Ground states for the massless Nelson model on a Lorentzian manifold

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Abstract: We consider the massless Nelson model on a Lorentzian manifold whose Hamiltonian is

$$H = K \otimes I + I \otimes d\Gamma(\omega) + \frac{1}{\sqrt{2}}(a(\omega^{-1/2}\rho_x) + a^*(\omega^{-1/2}\rho_X)).$$

Here K is the Hamiltonian of a confined particle acting on $L^2(\mathbb{R}^3, dX)$, $d\Gamma(\omega)$ the second quantization of ω and $a(f)$ and $a^*(f)$ the annihilation and creation operators satisfying the canonical commutation relations:

$$[a(f), a^*(g)] = \langle f, g \rangle, \quad [a(f), a(g)] = [a^*(f), a^*(g)] = 0$$

for any $f, g \in L^2(\mathbb{R}^3, dx)$. $\rho \in C_0^\infty(\mathbb{R}^3)$ is the charge distribution of the particle with the charge $g := \int \rho(x)dx < \infty$ and $\rho_X = \rho(\cdot - X)$. We assume that the elliptic operator

$$h := \sum_{i,j} \partial_{x_i} a_{ij}(x) \partial_{x_j} + c(x)$$

is positive self-adjoint and set $\omega = h^{1/2}$. On the Minkwoski space the operator h is the Laplacian $-\Delta$ and it is well-known that H has a ground state if and only if $g = 0$. This phenomena is the so-called infrared divergence. In this talk we assume that $g \neq 0$ and prove that:
(1) there exists a ground state if $c(x)$ decays like $|x|^{-\beta}$ with $\beta < 2$,
(2) there is no ground state if $c(x)$ decays like $\beta > 2$,
(3) for $\beta = 2$: (i) the ground state exists if $|c(x)| << 1$ and (ii) the ground state does not exist if $|c(x)| >> 1$.
In particular, there is no infrared divergence in the case of (1) or (3-i).

References

[1] C. Gérard, F. Hiroshima, A. Panati, A. Suzuki: Infrared divergence of a scalar quantum field model on a pseudo Riemann manifold, *Proceedings of the 8th Sendai WS IDA-QP* (to appear).