

Universal Character and q -Painlevé Equations

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The universal character, defined by K. Koike [1], is a polynomial attached to a pair of partitions and is a generalization of the Schur polynomial. In this talk, we introduce an integrable system of q -difference equations satisfied by the universal character, and call it the *(lattice) q -UC hierarchy*. We regard it as generalizing the q -KP hierarchy. Suitable similarity and periodic reductions of the hierarchy yield the q -difference Painlevé equations of types $A_{2g+1}^{(1)}$ ($g \geq 1$), $D_5^{(1)}$, and $E_6^{(1)}$. As its consequence, a class of algebraic solutions of the q -Painlevé equations is rapidly obtained by means of the universal character. The reduction procedure above is clearly understood in the level of τ -functions. So, we shall also present an algebro-geometric approach to τ -functions of the q -Painlevé equations.

References and Literature for Further Reading

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