Functional Integral Representation for Relativistic Schrödinger Operator Coupled to a Scalar Bose Field

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Abstract
In this paper the system of a quantum particle interacting with a scalar Bose field is investigated. The partilcle's Hamiltonian is given the relativistic Schrödinger operator with potential

$$H_{\mathrm{p}} = \sqrt{-\triangle + M^2} - M + V \tag{1}$$

A scalar Bose field is constructed by infinite dimensional stochastic process. The field operators $\{\phi(f)\}_{f \in \mathcal{K}_{\mathrm{b}}}$ are defined by the Gaussan random process indexed by a Hilbert space $\mathcal{K}_{\mathrm{b}}$ on a probability space $(Q_{\mathrm{b}}, \mathfrak{B}_{\mathrm{b}}, P_{\mathrm{b}})$. The state space is given by $L^2(Q_{\mathcal{K}_{\mathrm{b}}})$ and the free Bose Hamiltonian H_{b} is defined by the differential second quantization of $\omega_{\mathrm{b}}(-i\nabla)$ where ω_{b} is non-negative and continuous function. Physically $\omega_{\mathrm{b}}(\mathbf{k}) \geq 0$ denotes the one-particle energy of the field with momentum $\mathbf{k}$. Thus the triplet $\left(L^2(Q_{\mathcal{K}_{\mathrm{b}}}), H_{\mathrm{b}}, \{\phi(f)\}_{f \in \mathcal{K}_{\mathrm{b}}}\right)$ of the scalar Bose field is defined.

The state space of the interacting system is given by $\mathcal{H} = L^2(\mathbf{R}^d_{\mathbf{x}}) \otimes L^2(Q_{\mathrm{b}}) \simeq \int^{\oplus}_{\mathbf{R}^d} L^2(Q_{\mathrm{b}}) d\mathbf{x}$ where $\int^{\oplus}$ denotes the fibre direct integral. The free Hamiltonian is defined by $H_0 = H_{\mathrm{p}} \otimes I + I \otimes H_{\mathrm{b}}$ and the total Hamiltonian by

$$H_{\kappa} = H_0 \dot{+} \kappa \int^{\oplus}_{\mathbf{R}^d} P(\phi(\rho_{\mathbf{x}})) d\mathbf{x} \tag{2}$$

where $\dot{+}$ denotes the form sum, $P(\lambda) = \sum_{j=1}^{2n} c_j \lambda^j$, $c_j \in \mathbf{R}$, $j = 1, \cdots, 2n-1$, $c_{2n} > 0$, and the ultraviolet cutoff condition $\rho_x \in \mathcal{S}'_{\mathrm{real}}$ for each $\mathbf{x} \in \mathbf{R}^d$ is supposed.

In the main theorem, the functional integral representation of $e^{-tH_{\kappa}}$ is derived.