Study of 2D O(N) Sigma Model by Renormalization Group Method

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It is a longstanding problem to prove quark confinement in the framework of 4D latttice gauge field theory. This is unsolved yet, and a simiar 2D O(N) model is also a very hard problem. Here I show that a rigorous analysis of this model is possible, and establish our longstanding conjecture for large N. We start with the action

$$\begin{aligned} Z &= \int \exp[-W_0(\phi,\psi)] \prod d\psi d\phi \\ W_0 &= \frac{1}{2}\langle \phi, G_0^{-1}\phi\rangle + \frac{i}{\sqrt{N}}\langle(\phi^2 - N\beta, \psi\rangle \end{aligned}$$

where $G_0^{-1} = -\Delta + m_0^2$. The BST is the method to integerate e^{-W_n} recursively (thus $e^{-W_n} \to e^{-W_{n+1}} \to \cdots$) by high-momentum part ξ_n of ϕ_n and $\tilde{\psi}_n$ of ψ_n by decomposing

$$\phi_n = A_{n+1}\phi_{n+1} + Q\xi_n, \quad \psi_n = \tilde{A}_{n+1}\psi_{n+1} + Q\tilde{\psi}_n$$

where

$$\begin{aligned} \phi_{n+1}(x) &= (C\phi)_n(x) = \frac{1}{L^2}\sum_{\zeta\in\Delta_0})\phi_n(Lx+\zeta) \\ \psi_{n+1}(x) &= (C'\psi)_n(x) = L^2(C\psi)(x) = \sum_{\zeta\in\Delta_0})\psi_n(Lx+\zeta) \end{aligned}$$

and Δ_0 is the square of size $L\times L$ centered at the origin, $2 \le L$. This consists of averaging of spins over the blocks Δ_{Lx} (box centered at Lx) and scaling down ($Lx \to x$) of the coordinate $\Lambda_0 \subset Z^2 \to \Lambda_1 = L^{-1}\Lambda_0 \cap Z^2$. Q is the matrix to form zero-average fluctuations $Q\xi_n$ and $Q\tilde{\psi}_n$ which we integarte out (Wilson's idea). A_n and $\tilde{A}$ are chosen so that the main gaussian terms of W_n are diagobalized:

$$\begin{aligned} \langle \phi_n, G_n^{-1}\phi_n\rangle &= \langle \phi_{n+1}, G_{n+1}^{-1}\phi_{n+1}\rangle + \langle \xi_n, Q^+G_n^{-1}\xi_n\rangle \\ \langle \psi_n, H_n^{-1}\psi_n\rangle &= \langle \psi_{n+1}, H_{n+1}^{-1}\psi_{n+1}\rangle + \langle \tilde{\psi}_n, Q^+H_n^{-1}Q\tilde{\psi}_n\rangle \end{aligned}$$

We prove that W_n is given by the following form *outside the domain-walls D_w which have high-energy (thus with small probability)* –subtle to define–:

$$\begin{aligned} W_n &= \frac{1}{2}\langle \phi_n, G_n^{-1}\phi_n\rangle + \frac{i}{\sqrt{N}}\langle(\phi_n^2 - N\beta_n, \psi\rangle \\ &\quad + \frac{1}{2}\langle \psi_n, H_n^{-1}\psi_n\rangle + \gamma_n\langle \phi_n^2, E^\perp G_n^{-1}E^\perp\phi_n^2\rangle \end{aligned}$$

where $E^\perp$=projection to the block-wise zero average functions. Thus the last two terms are shown to be irrelevant since $E^\perp$ acts as a differentiation. Moreover we can show $\beta_n = \beta - cn(\to 0)$ and $\gamma_n = O((N\beta)^{-1}$. This establishes non-existence of phase transition in the model.