

## 研究集会 Intelligence of Low-dimensional Topology

京都大学数理解析研究所 RIMS 研究集会として、また、大阪市立大学数学研究所から後援をうけて、トポロジープロジェクトの一環として、標記の研究集会を開催いたします。また、この研究集会は科学研究費補助金 基盤研究 B 「グラフィクスとカンドル理論の観点からの 4 次元トポロジーの研究」( 課題番号 26287013、研究代表者 鎌田聖一氏 ( 大阪市立大学 )) と科学研究費補助金 基盤研究 B 「結び目と 3 次元多様体のトポロジー」( 課題番号 24340012、研究代表者 大槻知忠 ( 京都大学 )) の援助を受けています。

日程： 2014 年 5 月 21 日 ( 水 ) ~ 23 日 ( 金 )

場所： 京都大学 数理解析研究所 420 大講演室

アクセス： <http://www.kurims.kyoto-u.ac.jp/ja/access-01.html>

5 月 21 日 ( 水 )

13:20 ~ 14:10 中村 拓司 ( 大阪電気通信大学 工学部 )

The state numbers of a knot

14:30 ~ 15:20 水澤 靖 ( 名古屋工業大学 ) , 門上 晃久 ( 華東師範大学 )

Iwasawa invariants of links

15:40 ~ 16:30 原田新也 ( 東京工業大学 )

Hasse-Weil zeta functions of  $SL_2$ -character varieties of 3-manifolds

5 月 22 日 ( 木 )

10:00 ~ 10:50 松下 尚弘 ( 東京大学大学院数理科学研究科 )

The topologies of box complexes and the chromatic numbers of graphs

11:10 ~ 12:00 Hwa Jeong Lee ( Korea Advanced Institute of Science and Technology )

On the arc index of knots and links

13:20 ~ 14:10 原 隆 ( 東京電機大学 )

Concerning actions of 3-manifold groups: from topological and arithmetic viewpoints

14:30 ~ 15:20 古庄 英和 ( 名古屋大学大学院多元数理科学研究科 )

Galois action on knots

15:40 ~ Problem Session

5月23日(金)

10:00 ~ 10:50 Victoria Lebed (Osaka City University / JSPS)

Knotted 3-valent graphs, branched braids, and multiplication-conjugation relations in a group

11:10 ~ 12:00 Arnaud Mortier (Osaka City University)

Finite-type 1-cocycles of knots

13:20 ~ 14:10 櫻井 みぎ和 (茨城工業高等専門学校)

A polynomial invariant and the forbidden move of virtual knots

14:30 ~ 15:20 清水 達郎 (京都大学数理解析研究所)

An alternative construction of Kontsevich-Kuperberg-Thurston's universal finite type invariant of homology 3-spheres

組織委員：河内明夫、河野俊丈、金信泰造、鎌田聖一、大槻知忠

世話人：大槻知忠(京大数理研)、北山貴裕(東工大)

# Intelligence of Low-dimensional Topology

May 21–23, 2014

Room 420, RIMS, Kyoto University

Access: <http://www.kurims.kyoto-u.ac.jp/en/access-01.html>

## Program

### May 21 (Wed)

13:20–14:10 Takuji Nakamura (Faculty of Engineering, Osaka Electro-Communication University)

The state numbers of a knot

14:30–15:20 Yasushi Mizusawa (Nagoya Institute of Technology), Teruhisa Kadokami (East China Normal University)

Iwasawa invariants of links

15:40–16:30 Shinya Harada (Tokyo Institute of Technology)

Hasse-Weil zeta functions of  $SL_2$ -character varieties of 3-manifolds

### May 22 (Thu)

10:00–10:50 Takahiro Matsushita (Graduate School of Mathematical Science, University of Tokyo)

The topologies of box complexes and the chromatic numbers of graphs

11:10–12:00 Hwa Jeong Lee (Korea Advanced Institute of Science and Technology)

On the arc index of knots and links

13:20–14:10 Takashi Hara (Tokyo Denki University)

Concerning actions of 3-manifold groups: from topological and arithmetic viewpoints

14:30–15:20 Hidekazu Furusho (Graduate School of Mathematics, Nagoya University)

Galois action on knots

15:40– Problem Session

**May 23 (Fri)**

10:00–10:50 Victoria Lebed (Osaka City University / JSPS)

Knotted 3-valent graphs, branched braids, and multiplication-conjugation relations in a group

11:10–12:00 Arnaud Mortier (Osaka City University)

Finite-type 1-cocycles of knots

13:20–14:10 Migiwa Sakurai (Ibaraki National College of Technology)

A polynomial invariant and the forbidden move of virtual knots

14:30–15:20 Tatsuro Shimizu (RIMS, Kyoto University)

An alternative construction of Kontsevich-Kuperberg-Thurston's universal finite type invariant of homology 3-spheres

Scientific Committee: Akio Kawauchi, Toshitake Kohno, Taizo Kanenobu,  
Seiichi Kamada, Tomotada Ohtsuki

Organizers: Tomotada Ohtsuki (RIMS, Kyoto University),  
Takahiro Kitayama (Tokyo Institute of Technology)

# Intelligence of Low-dimensional Topology

May 21–23, 2014

RIMS, Kyoto University

## Abstract

**Hidekazu Furusho (Graduate School of Mathematics, Nagoya University)**  
**Galois action on knots**

I will show that the absolute Galois group of the rational number field acts non-trivially on 'the space of knots' in a non-trivial way.

**Takashi Hara (Tokyo Denki University)**  
**Concerning actions of 3-manifold groups: from topological and arithmetic viewpoints**

Somewhat mysterious as it might seem, there is a close relation between the (topological) notion of essential surfaces and (algebraic) actions of 3-manifold groups on trees. Such a rather classical observation implies that it should be worth introducing sophisticated algebraic and arithmetic methods even to the study of topological objects such as essential surfaces. As a practical example, we first present an extension of Marc Culler and Peter B. Shalen's construction of essential surfaces based upon the theory of Bruhat-Tits' buildings (this is a joint work with Takahiro Kitayama, Tokyo Institute of Technology). If time permits, we shall discuss certain perspectives and problems concerning essential surfaces and actions of 3-manifold groups from an arithmetic viewpoint.

**Shinya Harada (Tokyo Institute of Technology)**  
**Hasse-Weil zeta functions of  $SL_2$ -character varieties of 3-manifolds**

We introduce the notion of local and Hasse-Weil type zeta functions of  $SL_2(C)$ -character varieties of 3-manifolds and A-polynomials. We compute them in some cases and show some relations between the descriptions of the zeta functions and other topological invariants.

**Victoria Lebed (Osaka City University / JSPS)**

**Knotted 3-valent graphs, branched braids, and multiplication-conjugation relations in a group**

Algebraically, a quandle is a set endowed with a binary operation  $\triangleleft$  which satisfies all the relations holding for the conjugation operation  $a \triangleleft b = b^{-1}ab$  in every group. Topologically, quandles can be seen as an algebraization of knots, via a procedure of coloring knot diagrams by quandles: Reidemeister moves correspond precisely to quandle axioms. In this talk we introduce the notion of qualgebra (= quandle + algebra), which is a quandle endowed with an additional binary operation, compatible with  $\triangleleft$  in a certain way. Algebraically, this compatibility is inspired by the relations between multiplication and conjugation in a group. Topologically, qualgebras can be seen as an algebraization of knotted 3-valent graphs, again via a coloring procedure. We also give some results on branched braids (which correspond to knotted 3-valent graphs via the closure operation), construct a qualgebra homology theory, present examples of qualgebras, and calculate the associated invariants for certain graphs.

**Hwa Jeong Lee (Korea Advanced Institute of Science and Technology)**

**On the arc index of knots and links**

Every knot or link  $L$  can be embedded in the union of finitely many half planes in  $R^3$  which have a common boundary line such that each half plane intersects  $L$  in a single arc. Such an embedding is called an arc presentation of  $L$ . The arc index of  $L$  is the minimal number of half planes needed in its arc presentations. In this talk, we present a small survey of known results on the arc index of knots and links. We also introduce some recent results which are the arc index of Kanenobu knots and Cable knots. This is a joint work with Hideo Takioka.

**Takahiro Matsushita (Graduate School of Mathematical Science, University of Tokyo)**

**The topologies of box complexes and the chromatic numbers of graphs**

The application of topology to the graph coloring problem started from the Lovász's striking proof of the Kneser's conjecture in 1978. He defined the neighborhood complex of a graph and proved that its homotopy invariant gives a lower bound for the chromatic number of graphs, and determined the chromatic numbers of the Kneser's graphs. The box complex of a graph was defined after that, which is a  $Z_2$ -complex whose  $Z_2$ -homotopy invariant is known to be closely related to the chromatic numbers of graphs.

In this talk, I start from the basic definitions about the graph coloring problem, and mention about the results obtained recently. I construct graphs whose box complexes are isomorphic as cell complexes, but whose chromatic numbers are different. To construct such examples, we observe that the cell structure of the box complex  $B(G)$  of a graph  $G$  is closely related with the double cover  $K_2 \times G$  of  $G$ . I also construct graphs whose box complexes are  $Z_2$ -homotopy equivalent but their chromatic numbers are different. To prove this, we show that some deformations of graphs which may change the chromatic numbers do not change the  $Z_2$ -homotopy types of box complexes.

**Yasushi Mizusawa (Nagoya Institute of Technology), Teruhisa Kadokami (East China Normal University)**

**Iwasawa invariants of links**

“Iwasawa invariants” describe the  $p$ -adic growth of the orders of first homology groups in a certain tower of cyclic branched coverings of a link. Based on analogies between Alexander-Fox theory and Iwasawa theory, Iwasawa invariants in knot theory were introduced by Masanori Morishita as analogues of original invariants in number theory. We start with a survey of these basic analogies and related facts, and then we focus on the computation of Iwasawa invariants. In particular, we consider the existence of a link (and a tower) with prescribed Iwasawa invariants. If time permits, we propose and consider a problem motivated by Greenberg’s conjectures in Iwasawa theory.

**Arnaud Mortier (Osaka City University)**

**Finite-type 1-cocycles of knots**

Vassiliev invariants of knots are the 0-codimensional part of a theory of “finite-type” cohomology. Since this theory was introduced by V.A.Vassiliev in 1990, only one example has been found in higher codimension, known as the Teiblum-Turchin 1-cocycle, and still today no formula is known to actually evaluate this cocycle over the integers (though V.A.Vassiliev gave a formula to evaluate it with mod 2 coefficients). In this talk, I will describe a simple method to find 1-cocycle formulas over the integers, whose first example coincides with Teiblum-Turchin’s cocycle. Conjecturally, all formulas found this way give finite-type cocycles.

**Takuji Nakamura (Faculty of Engineering, Osaka Electro-Communication University)**

**The state numbers of a knot**

A state of a knot diagram  $D$  introduced in the Kauffman’s model of the Jones polynomial is a collection of circles obtained from  $D$  by splicing all crossings in  $D$ . For a knot diagram  $D$ , we define a sequence of integers  $s_n(D)$  ( $n = 1, 2, \dots$ ) such that  $s_n(D)$  is the number of states of  $D$  consisting of  $n$  circles. The  $n$ -state number of a classical or virtual knot  $K$  is the minimal number of  $s_n(D)$  for all possible diagrams  $D$  presenting  $K$ , and denoted by  $s_n(K)$ . It is known that the inequality  $s_1(K) \geq \det(K)$  is satisfied for any classical knot  $K$  and the equality holds if  $K$  is alternating. In this talk, we study the  $n$ -state number of a classical or virtual knot, and provide some lower and upper bounds of  $s_1(K)$ ,  $s_2(K)$ , and  $s_3(K)$  by using the Jones polynomial, the Miyazawa polynomial, the crossing number, and so on.

**Migiwa Sakurai (Ibaraki National College of Technology)**

**A polynomial invariant and the forbidden move of virtual knots**

Satoh and Taniguchi introduced a numerical invariant called an  $n$ -writhe and we can define a polynomial invariant by using the  $n$ -writhes. In this talk, we provide the difference of the values obtained from the polynomial invariants between two virtual knots which can be transformed into each other by a single forbidden move. As a result, we make it possible for many virtual knots to determine the unknotting numbers by forbidden moves.

**Tatsuro Shimizu (RIMS, Kyoto University)**

**An alternative construction of Kontsevich-Kuperberg-Thurston's universal finite type invariant of homology 3-spheres**

G. Kuperberg and D. Thurston gave a topological invariant  $Z^{\text{KKT}}$  of rational homology 3-spheres based on the idea of Kontsevich's Chern-Simons perturbation theory. As with the LMO invariant,  $Z^{\text{KKT}}$  is a universal finite type invariant of homology 3-spheres. In this talk, we give an alternative construction of  $Z^{\text{KKT}}$  via vector fields. This construction also gives an invariant of vector fields on a rational homology 3-sphere. As an application of this construction, we show that  $Z^{\text{KKT}}$  is equivalent to the "Morse homotopy invariant" obtained by T. Watanabe and K. Fukaya.