## Block Spin Transformations of 2D Sigma Model, Toward Solving a Millennium Problem K.R.Ito

Abstract: We analyze 2D O(N) sigma model by block spin transformation:

$$\exp\left[-\frac{1}{2}\langle\phi,G_0^{-1}\phi\rangle-\frac{g_0}{2N}\sum_x(\phi^2(x)-N\beta)^2\right]$$

where  $\phi(x) = (\phi_1(x), \dots, \phi_N(x)), G_0^{-1} = -\Delta + m_0^2$  and  $-\Delta$  is the lattice laplacian on  $Z^2$ . Put  $\phi_0 = \phi$  and recursively define block spins

$$\phi_{n+1}(x) = \frac{1}{L^2} \sum_{|\zeta| < L/2} \phi_n(Lx + \zeta)$$

We represent  $\phi_n$  in terms of block-spin  $\phi_{n+1}$  of next order and fluctuations  $z_n$ .  $z_n$  is a massive Gaussian but gets strong effects of back ground field  $\phi_{n+1}$ . We define a new notion of domain walls in the sigma model which has O(N) symmetry. The domain wall regions  $D_w$  have high energies, and on  $(D_w)^c$  we can safely implement the block spin transformations. Thus we obtain the renormalization group flow of the 2D sigma model, which enables us to prove our long-standing claim.