Update: June 6, 2011

Problems on Low-dimensional Topology, 2011

Edited by T. Ohtsuki[1]

This is a list of open problems on low-dimensional topology with expositions of their history, background, significance, or importance. This list was made by editing manuscripts written by contributors of open problems to the problem session of the conference "Intelligence of Low-dimensional Topology" held at Research Institute for Mathematical Sciences, Kyoto University in May 25–27, 2011.

Contents

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1 The volume conjecture

In [16] R. Kashaev defined a series of invariants $\langle L\rangle_N \in \mathbb{C}$ of a link L for $N = 2, 3, \cdots$ by using the quantum dilogarithm. In [17] he observed, by formal calculations, that

$$2\pi \cdot \lim_{N\to\infty} \frac{\log\langle L\rangle_N}{N} \;=\; \mathrm{vol}(S^3 - L)$$

when L is the figure-eight knot, the 5_2 knot and the 6_1 knot, where "vol" denotes the hyperbolic volume. Further, he conjectured that this formula holds for any hyperbolic link L. In 1999, H. Murakami and J. Murakami [33] proved that $\langle L\rangle_N = J_N(L)$ for any link L, where $J_N(L)$ denotes the N-colored Jones polynomial of L evaluated at $e^{2\pi\sqrt{-1}/N}$; this is the invariant obtained as the quantum invariant of links associated with the N-dimensional irreducible representation of the quantum group $U_q(sl_2)$. The following conjecture makes a bridge between quantum topology and hyperbolic geometry.

Conjecture 1.1 (**The volume conjecture** [17, 33]). *For any knot K,*

$$2\pi \cdot \lim_{N\to\infty} \frac{\log|J_N(K)|}{N} \;=\; \mathrm{vol}(S^3 - K),$$

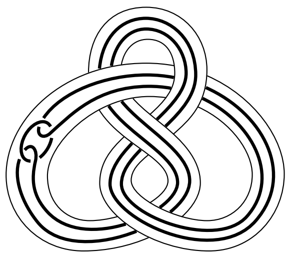
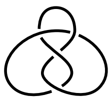
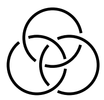
where "vol" *in this formula denotes the simplicial volume*[2] *(normalized by multiplying the hyperbolic volume of the regular ideal tetrahedron).*

As a complexification of the volume conjecture (Conjecture 1.1), it is conjectured in [34] that, for a hyperbolic link L,

$$2\pi\sqrt{-1} \cdot \lim_{N\to\infty} \frac{\log J_N(L)}{N} \;=\; \mathrm{cs}(S^3 - L) + \sqrt{-1}\,\mathrm{vol}(S^3 - L)$$

for an appropriate choice of a branch of the logarithm, where "cs" denotes the Chern-Simons invariant.

The volume conjecture has been rigorously proved for the following knots and links: torus knots, the figure-eight knot, Whitehead doubles of $(2, p)$-torus knots, positive iterated torus knots, Borromean rings, (twisted) Whitehead links, Borromean double of the figure-eight knot, Whitehead chains, and fully augmented links; for details see *e.g.* [32]. In particular, the volume conjecture for Borromean double of the figure-eight knot is proved in [52] by showing that

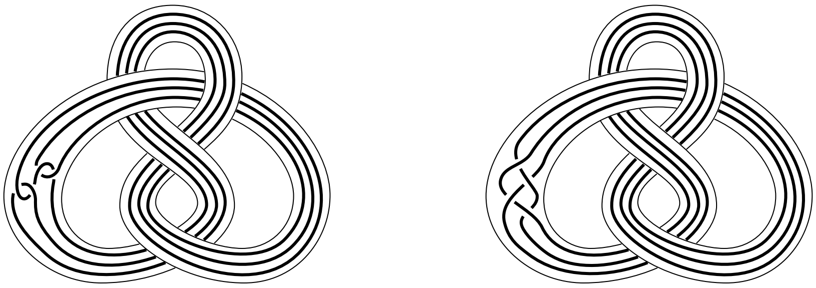
$$\lim_{N\to\infty} \frac{2\pi}{N} \log J_N \Bigg($$

$$\Bigg) \;=\; \mathrm{vol}\Big(S^3 -$$

$$\Big) + \mathrm{vol}\Big(S^3 -$$

$$\Big).$$

Section 1 was written by T. Ohtsuki, following Yoshiyuki Yokota's presentation in the problem session of the conference. He would like to thank Yokota for helpful comments.

[2] The (normalized) simplicial volume of a hyperbolic 3-manifold is equal to its hyperbolic volume. In general, the (normalized) simplicial volume of a 3-manifold is equal to the sum of volumes of hyperbolic pieces of the torus decomposition of the 3-manifold.

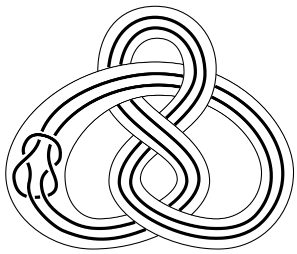
Modifying the above formula, it would be interesting for young researchers to consider the following problems.

Problem 1.2 (Y. Yokota)**.** *Prove the volume conjecture for the following knots.*



Note that the (standard) closure of the 3-braid of the pattern knot of the second figure is the figure-eight knot.

Problem 1.3 (Y. Yokota, A. Yasuhara)**.** *Prove the volume conjecture for the following knot.*



To make such problems relatively easier, it would be better to choose amphicheiral pattern knots, since the Chern-Simons invariant vanishes for the complements of amphicheiral knots. See also Section 2 for cabling the volume conjecture.

2 Twisting and cabling the volume conjecture

(Roland van der Veen)

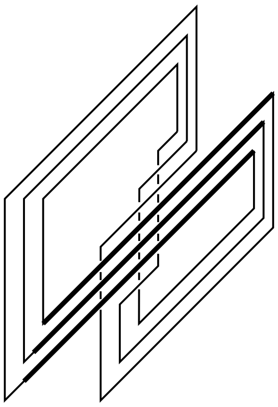
The volume conjecture states that the colored Jones polynomial determines the hyperbolic volume of the knot complement [17, 33]. Assuming hyperbolic volume is a good measure of complexity, it makes sense to approach this conjecture for knots of low volume first. Indeed the conjecture was verified first for torus knots [18] and later for all knots of zero volume [50]. It is also well known that the conjecture is true for smallest positive volume hyperbolic knot: the figure eight knot.

We can now proceed in two directions: First we can investigate other low volume hyperbolic knots. The majority of such knots are twisted torus knots [3], so we will discuss these briefly below. Second, we can consider ways to change a knot without changing its volume. The easiest way to do this is by cabling the knot, *i.e.* replacing the knot by a torus knot embedded into a tubular neighborhood of the knot. As we will see below both directions lead to questions interesting in their own right.

Twisted torus knots.
For positive integers a, b, c, d with $a > c$ we define a twisted torus knot $T(a^b c^d)$ to be the closure of the braid

$$(\sigma_1 \dots \sigma_a)^b (\sigma_1 \dots \sigma_c)^d.$$

In considering the volume conjecture for these knots, an immediate problem is the following. Some twisted torus knots turn out to be are actual torus knots, causing both colored Jones and volume to collapse, see for example the figure below.



The twisted torus knot $T(6^2 4^1)$ equals the torus knot $T(3^5)$. To see this, lift up the three shaded strands.

Question 2.1 (R. van der Veen). *Which twisted torus knots are actual torus knots?*

Some interesting patterns are the following: $T(a^b b^d) = T(b^{a+d})$ iff $b|a$ or $b|d$. Also interpreting the links as Lorentz links and flipping the template we get $T(a^b c^d) = T((b+d)^c b^{a-c})$, see [1]. $T(a^{k-a+2}(a-1)^{a-1}) = T(a^k)$ if $k = -1 \bmod a$. However these identities do not suffice to explain all the patterns observed experimentally.

Cabling the volume conjecture.
By cabling a knot we mean a particular case of the satellite construction in which the pattern is a torus knot. The behavior of the volume conjecture under cabling should be relatively mild because no volume is added. Indeed this was shown to be the case for the figure eight knot where one uses a $(2, p)$ torus knot as pattern [22]. The general case may be approached using the cabling formula [50]. This leads directly to the following question:

Question 2.2 (R. van der Veen). *Let K be a hyperbolic knot and let a_N be a linear form in N. Is it true that*

$$\frac{J_{N+a_N}(K) + J_{N-a_N}(K)}{2\,[N]}\Big|_{q=e^{2\pi\sqrt{-1}/N}}$$

grows exponentally in N and that the growth rate is maximal when $a_N = 0$?

Here $J_N(K)$ is the unnormalized colored Jones polynomial and $[N]$ is the value at the unknot. It follows from the Habiro expansion that the left hand side is always a Laurent polynomial. Note that for $a_N = 0$ Question 2.2 reduces to the ordinary volume conjecture. Apart from its importance in cabling the volume conjecture, the above question provides a new generalization of the volume conjecture. One wonders how the growth rate depends on K and a_N.

3 The complex volume of some hyperbolic knots

(Jinseok Cho)

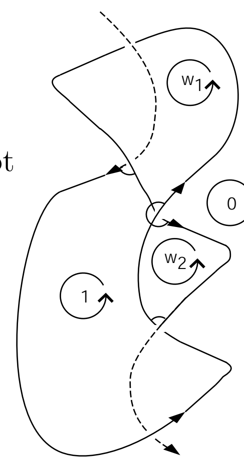
Let K be a hyperbolic knot. We consider parabolic representations $\rho : \pi_1(K) \to \mathrm{PSL}(2,\mathbb{C})$. It is known [53] that each parabolic representation ρ determines the complex volume $\mathrm{vol}(\rho) + \sqrt{-1}\,\mathrm{cs}(\rho)$ modulo $\sqrt{-1}\,\pi^2$, and if ρ is the geometric one, this complex volume equals the one of the hyperbolic knot. Although the complex volume of the hyperbolic knot is relatively well-known, the properties of complex volume of ρ are not yet explored much.

The potential function gives a convenient way to calculate this complex volume. Let $W(K; w_1, \ldots, w_n)$ be the potential function obtained from the optimistic limit of the colored Jones polynomial of the knot K. Then, it is known [47] that a parabolic representation $\rho_{\mathbf{w}}$ is induced by each essential solution $\mathbf{w} = (w_1, \cdots, w_n)$ of the hyperbolicity equations

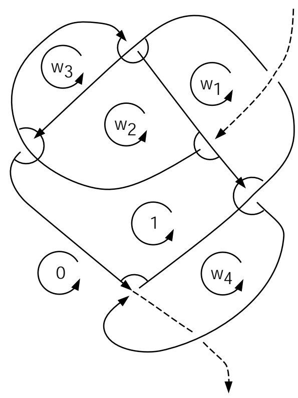
$$\exp\left(w_k \frac{\partial W(K; w_1, \ldots, w_n)}{\partial w_k}\right) = 1 \quad \text{for } k = 1, \ldots, n,$$

and the complex volume of this $\rho_{\mathbf{w}}$ is given by $\sqrt{-1}\,W(K; \mathbf{w})$; see [5].

5_2 knot



7_7 knot



One handy open problem related to the volume of the representation was suggested by Christian Zickert in his talk at Waseda University in summer 2010. He numerically confirmed that the volume of one representation of the 7_7 knot equals the hyperbolic volume of the 5_2 knot, but did not prove it rigorously.

Using potential functions, we can rewrite this problem as follows. From the above figure of the 5_2 knot, its potential function is presented by

$$W(5_2; w_1, w_2) = \mathrm{Li}_2(w_1) - \mathrm{Li}_2(\frac{1}{w_1}) + 2\,\mathrm{Li}_2(w_2) + \log\frac{1}{w_1}\log\frac{1}{w_2} - \frac{\pi^2}{6},$$

and the hyperbolicity equations are given by

$$\frac{w_2}{(1-w_1)(1-\frac{1}{w_1})} = 1, \qquad \frac{w_1}{(1-w_2)^2} = 1.$$

One of the essential solutions of these equations is

$$\mathbf{w} = \big(0.1226... - \sqrt{-1}\cdot 0.7449...,\ 1.6624... - \sqrt{-1}\cdot\ 0.5623...\big),$$

and the complex volume of the 5_2 knot is presented by

$$\begin{aligned}\mathrm{vol}(5_2) + \sqrt{-1}\ \mathrm{cs}(5_2) &\equiv \sqrt{-1}\ W(5_2;\mathbf{w}) \\ &\equiv 2.8281... + \sqrt{-1}\cdot\ 3.0241...\ \ (\mathrm{mod}\ \sqrt{-1}\,\pi^2).\end{aligned}$$

Further, from the above figure of the 7_7 knot, its potential function is presented by

$$\begin{aligned}W(7_7;w_1,\dots,w_4) = \mathrm{Li}_2(w_1) - \mathrm{Li}_2(\frac{1}{w_1}) + \mathrm{Li}_2(\frac{w_2}{w_1}) - \mathrm{Li}_2(\frac{w_1}{w_2}) - \mathrm{Li}_2(\frac{1}{w_2}) - 2\mathrm{Li}_2(\frac{w_3}{w_2}) \\ + \mathrm{Li}_2(w_4) - \mathrm{Li}_2(\frac{1}{w_4}) - \log\frac{w_1}{w_2}\log\frac{w_3}{w_2} - \log\frac{1}{w_2}\log\frac{w_3}{w_2} + \log\frac{1}{w_1}\log\frac{1}{w_4} + \frac{\pi^2}{6},\end{aligned}$$

and the hyperbolicity equations are given by

$$\frac{(1-\frac{w_2}{w_1})(1-\frac{w_1}{w_2})w_2w_4}{(1-w_1)(1-\frac{1}{w_1})w_3} = 1, \quad \frac{w_1w_3^2}{(1-\frac{w_2}{w_1})(1-\frac{w_1}{w_2})(1-\frac{1}{w_2})(1-\frac{w_3}{w_2})^2w_2^4} = 1,$$
$$(1-\frac{w_3}{w_2})^2\frac{w_2^2}{w_1} = 1, \quad \frac{w_1}{(1-w_4)(1-\frac{1}{w_4})} = 1.$$

Two of the essential solutions of these equations are

$$\begin{aligned}\mathbf{w}_1 = \big(&0.7649... - \sqrt{-1}\cdot 0.3611...,\ \ 0.8822... - \sqrt{-1}\cdot 0.2843..., \\ &-0.0153... - \sqrt{-1}\cdot 0.0831...,\ \ 0.4813... - \sqrt{-1}\cdot 0.6379...\big), \\ \mathbf{w}_2 = \big(&3.5598... - \sqrt{-1}\cdot 0.7635...,\ \ 1.3801... - \sqrt{-1}\cdot 5.6891..., \\ &3.2775... - \sqrt{-1}\cdot 5.8903...,\ \ -1.1437... + \sqrt{-1}\cdot 1.2001...\big).\end{aligned}$$

It would be easy to prove that $W(7_7;\mathbf{w}_1) \equiv W(7_7;\mathbf{w}_2)$ modulo π^2 by using some dilogarithm identities; we remark that $\rho_{\mathbf{w}_1}$ and $\rho_{\mathbf{w}_2}$ induce the same complex volume, though $\rho_{\mathbf{w}_1}$ and $\rho_{\mathbf{w}_2}$ are not conjugate. The complex volume of $\rho_{\mathbf{w}_i}$ $(i=1,2)$ is presented by

$$\begin{aligned}\mathrm{vol}(\rho_{\mathbf{w}_i}) + \sqrt{-1}\,\mathrm{cs}(\rho_{\mathbf{w}_i}) &\equiv \sqrt{-1}\,W(7_7;\mathbf{w}_i) \\ &\equiv 2.8281... - \sqrt{-1}\cdot 0.2657...\ \ (\mathrm{mod}\ \sqrt{-1}\,\pi^2).\end{aligned}$$

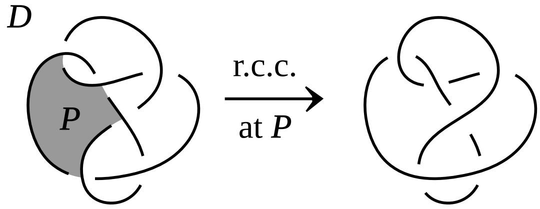
Problem 3.1 (J. Cho)**.** *Prove* $\mathrm{vol}(5_2) = \mathrm{vol}(\rho_{\mathbf{w}_i})$ $(i=1,2)$ *and* $\mathrm{cs}(5_2) \equiv \mathrm{cs}(\rho_{\mathbf{w}_i})$ *modulo* $\pi^2/6$ $(i=1,2)$ *rigorously.*

Remark. We can numerically verify that $\mathrm{cs}(5_2) \equiv \mathrm{cs}(\rho_{\mathbf{w}_i})$ modulo $\pi^2/3$ $(i=1,2)$. The author feels "modulo $\pi^2/6$" is reasonable for general cases.

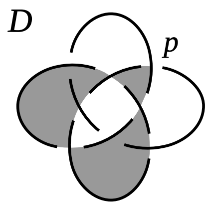
4 The region unknotting number and the crossing number

(Ayaka Shimizu)

Let D be a knot diagram on S^2, and let P be a region of D. A *region crossing change* at P is the crossing changes at all the crossing points on the boundary of P as shown in the following figure.



As shown in [43], we can make a crossing change at any crossing of a knot diagram by a sequence of region crossing changes; for example, we can make the crossing change at p of the following diagram



by a sequence of region crossing changes at the shaded regions, where such shaded regions are obtained as a checkerboard coloring of a "subdiagram" consisting of an arc from p to p. Hence, a region crossing change is an unknotting operation.

The *region unknotting number* $u_R(D)$ of a knot diagram D is the minimal number of region crossing changes on D which are needed to obtain a diagram of the trivial knot from D. The *region unknotting number* $u_R(K)$ of a knot K is the minimal $u_R(D)$ for all minimal crossing diagrams D of K. It is shown in [43] that $u_R(D) \leq c(D)/2 + 1$ for any reduced knot diagram D, and hence $u_R(K) \leq c(K)/2 + 1$ for any knot K, where $c(D)$ and $c(K)$ denote the crossing numbers of D and K. The former inequality implies that the region unknotting number is less than or equal to half the number of regions.

Problem 4.1 (A. Shimizu).
(1) *Is there a reduced knot diagram D whose region unknotting number is $c(D)/2+1$?*
(2) *Is there a knot K whose region unknotting number is $c(K)/2 + 1$?*

As mentioned in [43], if there exists such a diagram D, then $c(D)$ and the number of the black-colored regions of D with a checkerboard coloring are both even.

For example, for a twist knot K, $u_R(K) = 1$ (see [43]) and $c(K) \geq 3$. Further, for the $(2, 4m \pm 1)$-torus knot ($m = 1, 2, \dots$), $u_R(K) = m$ (see [43]) and $c(K) = 4m \pm 1$. Furthermore, for prime knots K with up to 9 crossings, $u_R(K) \leq 2$ (see [43]). These examples do not satisfy the condition of Problem 4.1.

Problem 4.2 (A. Shimizu). *Find a sharp upper bound of* $u_R(K)$.

For a given knot diagram D, we can determine $u_R(D)$ by checking the triviality of finitely many diagrams obtained from D by region crossing changes. In order to determine $u_R(K)$ for a given knot K, a sharp upper bound of $u_R(K)$ would be useful.

5 Killers of knot groups

(Masaaki Suzuki)

Let K be a knot in S^3, and let $G(K)$ be the fundamental group of the complement $S^3 - K$, called the *knot group* of K. Following [45], we call an element of a group a *killer*[3] if the group is normally generated by the element, *i.e.*, the group modulo the element is trivial. For instance, a meridian of a knot group is a killer. Further, the image of a meridian under any automorphism of the knot group is also a killer. Furthermore, there exist many killers of knot groups except meridians. Tsau [49] showed that in the knot group of a satellite knot, a meridian of its companion knot is a killer, if its pattern is of a certain special form. Further, Silver-Whitten-Williams [44] showed that, in the knot group of a two-bridge knot, which is of the form $\langle x, y \mid r\rangle$, $x(yx^{-1})^n$ is a killer, because, putting $y = ax$,

$$\left\langle\, x,\, y \;\middle|\; r,\, x(yx^{-1})^n \right\rangle = \left\langle\, x,\, a \;\middle|\; r|_{y=ax},\, xa^n \right\rangle = \left\langle\, a \;\middle|\; r|_{y=ax,\, x=a^{-n}} \right\rangle = \{e\}.$$

They also showed that there are many killers in the knot groups of torus knots and hyperbolic knots with unknotting number one. They also conjectured that every nontrivial knot group has infinitely many nonequivalent killers.

Let us consider the trefoil knot 3_1. We fix the following presentation of $G(3_1)$:

$$G(3_1) = \langle\, x, y \mid xyx = yxy \,\rangle.$$

Problem 5.1 (M. Suzuki). *Determine which word of* $G(3_1)$ *is a killer under the above presentation.*

The author verified that an element of $G(3_1)$ of word-length ≤ 5 is a killer if and only if its exponent sum is ± 1. Further, he found that $x^2yx^{-3}y$ is not a killer since there is a non-trivial homomorphism $G(3_1) \to SL(2; \mathbb{Z}/5\mathbb{Z})$ whose kernel contains it.

The above problem is the first model of the following problem.

Problem 5.2 (M. Suzuki). *Characterize the words of killers for given knot groups.*

[3] In this manuscript, we use this terminology following the literature, though we think that a less violent word would be appropriate as a mathematical terminology.

6 The unknotting conjecture

An *n-knot* is the image of a locally flat embedding of S^n into S^{n+2} in the differential or topological category. The *trivial n-knot* is the n-knot given by the standard embedding of S^n into S^{n+2}. Two n-knots K and K' are *equivalent* if there exists a diffeomorphism (or homeomorphism, depending on the category) f of S^{n+2} such that $f(K) = K'$. The following conjecture gives a homotopy theoretic characterization of the trivial n-knot; this conjecture is a long-standing classic problem in topology.

Conjecture 6.1 (**unknotting conjecture** ("unknotting theorem", in many cases))**.** *An n-knot K is equivalent to the trivial n-knot if and only if $S^{n+2} - K$ is homotopy equivalent to S^1.*

When $n = 1$, Conjecture 6.1 was proved by Papakyriakopoulos [40, Theorem 28.1] by showing that there exists a disk bounded by a knot whose complement is homotopy equivalent to S^1 by using Dehn's lemma proved by him.

When $n \geq 3$ in the topological category, Conjecture 6.1 was proved by Stallings [46] by showing that an n-knot whose complement is homotopy equivalent to S^1 is trivial when it is restricted to a compact set in $S^{n+2} - \{\text{point}\} \cong \mathbb{R}^{n+2}$, and considering an ascending sequence of such compact sets in $\mathbb{R}^{n+2}$.

When $n \geq 3$ in the differential category, Conjecture 6.1 was proved by Levine [23, 24] by choosing an $(n+1)$-dimensional submanifold $V \subset S^{n+2}$ bounded by an n-knot whose complement is homotopy equivalent to S^1, and eliminating elements of $\pi_k(V)$ for each k by modifying V.

When $n = 2$ in the topological category, Conjecture 6.1 was proved by Freedman, see [8, Theorem 11.7A], by making an s-cobordism between the exteriors of the trivial 2-knot and a 2-knot whose complement is homotopy equivalent to S^1, from which we obtain a homeomorphism between them by the s-cobordism theorem.

The remaining case of Conjecture 6.1 is the case $n = 2$ in the differential category, which is rewritten as follows.

Conjecture 6.2 (see [21, Problem 1.55 (A)])**.** *A smooth 2-knot K is smoothly equivalent to the trivial 2-knot if $\pi_1(S^4 - K) \cong \mathbb{Z}$.*

The condition $\pi_1(S^4 - K) \cong \mathbb{Z}$ implies that $S^4 - K$ is homotopy equivalent to S^1; see the proof of [8, Theorem 11.7A].

By the unknotting theorem [8, Theorem 11.7A] in the topological category, a 2-knot K is topologically unknotted if $\pi_1(S^4 - K) \cong \mathbb{Z}$. However, there might possibly exist a smooth 2-knot which is topologically unknotted, but smoothly knotted. Conjecture 6.2 means the non-existence of such a smooth 2-knot.

Note, see [21, Problems 1.55 (A) and 4.41], that Conjecture 1.2 might not hold for a smooth 2-knot in an exotic $\mathbb{R}^4$.

The first talk of the conference by Takao Matumoto is toward a proof of Conjecture 6.2.

Section 6 was written by T. Ohtsuki. He would like to thank Sadayoshi Kojima and Takao Matumoto for helpful comments.

7 Fundamental problems on surface-knots

(Shin Satoh)

All the following problems in this section have always been very famous.

In surface-knot theory, the ribbon 2-knots and deform-spun knots (including twist-spun knots and roll-spun knots) are popular families of knotted 2-spheres in 4-space. A surface-knot is often described by using a diagram, that is, a generic projection in 3-space equipped with crossing information.

Problem 7.1. *Construct a new family of* 2*-knots or orientable/non-orientable surface-knots and -links in any ways, in particular, by using a diagram, a motion picture, or a chart description of a* 2*-dimensional braid.*

The families of ribbon 2-knots and deform-spun knots are good families in the sense that they can be determined by relatively simple data and they include many non-trivial examples; such families are useful, for example, in order to make experimental checks of some given claims on 2-knots. It is a problem to find such good families of 2-knots or surface-knots and -links.

Whitney-Massey's theorem [28] states that $|e(F)| \leq 4 - 2\chi(F)$ for any non-orientable surface-knot F, where $e(F)$ denotes the normal Euler number of F and $\chi(F)$ denotes the Euler characteristic of F. In [28] this theorem was proved by using a corollary of the Atiyah-Singer index theorem. It is known that $e(F)$ is equal to the sum of the signs for all branch points of a diagram of F.

Problem 7.2. *Give an alternative proof of Whitney-Massey's theorem diagrammatically.*

A ribbon 2-knot is obtained from a trivial 2-link by surgery along several 1-handles on it. Three operations on such ribbon presentation — (1) adding a trivial pair of a 2-sphere and a 1-handle (2) sliding a 1-handle along another 1-handle and (3) passing a 1-handle through another 1-handle — do not change the 2-knot type.

Problem 7.3. *Are the three operations* (1), (2) *and* (3) *enough to deform one ribbon presentation of a ribbon* 2*-knot into another?*

For a non-orientable surface-knot F, it is an open problem whether $\pi_1(S^4 \setminus F) \cong \mathbb{Z}/2\mathbb{Z}$ implies that F is trivial. Let P_0 denote a trivial P^2-knot with $e(P_0) = +2$ or -2. Since $\pi_1(S^4 \setminus F \# P_0)$ is obtained from $\pi_1(S^4 \setminus F)$ by adding the relation $(\text{meridian})^2 = 1$, we have $\pi_1(S^4 \setminus \tau^{2n+1}K \# P_0) \cong \mathbb{Z}/2\mathbb{Z}$ for any odd-twist-spun knot.

Problem 7.4. *Is $\tau^{2n+1}K \# P_0$ trivial? In particular, is the connected sum of the* 3*-twist-spun trefoil and P_0 trivial?*

Problem 7.5. *Is there a P^2-knot which is not the connected sum of a* 2*-knot and P_0?*

We sometimes consider a 2-disk properly embedded in a 4-ball, which is appeared in the definition of a slice knot, for example. The notion of primeness for a 2-knot can be defined in a standard way. However, we have no example of a 2-knot which can be proved to be prime/composite.

Problem 7.6. *Is the trivial* 2*-knot prime?*

Problem 7.7. *Develop a tangle theory for surface-knots.*

In the conference, Akio Kawauchi gave us the following question.

Problem 7.8 (A. Kawauchi)**.** *For any ribbon* 2*-link* L*, is there a* 2*-link* L' *such that* L *is a sublink of* L' *and the link group of* L' *is a free group?*

He pointed out that the problem is true for any spun 2-link L; indeed, for any tangle in a 3-ball there is a set of tunnels such that the complement is a handlebody.

8 Torus-covering T^2-links and satellite T^2-links

(Inasa Nakamura)

A T^2*-link* is a smooth embedding of the disjoint union of tori into the Euclidean 4-space $\mathbb{R}^4$. Let T be the standard torus embedded in $\mathbb{R}^4$ which is the boundary of the standard solid torus embedded in $\mathbb{R}^3 \times \{0\} \subset \mathbb{R}^4$. Let $N(T)$ denote a tubular neighborhood of T in $\mathbb{R}^4$, and let p denote the projection $N(T) \to T$. A *torus-covering* T^2*-link* is a T^2-link F in $N(T) \subset \mathbb{R}^4$ such that $p|_F : F \to T$ is a covering map. We fix a base point of T, and fix a meridian μ and a longitude λ of T which intersects at the base point. A torus-covering T^2-link F is determined from two commutative m-braids $F \cap p^{-1}(\mu)$ and $F \cap p^{-1}(\lambda)$, called *basis braids* [36]. We denote by $\mathcal{S}_m(a, b)$ the torus-covering T^2-link with basis m-braids a and b.

The link group of a classical link or a T^2-link is the fundamental group of the link exterior. The link group of $\mathcal{S}_m(a, b)$ is presented as follows [36]:

$$\left\langle x_1, \ldots, x_m \;\middle|\; x_j = \mathcal{A}^a_*(x_j) = \mathcal{A}^b_*(x_j) \text{ for } j = 1, 2, \ldots, m \right\rangle.$$

Here, $\mathcal{A}^b_*$ denotes Artin's automorphism (see [12]) defined as follows. Let b be an m-braid in a cylinder $D^2 \times [0, 1]$, and let Q_m be the starting point set of b. Let $\{h_u\}_{u \in [0,1]}$ be an isotopy of D^2 rel ∂D^2 such that $\cup_{u \in [0,1]} h_u(Q_m) \times \{u\} = b$. Let $\mathcal{A}^b : (D^2, Q_m) \to (D^2, Q_m)$ be the terminal map h_1, and consider the induced map $\mathcal{A}^b_* : \pi_1(D^2 - Q_m) \to \pi_1(D^2 - Q_m)$, which is uniquely determined from b. We call $\mathcal{A}^b_*$ *Artin's automorphism* associated with b.

Problem 8.1 (I. Nakamura)**.** *Determine whether the link group of a torus-covering* T^2*-link has a non-trivial torsion element.*

Remark.
(1) For classical links, the classical link groups have no non-trivial torsion element ([19], see also [2]). Note that $\mathcal{S}_m(b, e)$ or $\mathcal{S}_m(e, b)$ is the link group of a classical link $\hat{b}$, where $\hat{b}$ denotes the closure of an m-braid b and e is the trivial braid; thus, for these cases it has no torsion element.
(2) For 2-knots, there are 2-knot groups with non-trivial torsion elements; for example, for any positive integer n, there exists a 2-knot group with an element of order n [15]. Here, a *2-knot* is a smooth embedding of S^2 into $\mathbb{R}^4$.

(3) A group π is a ribbon 2-knot group if and only if (i) $\pi/[\pi,\pi]$ is an infinite cyclic group and (ii) π has a Wirtinger presentation of deficiency one, where $[\pi,\pi]$ denotes the commutator subgroup of π [51]. The 1-knot groups are ribbon 2-knot groups. In [20], it is asked whether any ribbon 2-knot group has a non-trivial torsion element. Here, a surface link is called *ribbon* if it is obtained from a trivial 2-link F_0 (*i.e.* the split union of standard 2-spheres) by surgery along a finite number of mutually disjoint 1-handles attaching to F_0.

A T^2-*knot* is a smooth embedding of a torus into $\mathbb{R}^4$. Let T' be a T^2-knot. Consider a tubular neighborhood $N(T')$ of T' in $\mathbb{R}^4$, and the projection $p : N(T') \to T'$, regarding $N(T')$ as the normal bundle of $T' \subset \mathbb{R}^4$. Since this normal bundle is trivial, we fix a trivialization of the bundle. We fix a base point of T', and fix two simple closed curves μ and λ of T which intersects at the base point. We consider a T^2-link F in $N(T') \subset \mathbb{R}^4$ such that $p|_F : F \to T$ is a covering map. Such a T^2-link F is determined from T' and two commutative m-braids $a = F \cap p^{-1}(\mu)$ and $b = F \cap p^{-1}(\lambda)$. We denote this T^2-link by $\mathcal{S}_m(a, b; T')$. This is a kind of a "satellite" T^2-link of the "companion" T^2-knot T'.

Problem 8.2 (I. Nakamura)**.** *Find a presentation of the quandle cocycle invariant of $\mathcal{S}_m(a, b; T')$ in terms of some invariants of a, b and T'.*

Remark. For classical knots, the Alexander polynomial of a satellite knot can be presented in terms of those of its companion knot and its pattern; see, for example, [25, Theorem 6.15]. It might be an easier problem to find a presentation of the Alexander polynomial of $\mathcal{S}_m(a, b; T')$ in terms of some invariants of a, b and T'.

9 Surface-links and symmetric quandles

(Kanako Oshiro)

A *quandle* is a set X with a binary operation "$*$" satisfying that

- $x * x = x$ for any $x \in X$,
- for any $y, z \in X$ there exists a unique $x \in X$ such that $z = x * y$,
- $(x * y) * z = (x * z) * (y * z)$ for any $x, y, z \in X$.

A *symmetric quandle* [13, 14] is a pair (X, ρ) of a quandle X and a good involution ρ, where a map $\rho : X \to X$ is a *good involution* if it is an involution (*i.e.* $\rho \circ \rho = \mathrm{id}_X$) satisfying that $\rho(x * y) = \rho(x) * y$ and $(x * y) * \rho(y) = x$ for any $x, y \in X$. For an abelian group A, a *3-cocycle* of X is a map $\theta : X^3 \to A$ satisfying that

$$\theta(x,z,w) - \theta(x,y,w) + \theta(x,y,z) = \theta(x*y,z,w) - \theta(x*z,y*z,w) + \theta(x*w,y*w,z*w),$$
$$\theta(x,x,y) = \theta(x,y,y) = 0$$

for any $x, y, z, w \in X$. Further, a *symmetric 3-cocycle* of (X, ρ) is a 3-cocycle θ satisfying that

$$-\theta(x,y,z) = \theta\big(\rho(x),y,z\big) = \theta\big(x*y,\rho(y),z\big) = \theta\big(x*z,y*z,\rho(z)\big)$$

for any $x, y, z \in X$. A *surface-link* is a closed surface smoothly embedded in $\mathbb{R}^4$. Two surface-links F and F' are said to be *equivalent* if there exists an ambient isotopy $\{h_t\}_{0 \leq t \leq 1}$ of $\mathbb{R}^4$ such that $h_0 = \mathrm{id}_{\mathbb{R}^4}$ and $h_1(F) = F'$. A *surface-knot* is a surface-link of one component.

Given a symmetric quandle and a symmetric 3-cocycle of it, we can define a coloring and a cocycle invariant for surface-links in a similar way as the usual definition of them for oriented surface-links. An advantage of a symmetric quandle is that its coloring and cocycle invariants are available, not only for oriented surface-links, but also for non-orientable ones. Non-trivial examples and applications are known [4, 14, 38, 39] for colorings and cocycle invariants of non-orientable surface-links of 2 or more components. However, non-trivial examples of cocycle invariants of non-orientable surface-knots are not known so far.

Problem 9.1 (K. Oshiro)**.** *Find a symmetric quandle and a symmetric 3-cocycle of it whose cocycle invariant is non-trivial for non-orientable surface-knots.*

Remark. It is also a problem to construct various concrete examples of non-orientable surface-knots; see Section 7.

In order to approach Problem 9.1 concretely, we introduce the following terminology. For each $x \in X$, we define a map $S_x : X \to X$ by $S_x(y) = y * x$. For a symmetric quandle (X, ρ), we consider an orbit under the actions of S_x's and ρ. Such an orbit is a subquandle of (X, ρ). We call a symmetric quandle *connected* if it has only one orbit. When we consider colorings and cocycle invariants of non-orientable surface-knots, it is sufficient to consider connected symmetric quandles. In order to approach Problem 9.1, we consider the following problem.

Problem 9.2 (K. Oshiro)**.** *Find non-trivial symmetric 3-cocycles of a connected symmetric quandle.*

Remark. We review some simple cases below.
(1) When $\rho = \mathrm{id}_X$, X is a symmetric quandle if $(x * y) * y = x$ for any $x, y \in X$. Further, any symmetric 3-cocycle θ satisfies that $2\,\theta(x, y, z) = 0$ by definition, and hence, it is sufficient to consider symmetric 3-cocycles with $\mathbb{Z}/2\mathbb{Z}$ coefficients.
(2) For trivial quandles, any involution is a good involution [14], and cocycles are calculated in [39]. In particular, it follows that a connected symmetric trivial quandle is the quandle of one element or the quandle of two elements which are exchanged by ρ, and there are only trivial symmetric 3-cocycles for them.
(3) For dihedral quandles, all good involutions are determined in [14]. In particular, it follows that a connected symmetric dihedral quandle is of odd order, and its ρ is id_X. Any symmetric 3-cocycle of such a quandle is a 3-cocycle θ satisfying that $2\,\theta(x, y, z) = 0$. Such a 3-cocycle vanishes, since the $\mathbb{Z}/2\mathbb{Z}$-coefficient third cohomology group of a dihedral quandle of odd order vanishes [30]. Hence, there are only trivial symmetric 3-cocycles in this case.
(4) (T. Nosaka) For the Alexander quandle $\mathbb{F}_{p^m}[T]/(T+1)$ (which is isomorphic to the product of m copies of the dihedral quandle of prime order p), its ρ is id_X, and it has only trivial symmetric 3-cocycles.

(5) For quandles of order ≤ 5, such symmetric quandles and their 3-cocycles are classified in [37]. In particular, it follows that any connected symmetric quandle of order ≤ 5 has only trivial symmetric 3-cocycles.
(6) The quandle QS_6: it is the quandle consisting of the six vertices of an octahedron whose S_x is given by $90°$ rotation fixing x. A $\mathbb{Z}$-valued symmetric 3-cocycle of QS_6 is given in [4]. However, it is conjectured in [4] that the cocycle invariant derived from this 3-cocycle is trivial for non-orientable surface-knots.

Modifying the above problem, it would also be good problems to search non-trivial symmetric 3-cocycles with an (X, ρ)-set, non-trivial symmetric 3-cocycles with a twisted coefficient group A on which X acts, or non-trivial symmetric 4-cocycles (for shadow cocycle invariants).

A surface-link F is a *pseudo-ribbon* if there is a diagram of F without triple points. The *triple point canceling number* of a surface-link F is the smallest number of 1-handles attached to F to obtain a pseudo-ribbon. We denote it by $\tau(F)$. Iwakiri [11] gave a lower bound of triple point canceling numbers for orientable surface-links by using quandle cocycle invariants. And he gave some calculation examples.

Problem 9.3 (K. Oshiro)**.** *Can we give a lower bound of triple point canceling numbers for non-orientable surface-links by using symmetric quandle cocycle invariants?, and give a calculation example of triple point canceling numbers for non-orientable surface-links.*

The *triple point number* of a surface-link F is defined by the smallest number of the triple points among all the diagrams of F, and we denote it by $t(F)$. There are several studies, using quandle cocycle invariants, about triple point numbers of orientable surface-knots. For example, the triple point numbers of the 2 and 3-twist-spun trefoil knot were determined to be four and six, respectively, by using quandle cocycle invariants ([42]).

Problem 9.4 (K. Oshiro)**.** *Give a lower bound of triple point numbers for non-orientable surface-links by using symmetric quandle cocycle invariants.*

Remark. In [38], an evaluation of triple point numbers was given by using symmetric quandles. For 2-component non-orientable surface-links, there are some calculation examples. In particular, the following properties are known:

- For any positive integer n, there exists a 2-component surface-link $F = F_1 \cup F_2$ such that (i) F_1 and F_2 are (trivial) non-orientable surface-knots, (ii) $t(F) = 2n$. ([41])
- For any positive integer n, there exists a 2-component surface-link $F = F_1 \cup F_2$ such that (i) F_1 is a (trivial) orientable surface-knot, (ii) F_2 is a (trivial) non-orientable surface-knot, and (iii) $t(F) = 2n$.

Symmetric quandle cocycle invariants can be also used for orientable surface-links. The strength is not less than that of quandle cocycle invariants.

Problem 9.5 (K. Oshiro). *Can we give an analogous property for triple point numbers of 2-component orientable surface-links (by using symmetric quandle cocycle invariants)?*

We might be able to consider some other applications using symmetric quandles.

Problem 9.6 (K. Oshiro). *Give a new application of symmetric quandle invariants.*

10 Dehn surgery on 3-manifolds

(Kazuhiro Ichihara)

By the celebrated Perelman's works, where he announced an affirmative answer to the famous Geometrization Conjecture, raised by Thurston, we now have a classification theorem for compact 3-manifolds. Beyond the classification, one of the next directions in the study of 3-manifolds is to consider the relationships between 3-manifolds. One of the important operations describing such a relationship would be *Dehn surgery*; an operation to create a new 3-manifold from a given one and a given knot by removing an open tubular neighborhood of the knot, and gluing a solid torus back. This gives an interesting subject to study; because, for instance, it is known that any pair of closed orientable 3-manifolds are related by a finite sequence of Dehn surgeries on knots.

On the other hand, as a consequence of the Geometrization Conjecture, all closed orientable 3-manifolds are classified into; reducible (*i.e.*, containing essential 2-spheres), toroidal (*i.e.*, containing essential tori), Seifert fibered (*i.e.*, foliated by circles), or hyperbolic manifolds (*i.e.*, admitting a complete Riemannian metric with constant sectional curvature -1). Concerning the above four classes of 3-manifolds, many researchers would believe that the hyperbolic 3-manifolds are "ubiquitous" in a sense. This intuition can be justified in terms of Dehn surgery as follows.

Fact. Every closed orientable 3-manifold is related to a hyperbolic one via a Dehn surgery. Equivalently, every closed orientable 3-manifold contains a knot which admits a Dehn surgery yielding a hyperbolic manifold.

This can be obtained by using a result of Myers [35] and the Hyperbolic Dehn Surgery Theorem [48, Theorem 5.8.2] due to Thurston. It is also easily shown that every closed orientable 3-manifold contains a knot which admits a Dehn surgery yielding a reducible manifold and a knot which admits a Dehn surgery yielding a toroidal manifold.

Here we empirically know that the hyperbolic 3-manifold should be contrasted to the Seifert fibered one. Thus it seems interesting to consider:

Problem 10.1 (K. Ichihara). *Is every closed orientable 3-manifold related to a Seifert fibered 3-manifold via a Dehn surgery? Equivalently, in every closed orientable 3-manifold, is there a knot which admits a Dehn surgery yielding a Seifert fibered manifold?*

In my feeling, Seifert fibered ones are much “rarer” than the other classes of 3-manifolds, and so, the above problem should be answered negatively.

Furthermore it can be shown that the above problem is essentially equivalent to the next.

Problem 10.2 (K. Ichihara)**.** *In every closed orientable 3-manifold, is there a hyperbolic knot which admits a Dehn surgery yielding a Seifert fibered manifold?*

Actually, under the assumption that the knot is hyperbolic, the following are also open.

Problem 10.3 (K. Ichihara)**.**
(1) *In every closed orientable 3-manifold, is there a hyperbolic knot which admits a Dehn surgery yielding a reducible manifold?*
(2) *In every closed orientable 3-manifold, is there a hyperbolic knot which admits a Dehn surgery yielding a toroidal manifold?*

The former can be regarded as an extension of the famous unsolved conjecture; the Cabling Conjecture, originally conjectured in [9], and so, could be much difficult. See also [21, Problem 1.79].

On the other hand, the latter seems to be much easier, which should be answered affirmatively (or, can be already known).

We here remark that the following can be shown:

Fact.
(1) Every closed orientable 3-manifold contains a knot which admits a Dehn surgery yielding a non-hyperbolic manifold.
(2) Every closed orientable 3-manifold contains a knot which does not admit a Dehn surgery yielding a non-hyperbolic manifold.

The former can be obtained by showing the existence of hyperbolic knots of genus one based on [35]. The latter is shown by using the method used in [29].

If we consider suitable restrictions on kinds of knots, 3-manifolds, or surgeries, a lot of variations of the above problems can be obtained.

11 Mapping class groups of 3-dimensional handlebodies

(Susumu Hirose)

The oriented 3-dimensional handlebody H_g of genus g is an oriented 3-manifold constructed from a 3-ball by attaching g 1-handles. The boundary of H_g is homeomorphic to the orientable closed surface Σ_g of genus g.

In general, let X be a compact oriented manifold and $\mathrm{Diff}_+(X)$ (resp. $\mathrm{Homeo}_+(X)$) be be the group of orientation preserving diffeomorphisms (resp. homeomorphisms) over X, and $\mathcal{M}(X)$ be the group of isotopy classes of $\mathrm{Diff}_+(X)$. There is a natural surjection π_X from $\mathrm{Diff}_+(X)$ to $\mathcal{M}(X)$. We call a homomorphism s from $\mathcal{M}(X)$ to $\mathrm{Diff}_+(X)$ which satisfies $\pi_X \circ s = id_{\mathcal{M}(X)}$ a *section* for π_X. The problem is to determine whether there exists a section for the natural surjection or not. This problem is a kind of generalization of *the sections problem* introduced in [6, §6.3].

In the case where $X = \Sigma_g$, there is a complete solution for the problem. We remark here that the group of isotopy classes of $\mathrm{Diff}_+(\Sigma_g)$ and that of $\mathrm{Homeo}_+(\Sigma_g)$ are isomorphic, hence there is a natural surjection $\pi_{\Sigma_g} : \mathrm{Homeo}_+(\Sigma_g) \to \mathcal{M}(\Sigma_g)$. When $g = 1$, since $\mathcal{M}(\Sigma_1) = SL(2, \mathbb{Z})$, it is easy to construct a section. Morita [31] showed that the natural surjection from $\mathrm{Diff}^2_+(\Sigma_g)$, where 2 means C^2-class diffeomorphisms, to $\mathcal{M}(\Sigma_g)$ has no section when $g \geq 5$. Markovic [26] showed that $\pi_{\Sigma_g} : \mathrm{Homeo}_+(\Sigma_g) \to \mathcal{M}(\Sigma_g)$ has no section when $g \geq 6$. Franks and Handel [7] showed that $\pi_{\Sigma_g} : \mathrm{Diff}_+(\Sigma_g) \to \mathcal{M}(\Sigma_g)$ has no section when $g \geq 3$. Finally, Markovic and Saric [27] showed that $\pi_{\Sigma_g} : \mathrm{Homeo}_+(\Sigma_g) \to \mathcal{M}(\Sigma_g)$ has no section when $g \geq 2$.

In the case where $X = H_g$, there remain some cases to solve. We remark here that the group of isotopy classes of $\mathrm{Diff}_+(H_g)$ and that of $\mathrm{Homeo}_+(H_g)$ are isomorphic, hence there is a natural surjection from $\mathrm{Homeo}_+(H_g)$ to $\mathcal{M}(H_g)$. In the case where $g = 1$, it is easy to find a section for $\pi_{H_1} : \mathrm{Diff}_+(H_1) \to \mathcal{M}(H_1)$ and, since $\mathrm{Homeo}_+(H_1) \subset \mathrm{Diff}_+(H_1)$, this section is also a section for $\pi_{H_1} : \mathrm{Homeo}_+(H_1) \to \mathcal{M}(H_1)$. In the case where $g \geq 5$, it is shown in [10] that there is no section for $\pi_{H_g} : \mathrm{Diff}_+(H_g) \to \mathcal{M}(H_g)$.

Problem 11.1 (S. Hirose)**.**
(1) *Is there a section for* $\pi_{H_g} : \mathrm{Diff}_+(H_g) \to \mathcal{M}(H_g)$ *in the case where* $g = 2, 3, 4$ *?*
(2) *Is there a section for* $\pi_{H_g} : \mathrm{Homeo}_+(H_g) \to \mathcal{M}(H_g)$ *in the case where* $g \geq 2$ *?*

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