RG flow of 2D $O(N)$ Spin Models and Absence of Phase Transitions

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The 2D $O(N)$ spin model with large N is studied by the renormalization group (RG) method triggered by Wilson and founded mathematically by Gawedzki and Kupiainen [6]. Let $\phi(x) \in R^N$ be the $N-1$ dimensional sphere of radius $(N\beta)^{1/2}$. Using $\delta(\phi^2 - N\beta) = \int \exp[-i\psi(\phi^2 - N\beta)]d\psi/2\pi$ [4],

$$
\begin{aligned}
Z_\Lambda &= \int \cdots \int \exp[-W_0(\phi,\psi)] \prod \frac{d\phi_j d\psi_j}{2\pi} &(1)\\
W_0(\phi,\psi) &= \frac{1}{2} < \phi, (m^2 - \Delta)\phi > + \frac{i}{\sqrt{N}} < \phi^2 - N\beta, \psi > &(2)
\end{aligned}
$$

where $(-\Delta)_{xy} = 4\delta_{xy} - \delta_{|x-y|,1}$ is the 2D lattice laplacian, $\phi(x) \in R^N$ is dimensionless boson field and $\psi(x) \in R$ is the auxiliary field which has the dimension $(\text{length})^{-2}$. Let $G_0 = (-\Delta + m^2)^{-1}$. The mass parameter $m \sim e^{-2\pi\beta}$ is chosen so that $G_0(0) = \beta$. Define G_n and $C : R^{\Lambda_n} \to R^{\Lambda_{n+1}}$ by

$$
G_{n+1}(x,y) = (CG_nC^+)(x,y), \quad (Cf)(x) = \frac{1}{L^2}\sum_{z\in\Box_0} f(Lx+z) \qquad (3)
$$

where L is a positive integer and $\Box_0$ is the box of size $L \times L$ centered at the origin. We apply two types of block transformations C and $C' = L^2 C$ to $W_n(\phi_n, \psi_n)$. One is the block spin transformation $C : \phi_n(x) \to \phi_{n+1}(x) = (C\phi_n)(x)$, and the other is the block spin transformation $C' : \psi_n(x) \to \psi_{n+1}(x) = (C'\psi_n)(x)$. The main part of $W_n(\phi_n, \psi_n)$ is given by

$$
\begin{aligned}
W_n(\phi_n,\psi_n) &= \frac{1}{2}\langle \phi_n, G_n^{-1}\phi_n\rangle + \frac{1}{2}\gamma_n \langle \partial_\mu\phi_n, (\phi_n \otimes \phi_n)\partial_\mu\phi_n\rangle \\
&\quad + \langle \psi_n, H_n^{-1}\psi_n\rangle + \frac{i}{\sqrt{N}}\langle(\phi_n^2 - u_n), \psi_n\rangle \qquad (4)
\end{aligned}
$$

and the flow of W_n is parametrized by the mass parameters $m_n^2 \sim L^{2n} m_0^2$ in $G_n^{-1} \sim (-\Delta + m_n^2)$ and slowly changing parameters $\gamma_n \sim n/N$ and u_n. Here $H_n^{-1} = O(1) > 0$ is a local Hamiltonian of ψ_n and

$$(\phi_n \otimes \phi_n)_{(x,i),(y,j)} = \delta_{x,y} \phi_i(x) \phi_j(x) \tag{5}$$

$$u_n = N\beta_n, \quad \beta_n = \beta - \text{const.}\, n + o(n) \tag{6}$$

u_n is the position of the double-well of the potential. The most surprising term $\gamma_n \langle \partial_\mu \varphi_n, (\varphi_n \otimes \varphi_n) \partial_\mu \varphi_n \rangle$ means that the fluctuation field $\xi_n \sim \partial_\mu \phi_n$ is almost perpendicular to the block spin field ϕ_n since $\gamma_n \geq 0$ increases as $n \to \infty$. This term is a reminiscence of $\langle (\phi_k^2 - u_k), \psi_k \rangle$, $k \leq n$ and they sum up to yield γ_n. Fortunately, however, this term does not disturb the main stream of the RG flow. (This term was found in [2, 3]where it was not serious because of the hierarchical approximation. This was re-encountered in [5].)

These steps can be iterated [7] excluding large field regions. We expect [7] that this leads us to the conclusion given in the title of the talk. This idea may be applied to the study of the non-abelian lattice gauge theory [1].

References

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