A replica analysis of the 1D KPZ equation

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The Kardar-Parisi-Zhang equation is a well-known equation (nonlinear stochastic partial differential equation) which describes surface growth phenomena. The one-dimensionanl version reads

$$\partial_t h(x,t) = \tfrac{1}{2}\lambda(\partial_x h(x,t))^2 + \nu\partial_x^2 h(x,t) + \sqrt{D}\eta(x,t).$$

Here $x \in \mathbb{R}$ is a space coordinate, $t \geq 0$ is time and $h \in \mathbb{R}$ is the surface height at t and x. In addition $\eta(x,t)$ is a Gaussian white noise with $\langle \eta(x,t)\eta(x',t')\rangle = \delta(x-x')\delta(t-t')$. $\lambda, \nu, D > 0$ are the parameters of the equation. ν represents the strength of the diffusive relaxation, λ the nonliearity and D the noise.

Last year the height distribution for the solution of this equation was identified for the narrow wedge initial condition [1, 2]. The result was obtained by considering a certain weakly asymmetric limit of the asymmetric simple exclusion process (ASEP).

The same expression was derived by using the replica analysis of the related directed polymer problem [3]. The replica method has the advantage that it can be generalized to studies of multi point distributions and other initial conditions.

In this presentation we report on an analyis of the KPZ equation with the half Brownian motion initial condition [4].

References

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