Asymptotic behavior of the gyration radius for long-range oriented percolation and long-range self-avoiding walk

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Let $\alpha > 0$ and consider random walk and self-avoiding walk on $\mathbb{Z}^d$ whose 1-step distribution is given by $D(x) \approx |x/L|^{-d-\alpha}$ for some $L \in [1, \infty)$ such that

$$1 - \hat{D}(k) = v_\alpha |k|^{\alpha \wedge 2} \times \begin{cases} 1 + O(|k|^\epsilon) & (\alpha \neq 2), \\ \log \frac{1}{|k|} + O(1) & (\alpha = 2), \end{cases}$$

for some $v_\alpha = O(L^{\alpha \wedge 2})$. We also consider oriented percolation on $\mathbb{Z}^d \times \mathbb{Z}_+$ whose bond-occupation probability for each bond $((u, s), (v, s+1))$ is given by $pD(v - u)$, where $p \geq 0$, independently of $s \in \mathbb{Z}_+$. Let $\varphi_t(x)$ denote the two-point functions for those three models simultaneously.

In the recent joint work [2], we have proved, in a unified fashion, sharp asymptotics of the generating function of the sequence $\sum_{x \in \mathbb{Z}^d} |x_1|^r \varphi_t(x)$, where $x = (x_1, \ldots, x_d)$ and $r \in (0, \alpha)$, for random walk in any dimension with any L and for self-avoiding walk and critical/subcritical oriented percolation above the common upper-critical dimension $d_{\mathrm{c}} = 2(\alpha \wedge 2)$ with $L \gg 1$. As a result, we obtain that, for every $r \in (0, \alpha)$,

$$\frac{\sum_{x \in \mathbb{Z}^d} |x_1|^r \varphi_t(x)}{\sum_{x \in \mathbb{Z}^d} \varphi_t(x)} \underset{t \to \infty}{\sim} \frac{2 \sin \frac{r\pi}{\alpha \vee 2}}{(\alpha \wedge 2) \sin \frac{r\pi}{\alpha}} \, \frac{\Gamma(r+1)}{\Gamma(\frac{r}{\alpha \wedge 2} + 1)} \times \begin{cases} (C_\alpha t)^{\frac{r}{\alpha \wedge 2}} & (\alpha \neq 2), \\ (C_2 t \log t)^{r/2} & (\alpha = 2), \end{cases}$$

where the constant C_α depends only on the zeroth and first moments for the time variable of the lace-expansion coefficients when $\alpha \leq 2$, and only on those two quantities and the second moment for the spacial variable of the expansion coefficients when $\alpha > 2$. This solves the open problems in [1, 3].

References

[1] L.-C. Chen and A. Sakai. Critical behavior and the limit distribution for long-range oriented percolation. II: Spatial correlation. *Probab. Theory Relat. Fields* **145** (2009): 435–458.

[2] L.-C. Chen and A. Sakai. Asymptotic behavior of the gyration radius for long-range oriented percolation and long-range self-avoiding walk. In preparation.

[3] M. Heydenreich. Long-range self-avoiding walk converges to alpha-stable processes. Preprint, arXiv:0809.4333v1 (2008).