Non-Linear Evolution Equations Derived From the RG equatiosn of Wegner-Houghton-Aoki Type

M. Kuze and F.Hiroshima
Dept of Math., Kyushu Univ.
K.R.Ito
Dept. of Math. & Phys.
Setsunan Univ.

Block spin transformations (BSTs) are usually very complicated transformatiosns of functions $\{V_n\}$ which depend on many varibles $\{\phi(x); x \in Z^d\}$ (as many as space-time points Z^d). This difficulty mainly comes from the non-locality and the non-linearlity of the transformatiosns:

$$\exp[-V_n(\{\phi\})] \to \exp[-V_{n+1}(\{\phi\})], \quad n = 1, 2, 3, \cdots$$

The LPA (local potential approximation) is a technique to simplify these transformations (hopefully) without losing the essential properties of physics. Namely this is regarded as an infinitesimal form of the block spin transformation and is obtained by taking the limit $L \to 1$ and by discarding non-local terms (thus LPA), where L is the size of the blocks. Thus we obtain a series of non-linear partial differential equations of $V = V(t, x)$ which usually take the following form:

$$\frac{\partial}{\partial t} V = \Phi(V, V_x, V_{xx})$$

where e^t corresponds to the size $L \geq 1$ of the blocks (of BST) and x correspond to the field variable ϕ at the distance scale L.

We investigate these equatiosns for $d = 4$ and $d = 1$.

In the case of $d = 4$, this is the ϕ_4^4 problem and we expect $V(t, x) \to 0$ as $t \to \infty$.

In the case of $d = 1$, we expect that the difference $\Delta = E_2 - E_1$ (E_i is the energy of the i th excited state) may be described by an non-linear evolution equation (Aoki's conjecture). We discuss if the limit $\lim_{t\to\infty} V(t, x)$ exists.