T2] and [T3]. [HT]: local Langlands for GL_n by the "vanishing cycle side" in the sense of Carayol's program. [CHT]: Taylor-Wiles system for unitary groups. Proof of Sato-Tate conjecture assuming a generalization of Ihara's lemma. [T4]: By using Kisin's modified Taylor-Wiles arguments [K1], improvements are made so that we do not need level raising arguments and the generalization of Ihara's lemma. [HSBT]: Proof of Sato-Tate conjecture under mild conditions. Variant of $(3,5)$-trick replaced by a family of Calabi-Yau varieties.

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[Kh1] Khare, C. *Serre's modularity conjecture: the level one case.* Duke Math. J. **134** (2006), 534–567.
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[KW3] Khare, C., Wintenberger, J.-P. *Serre's modularity conjecture (II).* preprint.
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[KW1]: Constuction of compatible system of minimally ramified lifts by using Taylor's potential modularity ([T2] and [T3]) and Böckle's technique. Starting point of [Kh1], [KW2], and [KW3]. [Kh1]: Proof of Serre's conjecture for level one case. Construct more general compatible systems than [KW1]. [KW2]: Proof of Serre's conjecture Part 1. [KW3]: Proof of Serre's conjecture Part 2. [Kh2]: Survey of [Kh1]. [Kh3]: Serre's conjecture implies Artin's conjecture for two dimensional odd representations. [Ca2]: Carayol's lemma used in [KW1], and [KW2]. [Di1]: Existence of compatible system. [Di2]: Another proof of Serre's conjecture of level one case, not using the distribution of Fermat primes. [Bo]: The technique of the lower bound of the dimension of global deformation rings by using local deformation rings used in [KW1], and [Kh1]. [Sa]: Non-vanishing of certain local deformation rings and some calculations of strongly divisible modules are used in [Kh1], [KW2], and [KW3]. [Sc]: Non-existence of certain abelian varieties by using Fontaine's technique and Odlyzko's bound. Used in [KW1] to show Serre's conjecture for $p = 5$. [Ta2]: Proof of Serre's conjecture for $p = 2$. Minkowski's bound is used. [Se2]: Proof of Serre's conjecture for $p = 3$. Odlyzko's bound is used.

[BM] Breuil, C., Mézard, A. *Multiplicités modulaires et représentations de* $\mathrm{GL}_2(\mathbb{Z}_p)$ *et de* $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ *en* $\ell = p$. Duke Math. J. **115**(2) (2002), 205–310.

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[C1] Colmez, P. *Série principale unitaire pour* $\mathrm{GL}_2(\mathbb{Q}_p)$ *et représentations triangulines de dimension* 2. preprint.

[C2] Colmez, P. *Une correspondance de Langlands locale p-adique pour les représentations semi-stable de dimension* 2. preprint.

[BB2] Berger, L., Breuil, C. *Sur quelques représentations potentiellement cristallines de* $\mathrm{GL}_2(\mathbb{Q}_p)$. preprint.

[BM]: Breuil-Mézard conjecture, which says Hilbert-Samuel multiplicity of universal deformation rings is explicitly described by the terms of automorphic side. [B2]: Conjecture about mod p reduction of crystalline representations of intermediate weights, which is partially proved in [BLZ] and [BB1]. This conjecture comes from the insight of "mod p reduction" of p-adic local Langlands. Used in [BB1], and [K6]. [C1]: p-adic local Langlands. Construction of a bijection between trianguline irreducible two dimensional representations of $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ between "unitary principal series" of $\mathrm{GL}_2(\mathbb{Q}_p)$. Used in [BB1], and [K6]. [C2]: p-adic local Langlands. By using (φ, Γ)-modules, we construct a correspondence between two dimensional irreducible semistable representations of $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ between unitary representations of $\mathrm{GL}_2(\mathbb{Q}_p)$. Used in [BB1], and [K6]. [BB2]: p-adic local Langlands. We associate Banach representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ to two dimensional potentially crystalline representations of $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$. Used in [BB1], and [K6].