Ground States of the Two-Dimensional Spin Glass

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Abstract: This is joint work with Louis-Pierre Arguin, Michael Damron and Dan Stein (Commun. Math. Phys. 300 (2010) 641-657). It is an open problem to determine the number of infinite-volume ground states in the Edwards-Anderson (nearest neighbor) spin glass model on Z^d for $d \geq 2$ (with, say, mean zero Gaussian couplings). This is a limiting case of the problem of determining the number of extremal Gibbs states at low temperature. In both cases, there are competing conjectures for $d \geq 3$, but no complete results even for d=2. I report on results which go some way toward proving that (with zero external field, so that ground states come in pairs, related by a global spin flip) there is only a single ground state pair (GSP). Our result is weaker in two ways: First, it applies not to the full plane Z^2, but to a half-plane. Second, rather than showing that with probability one (with respect to the probability distribution of the quenched random coupling realization J) there is a single GSP, we show that there is a natural joint distribution on J and GSP's such that for a.e. J, the conditional distribution on GSP's given J is supported on only a single GSP. The methods used combine percolation-like geometric arguments with translation invariance (in one of the two coordinate directions of the half-plane) and uses as a main tool the "excitation metastate" which is a probability measure on GSP's and on how they change as one or more individual couplings vary.