【RIMS 合宿型セミナー】

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② 題　目 : シンプレクティック代数幾何

(英 文 名 : Symplectic Algebraic Geometry)

③実施期間 : 平成 25 年 9 月 30 日～平成 25 年 10 月 4 日(5 日間)

④参加者数 : 18 名 (内、外国人 9 名)

⑤講 演 数 : 18 コマ (内、英語で行なわれたもの 18 コマ)

⑥合宿型セミナーの概要 (開催目的、成果など) :

参加者全員が８０分の講演をした。今回の研究集会では、大雑把にいうと３つのトピックの研究者を招待した。トピックの１つは、非コンパクトな複素シンプレクティック多様体、２つ目は、コンパクトな複素シンプレクティック多様体、最後に、ポアソン幾何と generalized geometry である。 招待研究者はそれぞれ３つのトピックのどれかに重点を置いて研究しているが、今回の研究集会は良い研究交流の場を提供したと自負している。並河、Kaledin, Fu, 永井は、非コンパクト複素シンプレクティック多様体に関する講演をおこなった。 齋藤の講演も非コンパクト複素シンプレクティック多様体を扱ったものだが、パンルベ方程式の研究が動機になっている。 Markman, Addington は超ケーラー多様体の導来圏に関する講演をおこなった。Verbitsky、Hwang は各々、小林擬距離、VMRT(Variety of minimal rational tangent) の観点から複素シンプレクティック多様体を論じた。小木曽、向井は、超ケーラー多様体やＫ３曲面の自己同型群について論じた。松下は、超ケーラー多様体のラグランジアンファイブレーションに関して、吉岡は generalized Kummer variety の双有理幾何について話した。 Lehn とその共同研究者による、cubic 4-fold の twisted cubic の Hilbert 概形の研究も参加者の興味を引いていたようである。最後に、Gualtieri, Cavalcanti, 後藤, 藤木４人の（複素）微分幾何学者の参加によって、今回の合宿型セミナーはより多角的、重層的なものになったことを付記しておきたい。

研究成果の公表方法

⑦ 講究録を　☐ 発行する　☑ 発行しない
※発行する場合: 原稿完成予定時期　平成　年　月　日頃

⑧ 講究録以外の方法で報告集を発行する場合 :
タイトル:
出 版 社:　出版予定時期 : 平成　年　月　日頃

⑨ 専門誌等による場合 :
主要な論文リスト (掲載予定、プレプリントを含む。準備中も可)

※２～３ページ目に合宿型セミナー実施プログラム添付 (Ａ４版)

Symplectic Algebraic Geometry

1. Schedule

29, September

18:00- Dinner

30, September:

8:00 - Breakfast

9:00 - 10:20 **Shigeru Mukai**

10: 50 - 12:10 **Manfred Lehn**

12:30 - Lunch

14:30 - 15:50 **Ryushi Goto**

16:20 - 17:40 **Jun-Muk Hwang**

18:00- Dinner

1, October:

8:00 - Breakfast

9:00 - 10:20 **MasaHiko Saito**

10: 50 - 12:10 **Kota Yoshioka**

12:30 - Lunch

14:30 - 15:50 **Baohua Fu**

16:20 - 17:40 **Dmitry Kaledin**

18:00- Dinner

2, October:

8:00 - Breakfast

9:00 - 10:20 **Nicolas Addington**

10: 50 - 12:10 **Misha Verbitsky**

12:30 - Lunch

Free discussion

18:00- Dinner

3, October:

8:00 - Breakfast

9:00 - 10:20 **Marco Gualtieri**

10: 50 - 12:10 **Gil Cavalcanti**

12:30 - Lunch

14:30 - 15:50 **Yasunari Nagai**

16:20 - 17:40 **Yoshinori Namikawa**

18:00- Dinner

4, October:

8:00 - Breakfast

9:00 - 10:20 **Eyal Markman**

10: 50 - 12:10 **Daisuke Matsushita**

12:30 - Lunch

14:30 - 15:50 **Keiji Oguiso**

16:20 - 17:40 **Akira Fujiki**

18:00- Dinner

5, October

8:00- Breakfast

2. Titles and Abstracts

Nicolas Addington: On derived categories of moduli spaces of torsion sheaves on K3

If S is a K3 surface and M a moduli space of certain torsion sheaves on S, then using Arinkin's work on compactified Jacobians I can show that the functor $D(S) \to D(M)$ induced by the universal sheaf is a "$\mathbf{P}^n$-functor." This is a relative version of Huybrechts-Thomas's $\mathbf{P}^n$-objects, and like those it yields an autoequivalence of $D(M)$. More interestingly, it embeds $D(S)$ as a (non-full) subcategory of $D(M)$, and the Mukai lattice of S as a sublattice of the topological K-theory of M, and both may deform as M deforms into a general hyperkaehler manifold. In earlier work I showed that the analogous functor from $D(S) \to D(Hilb^n(S))$ from is a $\mathbf{P}^n$-functor, and conjectured that it would be so for any moduli space of sheaves on S. This is joint work with Ciaran Meachan.

Gil Cavalcanti: Examples and counter-examples of log-symplectic manifolds

We study topological properties of log-symplectic structures and produce examples of compact manifolds with such structures. Notably we show that several symplectic manifolds do not admit log-symplectic structures and several log-symplectic manifolds do not admit symplectic structures, for example $\sharp m\mathbf{C}P^2 \sharp n\mathbf{C}P^2$ has log-symplectic structures if and only if $m, n > 0$ while they only have symplectic structures for $m = 1$. We introduce surgeries that produce log-symplectic manifolds out of symplectic manifolds and show that for any simply connected 4-manifold M, the manifolds $M\sharp(S^2 \times S^2)$ and $M\sharp\mathbf{C}P^2\sharp\mathbf{CP}^2$ have log-symplectic structures and any compact oriented log-symplectic four-manifold can be transformed into a collection of symplectic manifolds by reversing these surgeries.

Baohua Fu: Generic singularities of nilpotent orbit closures and a conjecture of Lusztig

I shall report a joint work with Juteau, Levy and Sommers, in which we determined generic singularities of nilpotent orbit closures in exceptional Lie alegebras. As an application, we proved a sliced version of a conjecture of Lusztig on the geometry of special pieces.

Akira Fujiki: TBA

Ryushi Goto: Generalized complex structures and generalized Calabi-Yau metrics

I will discuss recent results on generalized geometry.

Marco Gualtieri: Holomorphic Dirac structures in generalized Kahler geometry

In studying generalized Kahler geometry from a holomorphic point of view, one finds a number of interesting structures, including holomorphic Poisson structures as well as holomorphic Courant algebroids and Dirac structures. I will explain this point of view and show how it can be used to study certain canonical deformations of a generalized Kahler structure.

Jun-Muk Hwang: Dual cones of varieties of minimal rational tangents

The varieties of minimal rational tangents play an important role in the geometry of uniruled projective manifolds. The goal of this talk is to exhibit their role in the symplectic geometry of the cotangent bundles of uniruled projective manifolds. More precisely, let X be a uniruled projective manifold satisfying the assumption that the VMRT at a general point is smooth. We show that the total family of dual cones of the varieties of minimal rational tangents is a coisotropic subvariety in $T^*(X)$. Furthermore, the closure of a general leaf of the null foliation of this coisotropic subvariety is an immersed projective space of dimension $d+1$ where d is the dual defect of the variety of minimal rational tangents at a general point. When $d = 0$, the symplectic reduction of the coisotropic variety can be realized as a subbundle of the cotangent bundle $T^*(M)$ of the parameter space M of the rational curves.

Dmitry Kaledin : Weil-Peterson metric, Simpson theory and mixed Hodge structures

I am going to describe an old subject that seems to be not well-known, so that it is perhaps worthwhile to revisit it. We know from the work of Carlos Simpson that the moduli space of flat connections on a projective algebraic variety X carries a natural $U(1)$-invariant hyperkahler metric. It is an observation of Deligne and Simpson that such a metric canonically corresponds to a mixed Hodge structure of a certain prescribed type on the algebra of germs of formal function near a point. On the other hand, as was realized later, $U(1)$-invariant hyperkahler metrics correspond to kahler forms on the set of $U(1)$-fixed points – in this case, to the Weil-Peterson metric on the moduli space of bundles on X. We thus obtain a Hodge-theoretic interpretation of the Weil-Peterson metric; how it is related to other standard construction of this metric seems to be an interesting open question.

Manfred Lehn: Twisted cubics on cubic fourfolds

The moduli space M of (generalised) twisted cubics on a smooth cubic fourfold Y that does not contain a plane admits a contraction $M \to Z$ to a projective smooth 8-dimensional irreducible holomorphic symplectic manifold Z. (This is joint work with Christian Lehn, Christoph Sorger and Duco van Straten).

Eyal Markman : A global Torelli theorem for rigid hyperholomorphic sheaves.

Let X be an irreducible holomorphic symplectic manifold of $K3^{[n]}$ deformation type. There exists over $X \times X$ a rank $2n - 2$ rigid and stable sheaf A of Azumaya algebras, constructed in arXiv:1105.3223. The characteristic classes of A were used to prove the standard conjectures when X is algebraic (joint with F. Charles). The sheaf A is also used to associate to X a generalized (non-commutative) deformation of the derived category of a K3 surface (joint work in progress with S. Mehrotra).

The sheaf A is constructed over $X \times X$ as a deformation of the following sheaf A_0. Let M be a smooth and compact moduli space of stable sheaves over a K3 surface S, and let A_0 be the reflexive sheaf, whose fiber over a pair (E, F) in $M \times M$, of non-isomorphic stable sheaves E and F, is $\mathrm{End}[\mathrm{Ext}^1(E, F)]$.

We will describe in this talk the following uniqueness result (joint with S. Mehrotra): Consider the forgetful (surjective) morphism from the moduli space of pairs (X, A) to the moduli space of X. We show that two points in the same fiber are inseparable. Furthermore, the generic fiber consists of a single pair (X, A).

Daisuke Matsushita: On base manifolds of Lagrangian fibration of singular symplectic manifolds.

Let X be a projetive symplectic variety which satisfies the following four conditions 1) X has only Q-factorial terminal singularities. 2) $h^1(X, O_X) = 0$, 3) $h^2(X, O_X) = 1$, and 4) $h^{1/2 \dim X}(X, O_X) \leq 1$. Assume that X admits a surjective morphism $f : X \to S$ which has only connected fibres. Extending Prof. Hwang's proof, we prove that f is Lagrangian fibration and S is isomorphic to a projective space under the additional assumption of smoothness of S.

Shigeru Mukai: Enriques surfaces and cubic 4-folds with action of M_{10}

An Enriques surface with action of M_{10} was found lattice theoretically in my joint work with Hisanori Ohashi, where M_{10} is a group of order 6! containing the alternating group A_6. Its universal cover is the K3 surface of Keum-Oguiso-Zhang constructed from Leech roots. In this talk I will describe the Enriques surface more explicitly and discuss the cubic 4-folds associated with it in the sense of Dolgachev-Markushevich.

Yasunari Nagai: Symmetric product of a semistable degeneration of surfaces

As a related topic of degenerations of holomorphic symplectic manifolds, we will discuss the symmetric product of a semistable degeneration of surfaces, in particular the structure of the singularity.

Yoshinori Namikawa: Poisson deformations and Mori dream spaces

Let $\pi : Y \to X$ be a crepant projective resolution of an affine symplectic variety X with a good $\mathbf{C}^*$-action. We prove that π is a relative Mori dream space in a different point of view from Hu and Keel. As usual the ample cones of different crepant resolutions of X determine chambers inside $H^2(Y, \mathbf{R})$. On the other hand, one can identify $H^2(Y, \mathbf{C})$ with the base spaceof the universal Poisson deformation of Y. Our main theorem asserts that the chambers determined by the ample cones completely agree with those determined by the discriminant locus of the universal Poisson deformation.

Keiji Oguiso: Automorphisms of compact hyperkähler manifolds - some recent progress

MasaHiko Saito: Apparent singularities and canonical coordinates for the moduli spaces of singular connections (a joint work with S. Szabo)

This is a part of joint works with S. Szabo. The moduli spaces of stable parabolic connections on a smooth projective curve with fixing the formal structures of singularities can be constructed as nonsingular quasi-projective symplectic schemes by works of Inaba and Saito. We will give a systematic way of introducing canonical coordinates for such moduli spaces. By these descirption, one can show that the moduli spaces are birational to the Hilbert schemes of the points of the total spaces of certain line bundles. Moreover for special case, one can also show that the more detail structures of the moduli spaces.

Misha Verbitsky: Ergodic complex structures and Kobayashi metric

Let M be a compact manifold. Consider the action of the diffeomorphism group $Diff(M)$ on the (infinite-dimensional) space $Comp(M)$ of complex structures. A complex structure is called ergodic if its $Diff(M)$-orbit is dense in the connected component of $Comp(M)$. I will show that on a hyperkaehler manifold or a compact torus, a complex structure is ergodic unless its Picard rank is maximal. This result has many geometric consequences; for instance, it follows that the Kobayashi pseudometric on any K3 surface or the deformations of its Hilbert scheme vanishes, solving a longstanding conjecture by Kobayashi.

Kota Yoshioka: Examples of the cones of generalized Kummer manifolds

I will explain the movable cone of the generalized Kummer manifolds. I also explain a few examples for a product of two non-isogeneous elliptic curves.