First-principle Derivation of Stable First-order Relativistic Dissipative Hydrodynamic Equation in a Generic Frame from Boltzmann Equation by Renormalization-group Method

Kyosuke Tsumura[1] and Teiji Kunihiro[2]

[1] *Analysis Technology Center, Fujifilm Corporation, Kanagawa 250-0193, Japan*
[2] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

Relativistic hydrodynamic equations are utilized in various fields of physics, especially in high-energy nuclear physics[1)] and astrophysics,[2)] and it seems that the study of the relativistic hydrodynamic equation with *dissipative* effects is now becoming a central interest in these fields.

We should, however, note that a full understanding of the theory of relativistic hydrodynamics for viscous fluids is yet to be done, although there have been many important studies since Eckart's pioneering work,[3)] because of the instability[4)] and causal problems.[5)–7)]

There have been continuous attempts to derive the phenomenological equations from the relativistic Boltzmann equation to give a microscopic foundation or denial to them;[5),7),8)] for instance, with use of the Chapman-Enskog expansion method[9)] and the Maxwell-Grad moment method.[10)] Although the past works certainly succeeded in identifying the assumptions and/or approximations to reproduce the known hydrodynamic equations by Eckart, Landau and Lifshitz, Stewart, and Israel, the physical meaning and foundation of these assumptions/approximations remain obscure, and thus the uniqueness of those hydrodynamic equations has never been elucidated as the long-wavelength and low-frequency limit of the underlying dynamics. Their validity or the fundamental compatibility with the underlying Boltzmann equation has never been questioned nor addressed. This unsatisfactory situation rather reveals the incompleteness of the Chapman-Enskog expansion method and the Maxwell-Grad moment methods themselves as a reduction theory of the dynamics.

In this contribution, which is based on 11), we report a derivation first-order relativistic dissipative hydrodynamic equations[12),13)] from relativistic Boltzmann equation on the basis of the renormalization-group (RG) method.[14),15)]

We introduce a macroscopic-frame vector (MFV), which does not necessarily coincide with the flow velocity, to specify the local rest frame on which the macroscopic dynamics is described. The five hydrodynamic modes are naturally identified with the same number of the zero modes of the linearized collision operator, i.e., the collision invariants.

After defining the inner product in the function space spanned by the distribution function, the higher-order terms, which give rise to the dissipative effects, are constructed so that they are precisely orthogonal to the zero modes in terms of the inner product: Here, any ansatz's, such as the so-called conditions of fit or matching conditions which have to be imposed in the standard methods in an ad-hoc way, are not necessary.

We elucidate that the Burnett term dose not affect the hydrodynamic equations owing to the very nature of the hydrodynamic modes as the zero modes.

Then, applying the RG equation, we obtain the hydrodynamic equation in a generic frame specified by the MFV, as the coarse-grained and covariant equation. Our generic hydrodynamic equation reduces to hydrodynamic equations in various local rest frames, including the energy and particle frames with a choice of the MFV. We find that our equation in the energy frame coincides with that of Landau and Lifshitz, while the derived equation in the particle frame is slightly different from that of Eckart, owing to the presence of the dissipative internal energy.

We prove that the Eckart equation can not be compatible with the underlying relativistic Boltzmann equation. The proof is made on the basis of the observation that the orthogonality condition to the zero modes coincides with the ansatz's posed on the dissipative parts of the energy-momentum tensor and the particle current in the phenomenological equations.

We also present an analytic proof that all of our equations have a stable equilibrium state owing to the positive definiteness of the inner product.

References

1) See review articles, P. Huovinen, in *Quark gluon plasma*, ed. R.C. Hwa, and X.N. Wang, (World Scientific, Singapore), p.600 [arXiv:nucl-th/0305064];
P. F. Kolb and U. W. Heinz, in "Quark gluon plasma", ed. R.C. Hwa, and X.N. Wang, (World Scientific, Singapore), p.634 [arXiv:nucl-th/0305084].
2) As a review article, see, for example, D. Balsara, Astrophys. J. Suppl. **132** (2001), 83.
3) C. Eckart, Phys. Rev. **58** (1940),919; L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, (Pergamon Press, London, 1959).
4) W. A. Hiscock and L. Lindblom, Phys. Rev. D **31** (1985),725.
5) S. R. de Groot, W. A. van Leeuwen, and Ch. G. van Weert, *Relativistic Kinetic Theory* (Elsevier North-Holland, 1980.
6) W. Israel, Ann. Phys. **100** (1976),310.
7) W. Israel and J. M. Stewart, Ann. Phys. **118** (1979),341.
8) W. Israel, J. Math. Phys. **4** (1963),1163; J. M. Stewart, *Non-Equilibrium Relativistic Kinetic Theory*, Lecture Notes in Physics No. 10, (Springer, Berlin, 1971); Ch. G. van Weert, *Some Problems in Relativistic Hydrodynamics*, in Lecture Notes in Mathematics **1385**, p.290, ed. by A. Anile and Y. Choquet-Bruhat (Springer, 1987); N. G. van Kampen, J. of Stat. Phys. **46** (1987),709;
9) S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases*, (Cambridge U.P., 1939).
10) H. Grad, Comm. Pure Appl. Math. **2** (1949),331.
11) K. Tsumura, T. Kunihiro, [arXiv:1108.1519 [hep-ph]].
12) K. Tsumura, T. Kunihiro, and K. Ohnishi, Phys. Lett. B **646** (2007),134 [arXiv:hep-ph/0609056].
13) K. Tsumura and T. Kunihiro, Phys. Lett. B **668** (2008),425 [arXiv:0709.3645].
14) L. Y. Chen, N. Goldenfeld, and Y. Oono, Phys. Rev. Lett. **73** (1994),1311; Phys. Rev. E **54** (1996),376.
15) T. Kunihiro, Prog. Theor. Phys. **94** (1995),503; **95**(1996),835 (E); Jpn. J. Ind. Appl. Math. **14** (1997),51; T. Kunihiro, Prog. Theor. Phys. **97** (1997),179; S.-I. Ei, K. Fujii, and T. Kunihiro, Ann. Phys. **280** (2000),236; Y. Hatta and T. Kunihiro, Ann. Phys. **298** (2002), 24;. T. Kunihiro and K. Tsumura, J. Phys. A **39** (2006),8089; Teiji Kunihiro, *Buturi*, **65**(2010), 683 [in Japanese].