On the existence of ground states for the Pauli-Fierz model on a pseudo Riemannian manifold

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Abstract

The existence of ground states of the Pauli-Fierz Hamiltonian on a psudo-Riemannian manifold is considered with an arbitrary coupling constant α.
The dispersion relation $\hat{\omega}$ is written by

$$\omega(x) = \sqrt{-\Delta + v(x)}.$$

The quantized radiation field A is written by

$$A(x) := \frac{1}{\sqrt{2}} \left(a^\dagger(\overline{\rho_x \hat{\omega}^{-1/2}}) + a(\rho_x \hat{\omega}^{-1/2}) \right),$$

where

$$\rho_x^{\mu,j}(y) = (2\pi)^{-3/2} \int \overline{\Psi(k,x)} \Psi(k,y) e_j^\mu(k)\, dk$$

and e_μ^j, $\mu = 1, 2, 3$, $j = 1, 2$ are porlarization vectors. Ψ satisfies the Lippman-Schwinger equation: for $k \neq 0$

$$(-\Delta_x + v(x))\Psi(k,x) = |k|^2 \Psi(k,x).$$

The free Hamiltonian H_f is written by

$$H_f := d\Gamma(\hat{\omega}).$$

The Pauli-Fierz Hamiltonian is written by

$$H^V := \frac{1}{2} \sum_{\mu,\nu} (p_\mu + \sqrt{\alpha} A_\mu) a_{\mu\nu} (p_\nu + \sqrt{\alpha} A_\nu) + H_f + V.$$

We consider $a_{\mu\mu} = 1$ and $a_{\mu\nu} = 0$ if $\mu \neq \nu$. Under some assumptions, we prove the existence of ground states of H^V by applying the method due to Griesemer, Lieb and Loss [1].

References

[1] M. Griesemer, E. H. Lieb, and M. Loss, "Ground states in non-relativistic quantum electrodynamics", Invent.Mass. **145**, 557-595(2001)