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Problems on Low-dimensional Topology, 2026

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This is a list of open problems on low-dimensional topology with expositions of their history, background, significance, or importance. This list was made by editing manuscripts written by contributors of open problems to the problem session of the conference "Intelligence of Low-dimensional Topology" held at Research Institute for Mathematical Sciences, Kyoto University in May 20–22, 2026.

Contents

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1 Real Seiberg–Witten theory and exotic P^2-knots

(Jin Miyazawa)

In this section, all embeddings of surfaces are smooth. A *2-knot* is an embedding of S^2 into S^4. For a closed surface Σ, a Σ-*knot* is an embedding of Σ into S^4. A pair of two Σ-knots is *exotic*, if they are topologically ambient isotopic and they are not smoothly ambient isotopic.

Exotic embeddings

The origin of the motivation to consider the degree invariant of Real Seiberg–Witten degree invariant is to give an example of an exotic 2-knot.

Problem 1.1. *Find an exotic pair of* 2*-knots.*

When one of the 2-knots is the standard unknot, it is conjectured that there does not exist such a pair; this conjecture is called the *unknotting conjecture*. One of the difficulty to give an exotic pair of 2-knots is to prove that two 2-knots are topologically isotopic when the fundamental group is not isomorphic to $\mathbb{Z}$.

Problem 1.1 should be quite difficult so I think one can begin with other related problems.

There are no known examples of exotic embeddings of orientable surfaces to S^4. Let us consider the embedding of the orientable surface Σ_g with genus g. If we chose a certain Real spin^c structure, the virtual dimension of Real Seiberg–Witten moduli space is 0. However, we have no nontrivial example so far. For example, if we consider the connected sum of a 2-knot and the standard embedding of Σ_g, then the moduli space is empty for a certain $\mathbb{Z}/2\mathbb{Z}$-equivariant metric on $\Sigma_2(S^4, \Sigma_g)$.

Problem 1.2 (J. Miyazawa)**.** *Set $g \geq 1$. Give an example of an embedding of Σ_g whose degree invariant is nontrivial.*

There is another opportunity to detect exotic embeddings. Let us consider a pair of embeddings of T^2. We assume they are pairwise diffeomorphic and let us denote by f the diffeomorphism. The dimension of Real Seiberg–Witten equations for the Real spin structure is -1. Then if we consider a mapping torus of f and consider the double branched cover along $T^2 \times S^1$, the formal dimension of family moduli space is 0.

Problem 1.3 (J. Miyazawa)**.** *Define canonical framing of the index bundle of the Real Dirac operator and define family Real Seiberg–Witten theory.*

Problem 1.4 (J. Miyazawa)**.** *Find exotic embeddings of T^2 such that they are pairwise diffeomorphic.*

Real Seiberg–Witten Floer homotopy type

Real Seiberg–Witten theory is applied not only to exotic surfaces, but also to knot theory. Let K be a knot in a homology 3-sphere Y. Then we can consider Real

Seiberg–Witten Floer homotopy type on $\Sigma_2(Y,K)$ for some Real spin^c structure. When Y is S^3 and $\mathfrak{s}$ is the Real spin structure, let us denote $SWF_R(\Sigma_2(Y,K),\mathfrak{s})$ by $SWF_R(K)$. By applying equivariant K-group to this $\mathbb{Z}/4\mathbb{Z}$-equivariant spectrum, we obtain kappa invariant $\kappa_R(K)$. This is a Real version of Manolescu's kappa invariant and this behaves quite different manner from other Floer theoretic knot invariants. Therefore we can obtain some different kinds of information of knots. For example, a lower bound of stabilizing number [16].

The following problem is submitted by Masaki Taniguchi:

Problem 1.5 (M. Taniguchi)**.** *Compute the lower bound of the stabilizing number of torus knots.*

Using Real kappa invariant and its variants, Kang–Park–Taniguchi [14] proved non-sliceness of any nontrivial cable knots of the figure-eight knot. However, the sliceness problem for other satellite operations on the figure-eight knot, for example Whitehead double, is still open.

Problem 1.6 (J. Miyazawa)**.** *Estimate $\kappa_{n,R}(Wh((4)_1))$. Can we prove that $Wh((4)_1)$ is not slice?*

In [18], we proved satellite formula of Real Seiberg–Witten Floer homotopy type of knots for odd satellite operations.

Problem 1.7 (J. Miyazawa)**.** *Compute $SWF_R(P(K))$ for even satellite P.*

To solve above problem, in my opinion, we have to consider the information on critical points and flows of ordinary Seiberg–Witten theory on the branched double cover along K.

Finally, let me submit an abstract question. In Real theory, we observe that some indefinite cobordism X with $\mathbb{Z}/2\mathbb{Z}$-action look like negative definite cobordism since we can ignore the elements in $H^+(X)$ fixed by the involution. The satellite formula in [18] is a consequence of this phenomena.

Question 1.8 (J. Miyazawa)**.** *Are there any other non-trivial relations on $SWF_R(K)$?*

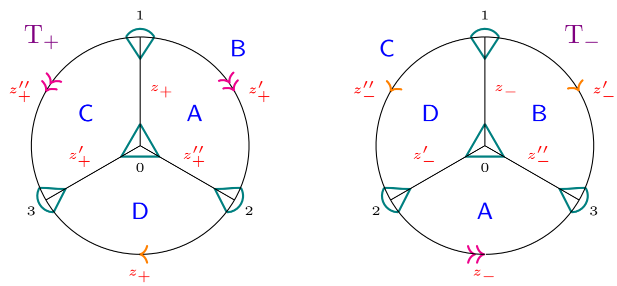
For example, our satellite formula can be viewed as one of a A_∞-relation with three inputs. How about other A_∞-relations.

2 Ordered ideal triangulations of hyperbolic knot complements and Andersen-Kashaev volume conjecture

(Ka Ho Wong)

In [5], Ben Aribi and the author introduced the notion of FAMED triangulations, which play an important role in the study of the Andersen–Kashaev volume conjecture for Teichmüller TQFT. This combinatorial condition on ordered ideal triangulations establishes a correspondence between the critical point equations of the potential function associated with the partition function in Teichmüller TQFT and Thurston's gluing equations for the ideal triangulation. The main result shows that if the triangulation is both FAMED and geometric, then the Andersen–Kashaev volume conjecture holds, as do its generalizations.

The condition describes a concrete relationship between two types of matrices with different origins: the face adjacency matrices arising from the construction of the partition function, and the Neumann–Zagier matrices arising from Thurston's gluing equations. We illustrate the definitions of these matrices using Thurston's ideal triangulation of the figure-eight knot complement, as shown in Figure 1. Note that each tetrahedron can be endowed with an ordering of its four vertices such that the face gluings respect the relative vertex order. This makes the triangulation an ordered triangulation. We say that a tetrahedron is *positive* (respectively, *negative*) if it has the configuration shown in the top left (respectively, top right) of Figure 1.



$$\mathcal{X}_0 = \begin{array}{c} \\ T_+ \\ T_- \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\mathcal{X}_1 = \begin{array}{c} \\ T_+ \\ T_- \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$\mathcal{X}_2 = \begin{array}{c} \\ T_+ \\ T_- \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$\mathcal{X}_3 = \begin{array}{c} \\ T_+ \\ T_- \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$\mathcal{A} = \begin{pmatrix} \mathcal{X}_0 - \mathcal{X}_1 + \mathcal{X}_2 \\ \mathcal{X}_2 - \mathcal{X}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & -1 \\ -1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ (matrix of tetrahedra signs)}$$

Figure 1: Thurston's triangulation of $M = S^3 \smallsetminus 4_1$, and the face adjacency matrices

For the face adjacency matrices, let $\mathcal{E}$ be the diagonal matrix whose diagonal entries encode the signs ± 1 of the tetrahedra. For each tetrahedron T and each $k = 0, 1, 2, 3$, let $x_k(T)$ denote the face opposite the k-th vertex of T, and let $\mathcal{X}_k$ be the coefficient matrix with entries

$$(\mathcal{X}_k)_{i,j} := \delta_{j\text{-th face},\, x_k(T_i)}.$$

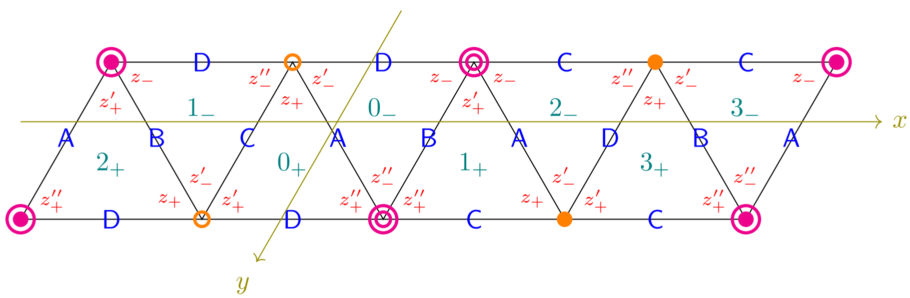
Define

$$\mathcal{B} := \begin{pmatrix} 0_N \\ \mathcal{E} \end{pmatrix} \quad \text{and} \quad \mathcal{A} := \begin{pmatrix} \mathcal{X}_0 - \mathcal{X}_1 + \mathcal{X}_2 \\ \mathcal{X}_2 - \mathcal{X}_3 \end{pmatrix}.$$

See Figure 1 for these face adjacency matrices of Thurston's ideal triangulation.

For the Neumann–Zagier matrices, Figure 2 shows the cusp triangulation of the torus obtained from the triangulation in Figure 1 by truncating the vertices. The gluing equations corresponding to the orange single arrow and the pink double arrow can be read off from the cusp triangulation by going around a vertex, which represents a truncated edge. Similarly, the holonomy equations associated with the meridian y and the preferred longitude $l = x+2y$ can be read from the corresponding peripheral curves. The associated Neumann–Zagier matrices $\mathbf{G}, \mathbf{G}', \mathbf{G}''$ with respect to the preferred longitude l are detailed in Figure 2. We define

$$\mathbf{A} := \mathbf{G} - \mathbf{G}' \quad \text{and} \quad \mathbf{B} := \mathbf{G}'' - \mathbf{G}'.$$



$(\rightarrow)$ starts at ○ and ends at ● : $2\mathrm{Log}(z_+) + \mathrm{Log}(z'_+) + 2\mathrm{Log}(z'_-) + \mathrm{Log}(z''_-) = 2i\pi$

$(\twoheadrightarrow)$ starts at ◎ and ends at ⊙ : $\mathrm{Log}(z'_+) + 2\mathrm{Log}(z''_+) + 2\mathrm{Log}(z_-) + \mathrm{Log}(z''_-) = 2i\pi$

Meridian y : $\mathrm{H}^{\mathbb{C}}_{X,y}(\mathbf{z}) = -\mathrm{Log}(z'_-) + \mathrm{Log}(z''_+)$

Preferred longitude $x+2y$: $\mathrm{H}^{\mathbb{C}}_{X,x+2y}(\mathbf{z}) = 2i\pi - 4\mathrm{Log}(z'_-) - 2\mathrm{Log}(z''_-)$

$$\mathbf{G} = \begin{array}{c} \\ (\rightarrow) \\ x+2y \end{array} \begin{array}{c} \begin{matrix} z_+ & z_- \end{matrix} \\ \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \end{array} \qquad \mathbf{G}' = \begin{array}{c} \\ (\rightarrow) \\ x+2y \end{array} \begin{array}{c} \begin{matrix} z'_+ & z'_- \end{matrix} \\ \begin{pmatrix} 1 & 2 \\ 0 & -4 \end{pmatrix} \end{array} \qquad \mathbf{G}'' = \begin{array}{c} \\ (\rightarrow) \\ x+2y \end{array} \begin{array}{c} \begin{matrix} z''_+ & z''_- \end{matrix} \\ \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \end{array}$$

Figure 2: Cusp triangulation of $S^3 \smallsetminus 4_1$, and the Neumann-Zagier matrices

Definition. *Let M be a compact 3-manifold with one toroidal boundary component and trivial second homology, such as a knot exterior. Let X be an ordered ideal triangulation of M with N tetrahedra, and let $\mathcal{A}$ be the face adjacency matrix associated with X as above.*

Let $l \in \pi_1(\partial M)$ be a peripheral curve, such as the preferred longitude of K if $M = S^3 \smallsetminus \nu(K)$. Let $\mathbf{A}$ and $\mathbf{B}$ be the Neumann–Zagier matrices with respect to l as above. We say that X is FAMED, for "Face Adjacency Matrices with Edge Duality," with respect to l if:

(1) *the space of angle structures is nonempty, so that the partition function is well-defined;*

(2) $\det \mathcal{A} \neq 0$*;*

(3) $\det \mathbf{B} \neq 0$*;*

(4) *we have*

$$\mathbf{B}^{-1}\mathbf{A} = \mathcal{X}_0 \mathcal{A}^{-1}\mathcal{B} + \left(\mathcal{X}_0 \mathcal{A}^{-1}\mathcal{B}\right)^{\top} + \frac{\mathcal{E} + \mathrm{Id}_N}{2}.$$

In [4], together with Ben Aribi and Guilloux, we observed numerically that whenever condition (2) holds, conditions (3) and (4) also hold. In all computed examples, one has

$$\det \mathcal{A} = \pm 1 \quad \text{and} \quad \det \mathbf{B} = \pm 2.$$

As a first step toward understanding the FAMED condition, one can try to prove the following statement.

Problem 2.1 (K. H. Wong). *Show that* $\det \mathcal{A} \neq 0$ *if and only if* $\det \mathbf{B} \neq 0$*. Moreover, for each vector in the kernel of* $\mathcal{A}$*, find a corresponding vector in the kernel of* $\mathbf{B}$*.*

The partition function in Teichmüller TQFT can be regarded as an infinite-dimensional analogue of the quantum hyperbolic invariants constructed by Baseilhac–Benedetti in [3]. Thus, it is natural to expect the two theories to share similar properties.

Problem 2.2 (K. H. Wong). *Formulate an analogue of the FAMED condition for the quantum hyperbolic invariants of Baseilhac–Benedetti. Compare the potential function arising from the partition function in Teichmüller TQFT with the corresponding potential function for quantum hyperbolic invariants.*

On the one hand, in the construction of the partition function in Teichmüller TQFT, variables are assigned to the faces of the triangulation. On the other hand, in order to recover the underlying geometry, one usually assigns shape parameters to the edges. Notice that each glued face of the ideal triangulation corresponds to an edge in the dual spine.

Problem 2.3 (K. H. Wong). *Find a way to express Thurston's gluing equations in terms of face variables.*

3 Weight systems of trivalent graphs for even dimensional manifolds

(Tadayuki Watanabe)

There are two versions of the Kontsevich graph homology: one for odd dimensional manifolds, and the other for even dimensional manifolds ([17]). Let $\mathcal{A}_n^{\mathrm{odd}}$ (resp. $\mathcal{A}_n^{\mathrm{even}}$) denote the trivalent part of the graph homology. It is defined by the vector space over $\mathbb{Q}$ spanned by finite connected trivalent graphs with $2n$ vertices, quotiented by the relation AS and IHX, which are diagrammatic analogues of the relations in Lie algebras (*e.g.* [19, 9]). A linear function $W\colon \mathcal{A}_n^{\mathrm{odd}} \to \mathbb{Q}$ is called a *weight system.* Weight systems are useful to find nontrivial classes in $\mathcal{A}_n^{\mathrm{odd}}$. For example, one can see that $\mathcal{A}_n^{\mathrm{odd}} \neq 0$ for infinitely many n.

Conjecture 3.1 (T. Watanabe). $\mathcal{A}_n^{even} \neq 0$ *for infinitely many* n*.*

Problem 3.2 (T. Watanabe). *Construct non-trivial weight systems* $\mathcal{A}_n^{even} \to \mathbb{Q}$ *for each* n*.*

Some computations of the dimensions of $\mathcal{A}_n^{\text{even}}$ for lower n are given in [2, 6]. For $\mathcal{A}_n^{\text{odd}}$, Bar–Natan invented an algorithm to construct many weight systems from semisimple Lie algebras ([1]). For even dimensional manifolds, clasper calculus and the Jacobi identity for Whitehead products suggest to consider a $\mathbb{Z}/2\mathbb{Z}$-graded Lie algebra $L_0 \oplus L_1$. Nevertheless, we do not know whether such an approach works.

4 Nielsen equivalence in mapping class groups

(Susumu Hirose[2] and Naoyuki Monden[3])

Nielsen equivalence is a natural equivalence relation on generating n-tuples of a finitely generated group G. Two n-tuples $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are Nielsen equivalent if one can be transformed into the other via a finite sequence of three elementary moves:
(1) replacing x_i with x_i^{-1} for some i;
(2) swapping x_i and x_j for $i \neq j$; and
(3) replacing x_i with $x_i x_j$ for $i \neq j$.

Let $d(G)$ be the minimum number of generators of G. For $d(G) = 2$ and a generating pair (x, y) of G, a classic invariant is the *Higman invariant*: the union of the conjugacy classes of the commutator $[x, y]^{\pm 1} = (xyx^{-1}y^{-1})^{\pm 1}$. While no such invariant word exists for $d(G) \geq 3$ (see [11]), the $d(G) = 2$ case provides a powerful tool for distinguishing equivalence classes. By a result of Wajnryb [20], the mapping class group $\mathcal{M}_g$ of a closed oriented surface of genus g is generated by two elements for $g \geq 1$; *i.e.*, $d(\mathcal{M}_g) = 2$. Moreover, the authors constructed infinitely many distinct generating pairs of $\mathcal{M}_g$ for $g \geq 9$ (see Theorem 1.2 of [12]). This led M. Linton and M. Sakuma to pose the following:

Question 4.1 (M. Linton, M. Sakuma)**.** *Are the generating pairs given in [12] Nielsen equivalent?*

The authors were unable to distinguish these pairs because their commutators are mutually conjugate. This motivates the search for a finer invariant:

Question 4.2 (S. Hirose, N. Monden)**.** *Does there exist an invariant of generating pairs of $\mathcal{M}_g$ under Nielsen equivalence that is finer than the Higman invariant?*

Although the authors showed in [13] that $\mathcal{M}_g$ has infinitely many Nielsen equivalence classes for $g \geq 8$, the case of small genera remains an open problem:

Question 4.3 (S. Hirose, N. Monden)**.** *For $2 \leq g \leq 7$, is the number of Nielsen equivalence classes of generating pairs of $\mathcal{M}_g$ finite or infinite?*

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Stabilization is the process of adding identity elements to a generating tuple. Any two generating n-tuples $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ is Nielsen equivalent after n stabilizations:

$$(x_1, \ldots, x_n, 1, \ldots, 1) \sim (x_1, \ldots, x_n, y_1, \ldots, y_n) \sim (y_1, \ldots, y_n, 1, \ldots, 1),$$

where $\sim$ denotes Nielsen equivalence. However, the behavior for $k < n$ stabilizations is non-trivial. Indeed, stabilization does not always trivialize the relation; Kapovich and Weidmann [15] proved that for certain groups, $n-1$ stabilizations are insufficient to achieve equivalence. Let (h_0, ρ_m) be a generating pair constructed in Theorem 1.3 of [13] for any integer m. Then, we have $(h_0, \rho_\ell, 1) \sim (h_0, \rho_m, 1)$ for any $\ell \neq m$ as they share the common element h_0 (an observation communicated to the authors by S. Kamada). This suggests the following question:

Question 4.4 (S. Hirose, N. Monden)**.** *For $g \geq 2$, is the number of Nielsen equivalence classes of generating 3-tuples of $\mathcal{M}_g$ finite or infinite?*

5 Goldman bracket on loops on a surface

(Aoi Wakuda)

Let Σ be a connected oriented surface of negative Euler characteristic, and let $\hat{\pi}$ denote the set of free homotopy classes of loops on Σ. Let K be a commutative ring. For any set X, let KX denote the free K-module generated by X.

Goldman [10] defined the Goldman bracket on $K\hat{\pi}$ by

$$[x, y] := \sum_{P \in x \cap y} \varepsilon_P(x, y)\, |x_P y_P|.$$

Here x and y are chosen so that they intersect transversely, $\varepsilon_P(x, y)$ denotes the sign of the intersection at P, and $|x_P y_P|$ denotes the free homotopy class represented by the loop product at P.

Goldman [10] proved that if one of $x, y \in \hat{\pi}$ is represented by a simple closed curve, then the vanishing of the Goldman bracket $[x, y] = 0$ is equivalent to the vanishing of the geometric intersection number $i(x, y) = 0$. Moreover, Chas [7] showed that if one of the loops is simple, then the number of terms appearing in the Goldman bracket, counted with multiplicity, is equal to the geometric intersection number. However, in the case of non-simple loops, many basic questions remain open.

Problem 5.1 (A. Wakuda)**.** *Find all pairs $x, y \in \hat{\pi}$ satisfying $[x, y] = 0$.*

If $i(x, y) = 0$, then the Goldman bracket vanishes. However, the converse does not hold in general. Chas [7] constructed infinitely many pairs satisfying

$$[x, y] = 0 \quad \text{and} \quad i(x, y) > 0.$$

What are all possible such examples?

For $x \in \hat{\pi}$, let $x^{-1} \in \hat{\pi}$ denote the free homotopy class represented by reversing the orientation of a representative of x.

Problem 5.2 (M. Chas, A. Kabiraj [8]). *Let $x, y \in \hat{\pi}$ satisfy $x \neq y^{\pm 1}$. Determine whether the condition $[x+x^{-1},\, y+y^{-1}] = 0$ is equivalent to the condition $i(x, y) = 0$.*

The condition $[x+x^{-1},\, y+y^{-1}] = 0$ is equivalent to the vanishing of the Thurston–Wolpert–Goldman bracket of the corresponding unoriented loops [8]. Computer experiments in [8] suggest that the equivalence holds for certain surfaces when the word lengths are at most 7. Moreover, Goldman [10] proved the equivalence when one of the loops is simple.

Question 5.3 (A. Wakuda). *Let $x, y \in \hat{\pi}$ and let $m \geq 2$. Suppose that $y \neq x^m$. Is the number of terms appearing in the Goldman bracket $[x^m, y]$, counted with multiplicity, equal to $m \times i(x, y)$?*

The vanishing case was proved by the author [21]. More precisely, if $[x^m, y] = 0$, then either $i(x, y) = 0$ or $y = x^m$ holds. In particular, Question 5.3 suggests that the Goldman bracket may detect geometric intersection numbers even when neither of the loops is simple.

References

[1] Bar-Natan, D., *On the Vassiliev knot invariants*, Topology **34** (1995) 423–472.

[2] Bar-Natan, D., McKay, B., *Graph cohomology - An overview and some computations*, draft, 2001. `https://drorbn.net/AcademicPensieve/Annotations/Bar-Natan/GCOC@.pdf`

[3] Baseilhac, S., Benedetti, R., *Quantum hyperbolic invariants of 3-manifolds with $PSL(2;)$-characters*, Topology **43** (2004) 1373–1423.

[4] Ben Aribi, F., Guilloux, A., Wong, K.H., *FAMED by computer: proving the Andersen-Kashaev volume conjecture for 42,000 knots*, arXiv:2512.17437.

[5] Ben Aribi, F., Wong, K.H., *The Andersen-Kashaev volume conjecture for FAMED geometric triangulations*, arXiv 2410.10776.

[6] Brun, S., Willwacher, T., *Graph homology computations*, New York J. Math. **30** (2024) 58–92.

[7] Chas, M., *Minimal intersection of curves on surfaces*, Geom. Dedicata **144** (2010) 25–60.

[8] Chas, M., Kabiraj, A., *The Lie bracket of undirected closed curves on a surface*, Trans. Amer. Math. Soc. **375** (2022) 2365–2386.

[9] Chmutov, S., Duzhin, S., Mostovoy, J., *Introduction to Vassiliev knot invariants*. Cambridge University Press, Cambridge, 2012. xvi+504 pp.

[10] Goldman, W. M., *Invariant functions on Lie groups and Hamiltonian flows of surface group representations*, Invent. Math. **85** (1986) 263–302.

[11] Guralnick, R., Pak, I., *On a question of B. H. Neumann*, Proc. Amer. Math. Soc. **131** (2003) 2021–2025.

[12] Hirose, S., Monden, N., *On generating mapping class groups by pseudo-Anosov elements*, Bull. Braz. Math. Soc. (N.S.) **57** (2026) Paper No. 11, 21 pp.

[13] Hirose, S., Monden, N., *On Nielsen equivalence classes of two-element generators of mapping class groups*, arXiv:2505.07284.

[14] Kang, S., Park, J., Taniguchi. M., *Smooth concordance of cables of the figure-eight knot* arXiv:2505.03720.

[15] Kapovich, I., Weidmann, R., *Nielsen equivalence in a class of random groups*, J. Topol. **9** (2016) 502–534.

[16] Konno, H., Miyazawa, J., Taniguchi, M., *Involutions, knots, and Floer K-theory*, Compos. Math. **161** (2025) 2852–2910.

[17] Kontsevich, M., *Feynman diagrams and low-dimensional topology*, First European Congress of Mathematics, Vol. II (Paris, 1992), Progr. Math. **120** (Birkhauser, Basel, 1994), 97–121.

[18] Miyazawa, J., Park, J., Taniguchi. M., *A satellite formula for real Seiberg-Witten Floer homotopy types*, arXiv:2504.03270.

[19] Ohtsuki, T., *Quantum invariants. A study of knots, 3-manifolds, and their sets*, Ser. Knots Everything, **29** World Scientific Publishing Co., Inc., River Edge, NJ, 2002. xiv+489 pp.

[20] Wajnryb, B., *Mapping class group of a surface is generated by two elements*, Topology **35** (1996) 377–383.

[21] Wakuda, A., *Separability criteria for loops via the Goldman bracket*, arXiv:2511.18503.