研究集会 Intelligence of Low-dimensional Topology

京都大学数理解析研究所 RIMS 共同研究(公開型)として、また、トポロジープロジェクトの一環として、標記の研究集会を開催いたします。また、この研究集会は科学研究費補助金基盤研究 A 「3次元双曲多様体上の量子トポロジー」(課題番号 21H04428、研究代表者大槻知忠(京都大学))の援助をうけています。

日程: 2025年 5月26日 (月)~ 5月28日 (水) 場所: 京都大学 数理解析研究所 420 大講演室 アクセス: https://www.kurims.kyoto-u.ac.jp/ja/access-01.html 研究集会ホームページ: https://www.kurims.kyoto-u.ac.jp/~ildt/

この研究集会は、ハイブリッド型(対面とオンライン(Zoom)の併用)で開催すること を計画しています。参加者数を事前に把握するために、参加される方(対面もオンライン も)は、5月12日までに、参加登録をお願いします。参加登録の際に「配信映像を録画・ 録音しないこと」のご同意をお願いします。参加登録方法について、研究集会ホームペー ジ(上記 URL)をご覧ください。

5月26日(月)

13:40~14:20 市原 一裕 (日本大学文理学部) Pure cactus groups and configuration spaces of points on the circle

14:40~15:20 佐藤 真衣 (津田塾大学大学院理学研究科数学専攻) On an R-equivalence relation on the set of quandle colorings

 $15:40 \sim 16:20$ Patrick Kinnear (University of Hamburg / JSPS Fellow) Skein modules of mapping tori of the 2-torus

5月27日(火)

10:30~11:10 中村 伊南沙 (佐賀大学理工学部) Knitted surfaces and their chart description: overview and the case of degree 2

11:30~12:10 櫻井 みぎ和 (芝浦工業大学) Infinitely many virtual knots whose virtual unknotting number equals one and a sequence of *n*-writhes

13:40 ~ 14:20 María Cumplido (University of Seville) An algorithm to compute canonical reduction systems for braids

14:40~15:20 藤原 耕二 (京都大学) The rates of growth in hyperbolic groups

15:40 ~ Problem Session

5月28日(水)

10:30~11:10 谷口 東曜 (東京大学大学院数理科学研究科) Modular vector fields in non-commutative geometry

11:30~12:10 安田 順平 (大阪公立大学) Plat presentation of surface-links and their invariants

13:40~14:20 新井 克典 (大阪大学大学院理学研究科数学専攻) MGR coloring invariants of Seifert surfaces

14:40~15:20 甲斐 涼哉 (大阪公立大学) On metrics for quandles

 $15{:}40 \sim 16{:}20~$ Marc Lacken by (University of Oxford) (online) Certifying knot hyperbolicity

> 組織委員:秋吉宏尚、大槻知忠、鎌田聖一、鎌田直子、河野俊丈 世話人:大槻知忠 (京大 数理研)、軽尾浩晃 (学習院大 理学部数学科)

Intelligence of Low-dimensional Topology

May 26–28, 2025

This conference is planned to be held at

Room 420, RIMS, Kyoto University,

whose live streaming is distributed online.

Program

May 26 (Mon)

13:40–14:20 Kazuhiro Ichihara (College of Humanities and Sciences, Nihon University) Pure cactus groups and configuration spaces of points on the circle

14:40–15:20 Mai Sato (Department of Mathematics, Tsuda University) On an R-equivalence relation on the set of quandle colorings

15:40–16:20 Patrick Kinnear (University of Hamburg / JSPS Fellow) Skein modules of mapping tori of the 2-torus

May 27 (Tue)

10:30–11:10 Inasa Nakamura (Saga University) Knitted surfaces and their chart description: overview and the case of degree 2

11:30–12:10 Migiwa Sakurai (Shibaura Institute of Technology) Infinitely many virtual knots whose virtual unknotting number equals one and a sequence of n-writhes

13:40–14:20 María Cumplido (University of Seville) An algorithm to compute canonical reduction systems for braids

14:40–15:20 Koji Fujiwara (Kyoto University) The rates of growth in hyperbolic groups

15:40– Problem Session

May 28 (Wed)

10:30–11:10 Toyo Taniguchi (Graduate School of Mathematical Sciences, The University of Tokyo) Modular vector fields in non-commutative geometry

11:30–12:10 Jumpei Yasuda (Osaka Metropolitan University) Plat presentation of surface-links and their invariants

13:40–14:20 Katsunori Arai (Department of Mathematics, Graduate School of Science, The University of Osaka) MGR coloring invariants of Seifert surfaces

14:40–15:20 Ryoya Kai (Osaka Metropolitan University) On metrics for quandles

15:40–16:20 Marc Lackenby (University of Oxford) (online) Certifying knot hyperbolicity

Scientific Committee: Hirotaka Akiyoshi, Naoko Kamada, Seiichi Kamada, Toshitake Kohno, Tomotada Ohtsuki

Organizers: Tomotada Ohtsuki (RIMS, Kyoto University), Hiroaki Karuo (Department of Mathematics, Gakushuin University)

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Abstract

Katsunori Arai (Department of Mathematics, Graduate School of Science, The University of Osaka)

MGR coloring invariants of Seifert surfaces

A. Ishii, S. Matsuzaki, and T. Murao introduced an algebraic system called a multiple group rack (MGR). MGRs are used to define invariants of compact oriented surfaces with boundaries embedded in the 3-sphere S^3 , known as MGR coloring invariants. In this talk, we present an infinite family of pairs of Seifert surfaces for a link, where each pair satisfies the following conditions:

- (1) Their regular neighborhoods in S^3 are ambiently isotopic,
- (2) Their Seifert matrices are unimodular-congruent, and
- (3) The two Seifert surfaces are not ambiently isotopic.

To prove (3), we distinguish the Seifert surfaces using MGR coloring invariants.

María Cumplido (University of Seville)

An algorithm to compute canonical reduction systems for braids

One definition of the braid group is as the mapping class group of the *n*-punctured disc. According to this viewpoint, braids admit a Nielsen-Thurston classification and can be categorized as periodic, reducible, or pseudo-Anosov. In many arguments, the periodic case is often straightforward, and the pseudo-Anosov case has a rich structure that can be exploited to derive results. In the reducible case, Birman, Lubotzky, and McCarthy showed that a braid preserves a family of curves that can be chosen canonically. By considering the subsurfaces defined by these curves, one can study the action of the reducible braid on each subsurface, which will be either periodic or pseudo-Anosov, hence the term reducible. In this work, in collaboration with Juan González-Meneses and Davide Perego, we combine tools from low-dimensional topology and Garside theory to present an algorithm for computing the canonical reduction system of a reducible braid.

Koji Fujiwara (Kyoto University)

The rates of growth in hyperbolic groups

Let e(G, S) be the exponential growth rate of a finite generated group G with a finite generating set S. Let $\xi(G)$ be the set $\{e(G, S)\}$ for all finite generating sets S of G. Sela and I proved that if G is a non-elementary hyperbolic group, then $\xi(G)$ is well-ordered. We will discuss some other properties of $\xi(G)$.

Kazuhiro Ichihara (College of Humanities and Sciences, Nihon University) Pure cactus groups and configuration spaces of points on the circle

The cactus group was introduced by Henriques and Kamnitzer as an analogue of the braid group. In this talk, I will present a new description of the pure cactus group of degree four. I will also discuss the relationship between the pure cactus groups and the configuration spaces of points on the circle. This talk is based on a joint work with Takatoshi Hama (Nihon University).

Ryoya Kai (Osaka Metropolitan University) On metrics for quandles

In geometric group theory, finitely generated infinite groups are studied from a geometric perspective. We aim to construct quandle theoretic analogue for geometric group theory, and to provide geometric interpretation of infinite quandles. In this talk, we define two types of quasi-isometric classes of metrics for a quandle with a certain finiteness property. Additionally, we provide some examples of quandles that are quasi-isometric to well-known metric spaces. This talk is based on a joint work with Kohei Iwamoto (Ritsumeikan University) and Yuya Kodama (Kagoshima University).

Patrick Kinnear (University of Hamburg / JSPS Fellow) Skein modules of mapping tori of the 2-torus

I will give a formula for the dimension of the Kauffman bracket skein module (at generic q) for mapping tori of the 2-torus, generalizing the well-known computation of Carrega and Gilmer. In the process, we will see a decomposition of the twisted Hochschild homology of the G-skein algebra for G = SLN or GLN, which is a direct summand of the whole skein module, and from which the dimensions follow easily in the cases $G = SL_2$ and $G = GL_1$.

Marc Lackenby (University of Oxford) (online) Certifying knot hyperbolicity

Snappy is remarkably good at finding hyperbolic structures on knot complements. It is natural to ask why this should be. One can make this question precise by asking: what is the computational complexity of determining whether a knot is hyperbolic, and if it is hyperbolic, how hard is it to find its hyperbolic structure? In my talk, I will address the first of these questions. I will outline a new theorem that states that the problem of determining whether a knot is hyperbolic is in NP. In other words, if a knot is hyperbolic, then there is a certificate that establishes this, which is verifiable in polynomial time as a function of the number of crossings in a given diagram for the knot. In the case of fibred knots, this is recent work of Filippo Baroni on the detection of pseudo-anosov surface automorphisms. For non-fibred knots, the proof uses hierarchies. A complementary result was established by Baldwin and Sivek in 2017, who showed that knot hyperbolicity is in co-NP assuming the Generalised Riemann Hypothesis.

Inasa Nakamura (Saga University)

Knitted surfaces and their chart description: overview and the case of degree 2 A knitted surface is a surface with boundary properly embedded in a bi-disk, which is a generalization of a braided surface. A knitted surface is described as a trace of transformation of knits, where a knit is a tangle obtained from a braid by splice at some crossings. A knitted surface has a graphical description called a chart description, which is a generalization of a chart of a braided surface. In this talk, we explain knitted surfaces and their chart description. Further, we consider knitted surfaces of degree 2. We show that the plat closure of any knitted surface of degree 2 is a trivial surface-link, and any trivial surface-link is ambiently isotopic to the plat closure of a knitted surface of degree 2. This is joint work with Jumpei Yasuda.

Migiwa Sakurai (Shibaura Institute of Technology) Infinitely many virtual knots whose virtual unknotting number equals one and a sequence of *n*-writhes

Satoh and Taniguchi introduced the *n*-writhe J_n for each non-zero integer *n*, which is an integer invariant for virtual knots. It is obvious that the virtualization of a real crossing is an unknotting operation for virtual knots. In our previous research, we showed that if $\{r_n\}_{n\neq 0}$ is a sequence of integers with $\sum_{n\neq 0} nr_n = 0$, then there exists a virtual knot *K* whose virtual unknotting number equals one and $J_n(K) = r_n$ for any $n \neq 0$. In this talk we show that there exist infinitely many virtual knots having such properties by using the vertex connected sum on Gauss diagrams.

Mai Sato (Department of Mathematics, Tsuda University) On an R-equivalence relation on the set of quandle colorings

A quandle is an algebraic system which has close relationships with knot theory. For each quandle X, we may consider X-colorings of an oriented knot diagram. It is wellknown that a Reidemeister move naturally relates the X-colorings of the original diagram to the ones of the diagram obtained from the original one by the Reidemeister move, one-to-one onto. Although we may deform an oriented knot diagram to itself by a finite sequence of Reidemeister moves, the X-colorings of the diagram related to each other by the sequence are not always the same. To study such colorings, we introduce the notion of R-equivalence relation on the set of X-colorings of an oriented knot diagram. We discuss some methods to determine R-equivalence classes of X-colorings with concrete examples.

Toyo Taniguchi (Graduate School of Mathematical Sciences, The University of Tokyo)

Modular vector fields in non-commutative geometry

A double bracket is a non-commutative analogue of a Poisson bracket. One important example emerges from a loop operation in two-dimensional topology: Kawazumi-Kuno and Massuyeau-Turaev independently defined the same double bracket on the group algebra over the fundamental group of an oriented surface with non-empty boundary, which is based on surgery of two loops at an intersection. The speaker introduces a noncommutative version of the modular vector field in Poisson geometry and shows that it coincides with Turaev's other loop operation in the case of the above group algebra.

Jumpei Yasuda (Osaka Metropolitan University)

Plat presentation of surface-links and their invariants

Plat presentation is a method of representing surface-links using braided surfaces. We will discuss plat presentation of surface-links and its applications to invariants of surface-links. Specifically, we provide algebraic characterizations of the knot groups and the knot symmetric quandles of surface-links. If time permits, we will also discuss results on the classification of 2-knots with 2-plat presentation.