New and Old Unsolved Problems in Mathematical Physics and Renormalization Group Methods

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Abstract. In this talk I first survey some recent studies where renormalization group type arguments are used, namely

1. fluid mechanics (Sinai)
2. Boltzmann equation (Erdös, Yau)
3. Pauli-Fierz model (Semi-Classical QED)
4. Perelman's theory (σ model)
5. traditional problems (Field theory, Statistical mechanics)

After this, I would like to show our recent study of 2D σ model by block-spin transformations. This is one of the unsolved problems which go back to 20 century as well as the problem of quark confinement.

In this study, we introduce auxiliary fields $\{\psi(x); x \in Z^2\}$ which keep $\phi(x) \in R^N$ on S^{N-1}:

$$\delta(\phi^2(x) - 1) = \int \exp[i(\phi^2(x) - 1)\psi(x)] \frac{d\psi(x)}{2\pi}$$

The right hand side is similar to the Yukawa model with pure imaginary coupling constant.

Thus we start with the system of variables $\{(\phi, \psi)\}$ living on Z^2, and show that the main stream of the renormalization group is consistent with the conventional wisdom for the 2D $O(N)$ σ model with large $N > 2$, namely absence of phase transitions. (This is partly a joint work with E.Seiler (Munich).)