# 研究集会 Intelligence of Low-dimensional Topology

京都大学数理解析研究所 RIMS 共同研究(公開型)として、また、トポロジープロジェ クトの一環として、標記の研究集会を開催いたします。また、この研究集会は科学研究費 補助金 基盤研究 B「グラフィクスとカンドル理論の観点からの4次元トポロジーの研究」 (課題番号 19H01788、研究代表者 鎌田聖一氏(大阪大学))と科学研究費補助金 基盤研 究 A「結び目と3次元多様体の量子トポロジー」(課題番号 16H02145、研究代表者 大槻 知忠(京都大学))と科学研究費補助金 挑戦的萌芽研究「ゲージ理論に関連する結び目 と3次元多様体の不変量と量子トポロジー」(課題番号 16K13754、研究代表者 大槻知忠 (京都大学))の援助をうけています。

日程: 2020年 5月13日 (水)~ 5月15日 (金) ホームページ: http://www.kurims.kyoto-u.ac.jp/~ildt/

この研究集会は、オンライン会議システム Zoom を用いて、オンラインで開催します。参加される方は、4月30日までに、参加登録をお願いします。参加登録の際に「誓約書」(配信映像を録画しない、などの内容)の提出をお願いします。オンライン会議の視聴のために「ネット環境(wifiなど)」と「パソコン」が必要です。Zoomの映像配信の通信量は、1時間あたり200MB~300MB程度の見込みです。(スマホでも、参加自体は可能ですが、講演スライドの文字を読むのは困難とおもわれ、また、通信量もかかるとおもわれることに、ご注意ください。)参加登録方法と参加方法について、詳細は、研究集会ホームページ(上記 URL)をご覧ください。

#### 5月13日 (水)

13:15~13:45 金信 泰造 (大阪市立大学大学院理学研究科) Classification of small ribbon 2-knots

14:00 ~ 14:30 中兼 啓太 (東京工業大学 理学院数学系 / 日本学術振興会特別研究員 DC2) Homfly and full twists

14:45~15:15 カールマン タマシュ (東京工業大学) Clock theorems for triangulated surfaces

# 5月14日(木)

10:30~11:00清水 達郎 (大阪市立大学数学研究所)Chern-Simons perturbation theory and Reidemeister-Turaev torsion

11:15~11:45 湯淺 亘 (京都大学数理解析研究所 / 日本学術振興会特別研究員 PD) Twist formulas for one-row colored  $A_2$  webs and  $\mathfrak{sl}_3$  tails of (2, 2m)-torus links 13:15~13:45 作間 誠 (大阪市立大学数学研究所 / 広島大学) "Monodromy groups" of Heegaard surfaces of 3-manifolds

14:00~14:30茂手木 公彦 (日本大学文理学部)The Strong Slope Conjecture for Whitehead doubles

# 5月15日(金)

10:30~11:00 原子 秀一 (東京大学大学院数理科学研究科) The symplectic derivation Lie algebra of the free commutative algebra

11:15 ~ 11:45 阿蘇 愛理 (東京都立大学 理学研究科) A note on the asymptotic behavior of the twisted Alexander polynomials of  $5_2$  knot

 $13:15 \sim 13:45$  Anderson Vera (Kyoto University / JSPS Research Fellow) Johnson-type homomorphisms, a conjecture by Levine, and the LMO invariant

14:00 ~ 14:30 村上 順 (早稲田大学) On quantum representation of knots via braided Hopf algebra

> 組織委員: 河内明夫、河野俊丈、金信泰造、鎌田聖一、大槻知忠 世話人:大槻知忠 (京大 数理研)、秋吉宏尚 (大阪市立大 理学研究科) 協力スタッフ:石川勝巳、石橋典、軽尾浩晃、清水達郎、辻俊輔、湯淺亘

# Intelligence of Low-dimensional Topology

May 13–15, 2020

This is an online conference whose live streaming is distributed from RIMS, Kyoto University.

# Program

#### May 13 (Wed)

13:15–13:45 Taizo Kanenobu (Department of Mathematics, Osaka City University) Classification of small ribbon 2-knots

14:00–14:30 Keita Nakagane (Department of Mathematics, Tokyo Institute of Technology / JSPS Research Fellow DC2) Homfly and full twists

14:45–15:15 Tamas Kalman (Tokyo Institute of Technology) Clock theorems for triangulated surfaces

#### May 14 (Thu)

10:30–11:00 Tatsuro Shimizu (Osaka City University Advanced Mathematical Institute) Chern-Simons perturbation theory and Reidemeister-Turaev torsion

11:15–11:45 Wataru Yuasa (RIMS, Kyoto University / JSPS Research Fellow PD) Twist formulas for one-row colored  $A_2$  webs and  $\mathfrak{sl}_3$  tails of (2, 2m)-torus links

13:15–13:45 Makoto Sakuma (Osaka City University Advanced Mathematical Institute / Hiroshima University) "Monodromy groups" of Heegaard surfaces of 3-manifolds

14:00–14:30 Kimihiko Motegi (Nihon University, College of Humanities and Sciences) The Strong Slope Conjecture for Whitehead doubles May 15 (Fri)

10:30–11:00 Shuichi Harako (Graduate School of Mathematical Sciences, the University of Tokyo)

The symplectic derivation Lie algebra of the free commutative algebra

11:15–11:45 Airi Aso (Graduate School of Science, Tokyo Metropolitan University) A note on the asymptotic behavior of the twisted Alexander polynomials of  $5_2$  knot

13:15–13:45 Anderson Vera (Kyoto University / JSPS Research Fellow) Johnson-type homomorphisms, a conjecture by Levine, and the LMO invariant

14:00–14:30 Jun Murakami (Waseda University) On quantum representation of knots via braided Hopf algebra

Scientific Committee: Akio Kawauchi, Toshitake Kohno, Taizo Kanenobu, Seiichi Kamada, Tomotada Ohtsuki

Organizers: Tomotada Ohtsuki (RIMS, Kyoto University), Hirotaka Akiyoshi (Graduate School of Science, Osaka City University)

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# Intelligence of Low-dimensional Topology

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# Abstract

## Airi Aso (Graduate School of Science, Tokyo Metropolitan University) A note on the asymptotic behavior of the twisted Alexander polynomials of $5_2$ knot

R. M. Kashaev conjectured that the asymptotics of the Kashaev invariant of hyperbolic links gives the hyperbolic volume of the link compliment. H. Murakami and J. Murakami extended Kashaev's conjecture (volume conjecture) and H. Murakami, J. Murakami, M. Okamoto, T. Takata, and Y. Yokota proposed the complexification of the volume conjecture. On the other hands, H. Goda gave a fomula of hyperbolic volume with twisted Alexander polynomials. In this talk, we try to give a formula of the complexification of the formula of twisted Alexander polynomials and hyperbolic volume of  $5_2$  knot. To this end, we observe the asymptotic behavior of the twisted Alexander polynomials of  $5_2$  knot.

# Shuichi Harako (Graduate School of Mathematical Sciences, the University of Tokyo)

#### The symplectic derivation Lie algebra of the free commutative algebra

A generalization of Vassiliev invariants using perturbative Chern-Simons theory was proposed by Kontsevich and Bar-Natan. This enables us to consider Vassiliev invariants of knots and links in arbitrary 3-manifold. Moreover, this invariant is described by the homology of the graph complex which Kontsevich referred as "commutative case". This graph complex is identified with a certain Lie algebra  $\mathfrak{c}_g$  with the canonical symplectic action. We concentrate on this Lie algebra.

In order to understand the homology of  $\mathfrak{c}_g$ , we consider a grading called *weight* on the Chevalley-Eilenberg chain space of  $\mathfrak{c}_g$  compatible with its differential and the symplectic action. The positive weight part of  $\mathfrak{c}_g$  is denoted by  $\mathfrak{c}_g^+$ . It is known that the *n*-th homology of  $\mathfrak{c}_g$  is isomorphic to the tensor product of the *k*-th homology of  $\mathfrak{sp}(2g;\mathbb{Q})$  and the symplectic invariant part of the (n-k)-th homology of  $\mathfrak{c}_g^+$  if g is sufficiently large. The symplectic invariant homology of  $\mathfrak{c}_g^+$  is also known as commutative graph homology.

So far, there are some computational results about commutative graph homology. For example, the dimensions of the homology group and the chain space were partially computed by Bar-Natan and McKay. Conant, Gerlits, and Vogtmann also computed this homology group up to weight 12. The generating function of the Euler characteristic was determined by Willwacher and Živković. However, the whole homology group of  $\mathfrak{c}_g^+$  itself is little known. We show that the weight w part of  $H_2(\mathfrak{c}_g^+)$  is zero for  $g, w \ge 4$ , so that  $H_2(\mathfrak{c}_g^+)$  is completely determined in this case.

## Tamas Kalman (Tokyo Institute of Technology) Clock theorems for triangulated surfaces

We investigate triangulations of the two-dimensional sphere and torus with the faces properly colored white and black, focusing on matchings between white triangles and incident vertices. On the torus our objects are perfect pairings, whereas on the sphere this is only true after removing one triangle and its vertices. In the latter case, such matchings (first studied by Tutte) extend the notion of state in Kauffman's treatment of the Alexander polynomial and we show that his Clock Theorem, in its form due to Gilmer and Litherland, also extends: the set of matchings naturally forms a distributive lattice. Here the role of state transposition is played by a simple local operation about black triangles. By contrast, on the torus, the analogous state transition graph is usually disconnected: some of its components still form distributive lattices with global maxima and minima, while other components contain directed cycles and are without local extrema. This is joint work with Camden Hine.

#### Taizo Kanenobu (Department of Mathematics, Osaka City University) Classification of small ribbon 2-knots

We consider classification of ribbon 2-knots with small ribbon crossing numbers. We show the difference by: the Alexander polynomial; the trace set, which is obtained from the representations of the knot group to  $SL(2, \mathbb{C})$ ; the twisted Alexander polynomial; the fundamental group of the branched cyclic covering space of  $S^4$ .

# Kimihiko Motegi (Nihon University, College of Humanities and Sciences) The Strong Slope Conjecture for Whitehead doubles

The Slope Conjecture proposed by Garoufalidis asserts that the degree of the colored Jones polynomial determines a boundary slope, and its refinement, the Strong Slope Conjecture proposed by Kalfagianni and Tran asserts that the linear term in the degree determines the topology of an essential surface that satisfies the Slope Conjecture. Under certain hypotheses, we show that cablings, connected sums, Whitehead doubles of a knot satisfies the Strong Slope Conjecture if the original knot does. This is joint work with Kenneth L. Baker and Toshie Takata.

#### Jun Murakami (Waseda University)

#### On quantum representation of knots via braided Hopf algebra

For a knot K and a group G, we have the space of G representations of K, which is the space of all homomorphisms from the fundamental group  $\pi_1(S^3 \setminus K)$  to G. This space is reconstructed from the view point of the fundamental quandle and its representation associated with a Hopf algebra. Here we extend this construction to any braided Hopf algebra with braided commutativity. The typical example of a braided Hopf algebra is BSL(2), which is the braided quantum SL(2) introduced by S. Majid. By applying the above construction to BLS(2), we get a quantized SL(2) representation of K.

# Keita Nakagane (Department of Mathematics, Tokyo Institute of Technology) Homfly and full twists

The Homfly polynomial is an invariant of oriented links, which specializes to the Alexander polynomial and the Jones polynomial. The full twist formula by Kálmán claims that the certain extreme parts of the Homfly polynomial are related by adding a full twist to a braid representation. In this talk, we show the categorified version of the formula, which is applied to the Khovanov–Rozansky Homfly homology. This is joint work with Eugene Gorsky, Matthew Hogancamp, and Anton Mellit. If time permits, we also give an extension of braid diagrams which admits the polynomial version of the full twist formula.

# Makoto Sakuma (Osaka City University Advanced Mathematical Institute / Hiroshima University)

#### "Monodromy groups" of Heegaard surfaces of 3-manifolds

For a Heegaard surface S in a 3-manifod M, the "monodromy group" of S is defined to be the subgroup of the (full) mapping class group Mod(S) of S, consisting of the elements which are homotopic to the identity map when regarded as maps into M. We propose various natural questions related this group. This is a joint work with Yuya Koda.

#### Tatsuro Shimizu (Osaka City University Advanced Mathematical Institute) Chern-Simons perturbation theory and Reidemeister-Turaev torsion

Let  $\rho$  be a representation of the fundamental group of a closed oriented 3-manifold M such that the corresponding local system is acyclic. We give an invariant  $d(\rho)$  of  $\rho$  as a 1-dimensional cohomology class of M with twisted coefficient. This invariant is deeply related to the Chern-Simons perturbation theory. In this talk, when  $\rho$  is an abelian representation, we show that  $d(\rho)$  can be computed from Reidemeister-Turaev torsion of M.

#### Anderson Vera (Kyoto University / JSPS Research Fellow) Johnson-type homomorphisms, a conjecture by Levine, and the LMO invariant

In this talk we consider several filtrations of the mapping class group of a surface and of the monoid of homology cylinders of a surface. In particular, we give an answer to a weak version of a question by Levine with respect to a comparison of two of these filtrations. We also give an explicit relationship between the Johnson-type homomorphisms and the functorial extension of the Le-Murakami-Ohtsuki invariant.

# Wataru Yuasa (RIMS, Kyoto University / JSPS Research Fellow PD) Twist formulas for one-row colored $A_2$ webs and $\mathfrak{sl}_3$ tails of (2, 2m)-torus links

The  $\mathfrak{sl}_3$  colored Jones polynomial  $J_{(s,t)}^{\mathfrak{sl}_3}(L)$  is obtained by coloring the link components with two-row Young diagram (s,t). Although it is difficult to compute  $J_{(s,t)}^{\mathfrak{sl}_3}(L)$  in general, we can calculate it by using Kuperberg's  $A_2$  skein relation. In this talk, we introduce some formulas for twisted two strands colored by one-row Young diagram in the  $A_2$  web space and compute  $J_{(n,0)}^{\mathfrak{sl}_3}(T(2,2m))$  for an oriented (2,2m)-torus link. These explicit formulas derive the  $\mathfrak{sl}_3$  tail of T(2,2m). They also give explicit descriptions of the  $\mathfrak{sl}_3$  false theta series with one-row coloring because the  $\mathfrak{sl}_2$  tail of T(2,2m) is known as the false theta series.