RIMS Workshop

Discrete Convexity and Optimization

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Research Institute for Mathematical Sciences
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Minimization Algorithms for Discrete Convex Functions

Akiyoshi Shioura (Tohoku University)

The two discrete convexity concepts called M-convexity and L-convexity play primary roles in the theory of discrete convex analysis. In this talk, we consider (unconstrained and constrained) minimization of M-/L-convex functions on the integer lattice points. We explain various algorithmic approaches for unconstrained minimization of M-/L-convex functions. We also present some recent results on the approximation algorithms for constrained minimization problems of M-convex functions based on continuous relaxation approach.

Discrete Convexity in Supply Chain Models

S. Thomas McCormick (University of British Columbia)

Discrete convexity has found application in several supply chain models. We survey some of these models, with a view towards making the supply chain context clear to non-specialists. In particular, we look at various attempts to model inventory problems in supply chains using discrete convexity. We focus in particular on the paper "Order-Based Cost Optimization in Assemble-to-Order Systems" by Lu and Song (OR 2005). Joint work with Aref Bolandnazar, Tim Huh, and Kazuo Murota.

On the 3-Dimensional Rigidity Matroid

Tibor Jordán (Eötvös Loránd University, Budapest)

In this survey talk we study those families of graphs for which the rank in the 3-dimensional rigidity matroid is well characterized. We also give an overview of general lower and upper bounds for the rank and discuss several related conjectures.

Highly Connected Rigidity Matroids and Rigidity of Hypergraphs in Various Spaces

Viktória E. Kaszanitzky (Eötvös Loránd University, Budapest)

Let $\mathcal{M}$ be a d-dimensional generic rigidity matroid of some graph G. We consider the following problem, posed by Brigitte and Herman Servatius in 2006: is there a (smallest) integer k_d such that the underlying graph G of $\mathcal{M}$ is uniquely determined, provided that $\mathcal{M}$ is k_d-connected? Since the one-dimensional generic rigidity matroid of G is isomorphic to its cycle matroid, a celebrated result of Hassler Whitney implies that $k_1 = 3$. We extend this result by proving that $k_2 \leq 11$.

Consider a hypergraph $H = (V, E)$ and two maps $p, q : V \to \mathbb{R}^d$. The pair (H, p) is said to be equivalent to (H, q) if the positions in p of the vertices of each hyperedge can be mapped to their positions in q by an affine map of $\mathbb{R}^d$. The pairs are congruent if the positions of all the vertices in p can be mapped to their positions in q by a single affine map of $\mathbb{R}^d$. The pair (H, p) is *affinely rigid* in $\mathbb{R}^d$ if there is a small neighbourhood in the configuration space so that for any (H, q) in this neighbourhood we have that if (H, q) is equivalent to (H, p) then (H, p) and (H, q) are congruent. For generic maps p affine rigidity depends only on H. Relying on a result of W. Whiteley, we characterize the generically affinely rigid hypergraphs. We also give inductive constructions for low-dimensional minimally generically affinely rigid hypergraphs.

The rigidity of hypergraphs in projective spaces can be defined similarly. George and Ahmed conjectured that a 4-uniform hypergraph H is rigid in the one-dimensional projective space if and only if it satisfies a certain sparsity condition. Using the inductive construction above we can prove their conjecture.

M-Convex Functions on Jump Systems: A Survey

Kazuo Murota (University of Tokyo)

In discrete convex analysis, the concept of M-convex function was defined originally for functions on base polyhedra, as a natural extension of valuated matroids, which are well-behaved nonlinear functions defined on matroid bases introduced by Dress and Wenzel. The subsequent generalization to $M^\natural$-convex functions, due to Murota and Shioura, turned out to be convenient both in theory and in application, but these two concepts are to be understood as equivalent variants, transformed to each other through a simple projection operation.

A further generalization of M-convex functions to functions on even-parity jump systems makes an essential difference. This generalization sheds a new light on some graph-theoretic problems. For example, odd-cycle symmetry condition in the even factor problem can be formulated as M-convexity on jump systems (due to Kobayashi and Takazawa). Some important properties of M-convex functions on base polyhedra survive in this generalization: a greedy (or steepest descent) algorithm works, and operations such as infimal convolution and network transformation preserve M-convexity. Some other properties related to discrete duality are lost in this generalization.

Scaling Limits for Optimization in Random Graphs Using Combinatorial Interpolation

Prasad Tetali (Georgia Institute of Technology)

For a variety of models, including independent sets, MAX-CUT, Coloring and K-SAT, we prove that the free energy both at a positive and zero temperature, appropriately rescaled, converges to a limit as the size of the underlying graph diverges to infinity. In the zero temperature case, this is interpreted as the existence of the scaling limit for the corresponding combinatorial optimization problem. For example, we prove that the size of a largest independent set in these graphs, normalized by the number of nodes converges to a limit w.h.p. This resolves an open problem proposed by Aldous, and conjectures by Bollobas-Riordan, and Janson-Thomason. Computing the limit however remains a challenging open problem in most cases.

Our approach is based on extending and simplifying the interpolation method of Guerra and Toninelli, Franz and Leone. We also establish the large deviations principle for the satisfiability property of the constraint satisfaction problems – Coloring, K-SAT and NAE-K-SAT – in sparse random graphs. This is joint work with Mohsen Bayati (Stanford) and David Gamarnik (MIT).

Restricted b-Matchings

Kristóf Bérczi (Eötvös Loránd University, Budapest)

A $C_{(\leq)k}$-free 2-matching/factor is a 2-matching/factor not containing cycles of length (at most) k. The problem of finding a maximum subgraph with this property has been extensively studied because of its many applications, such as the undirected node-connectivity augmentation or the Hamiltonian cycle problem. However, we face NP-hard problems even for small values of k. Papadimitiu showed that deciding the existence of a $C_{\leq k}$-free 2-factor is NP-complete for $k \geq 5$. Vornberger proved that the problem is NP-complete even in cubic graphs, and the weighted version is NP-hard for cubic graphs and $k = 4$.

From now on, C_3-and C_4-free 2-matchings are called triangle- and square-free, respectively. Hartvigsen proposed an algorithm for the triangle-free case (in his PhD thesis), while Nam gave a polynomial algorithm for the square-free 2-matching problem in the special case when the squares are node-disjoint. However, the existence of a $C_{\leq 4}$-free or C_4-free 2-matching in general is still open.

Considering the maximum weight version of the problems, there is a firm difference between triangle- and square-free 2-factors. Király showed that finding a maximum weight square-free 2-factor is NP-complete even in bipartite graphs with $0-1$ weights. This result implies that we should not expect a nice polyhedral description of C_k-free or $C_{\leq k}$-free 2-factor polytope when $k \geq 4$.

According to the above, two cases remain: the unweighted $C_{\leq 4}$-free 2-factor and the weighted C_3-free 2-factor problems. These problems are unsolved to the present day. Yet imposing the condition that the graph is subcubic (that is, the maximum degree of G is 3) or bipartite, these problems become solvable.

Concerning subcubic graphs, polynomial-time algorithms were given by Hartvigsen and Li, and by Kobayashi for the weighted C_3-free 2-factor problem with an arbitrary weight function. Sqaure-free 2-matching in bipartite graphs have been studied by Hartvigsen, Kir疝y, Pap and Takazawa. By observing that a square is the complete bipartite graph $K_{2,2}$, Frank and Makai extended the results in bipartite graphs to $K_{t,t}$-free t-matchings. This was further generalized to non-bipartite graphs by Bérczi and Végh, and Kobayashi and Yin. The square-free 2-matching problem in subcubic graphs was solved by Bérczi and Kobayashi.

The proof of the latter result is based on the fact that the square-free 2-matchings are endowed with a matroid structure called a jump ystem. It was further showed that the weighted square-free 2-matching problem in simple subcubic graphs can be solved in polynomial time if the weight function is vertex-induced on every square. On the other hand, the problem is NP-hard for general weights. The algorithm for the weighted problem uses the theory of M-concave (M-convex) functions on constant-parity jumpsystems introduced by Murota.

Discrete Convexity in Network Optimization: Matching Forests and Bibranchings

Kenjiro Takazawa (Kyoto University)

Discrete convexity often appears in combinatorial optimization problems and is recognized as an indicator of tractability of those problems. In this talk, we present new examples of network optimization problems which involve discrete convexity.

The matching forest problem, introduced by Giles (1982), is a common generalization of the matching and branching problems. The bibranching problem, introduced by Schrijver (1982), is a common generalization of the edge cover problem in bipartite graphs and the branching problem. For both of these problems, polyhedral description with total dual integrality and polynomial time algorithms for the weighted case are known.

In this talk, beginning with the exchange property of branchings by Schrijver, we reveal discrete convexity underlying these two problems. For matching forests, we show that the degree sequences of matching forests form a delta-matroid and the weighted matching forests induce a valuated delta-matroid. For bibranchings, we show that the shortest bibranching problem is polynomially reducible to the valuated matroid intersection problem. We also discuss algorithmic aspects of these facts.

The Complexity of Valued CSPs

Stanislav Živný (University of Warwick)

The topic of this talk is Valued Constraint Satisfaction Problems (VCSPs) and the question of how VCSPs can be solved efficiently. This problem can also be cast as how to minimise separable functions efficiently. I will present algebraic tools that have been developed for this problem and will also mention a recent result on the connection between linear programming and VCSPs (based on a paper with J. Thapper, to appear in FOCS'12).

Discrete Convexity and Polynomial Solvability in Minimum 0-Extension Problems

Hiroshi Hirai (University of Tokyo)

The minimum 0-extension problem **0-Ext**$[\Gamma]$ on a graph Γ is: given a set V including the vertex set V_Γ of Γ and a nonnegative cost function c defined on the set of all pairs of V, find a 0-extension d of the path metric d_Γ of Γ with $\sum_{xy} c(xy)d(x,y)$ minimum, where a 0-extension is a metric d on V such that the restriction of d to V_Γ coincides with d_Γ and for all $x \in V$ there exists a vertex s in Γ with $d(x,s) = 0$. **0-Ext**$[\Gamma]$ includes a number of basic combinatorial optimization problems, such as minimum (s,t)-cut problem and multiway cut problem.

Karzanov proved the polynomial solvability for a certain large class of modular graphs in connection with multicommodity flow problems, and raised the question: What is Γ for which **0-Ext**$[\Gamma]$ can be solved in polynomial time? He also proved that **0-Ext**$[\Gamma]$ is NP-hard if Γ is not modular or not orientable (in a certain sense).

In this paper, we prove the converse: if Γ is orientable modular, then **0-Ext**$[\Gamma]$ can be solved in polynomial time.

This completes the classification of the tractable graphs for the 0-extension problem.

To prove our main result, we develop a theory of discrete convex functions on orientable modular graphs, analogous to discrete convex analysis by Murota, and utilize a recent result of Thapper and Zivny on Valued-CSP.

Generic Rigidity with Forced Symmetry

Louis Theran (FU Berlin)

This talk will survey some recent advances in characterizing generic rigidity properties of 2d-diensional frameworks with forced symmetry. Along the way, I will discuss direction networks with symmetry constraints and their connections to matroids.

Joint work with Justin Malestein.

Rigidity of Graphs with Symmetry

Shin-ichi Tanigawa (Kyoto University)

Laman's theorem is one of fundamental results in rigidity theory, which combinatorially characterizes the rigidity of two-dimensional frameworks on generic points (i.e., graphs generically embedded on the plane). The generic assumption is however too restrictive in many applications, and relaxing "genericity" is getting much attention these days. In particular, toward understanding the rigidity/flexibility of biomolecules, graphs (and frameworks) with certain group symmetry are recognized as important cases. In the talk I will discuss some recent progress on this topic, with focus on matroids of group-labeled graphs and symmetry-preserving operations.

Discrete Convexity and Unimodularity

Gleb Koshevoy (CEMI, Russian Academy of Sciences)

For a maximal unimodular collection $\mathcal{R}$ of vectors in $\mathbb{R}^n$ we define two classes of discrete convex sets. One class is constituted from integer polyhedra with direction of faces being flats of $\mathcal{R}$, and another class is constituted from polyhedra those facets have normal vectors in $\mathcal{R}$. For each class of discrete convex sets we define a corresponding class of discretely convex functions $\mathcal{R}F$ and $\mathcal{R}^*F$, respectively. For the graphical unimodular collection $\mathbb{A}_n = \{\pm e_i, e_i - e_j\}$ such theory developed in depth by Kazuo Murota. Subclasses of integer-valued functions form a dual pair with respect to the Fenchel duality. For both classes, there holds the property that a local minimum coincides with a global one. For a function $F \in \mathcal{R}F$ and a vector $m \in \mathbb{Z}^n$, a task of finding a vector $r \in \mathcal{R}$ such that $F(m+r) \leq F(m+r')$ for any $r' \in \mathcal{R}$

is of complexity $O(n^2)$. For a function $G \in \mathcal{R}^*F$ and a vector $z \in \mathbb{Z}^n$, a task of finding a crossing $\xi \in \mathcal{R}^\vee$ (crossings are primitive vectors of 1-dimensional chambers of arrangement of hyperplanes $r(x) = 0$, $r \in \mathcal{R}$) such that $G(z + \xi) \leq G(z + \xi')$ is equivalent to minimization of an $\mathcal{R}$-submodular function.

If time permit, I will explain a relation of a subclass of TP-functions of $\mathbb{A}_n F$ and Kashiwara's crystlas.

Even Cycle Decompositions of Graphs with No Odd K_4-Minor

Sang-il Oum (KAIST)

An even cycle decomposition of a graph G is a partition of E(G) into cycles of even length. Evidently, every Eulerian bipartite graph has an even cycle decomposition. Seymour [circuits in planar graphs. J. Combin. Theory Ser. B, 31(3): 327–338, 1981] proved that every 2-connected loopless Eulerian planar graph with an even number of edges also admits an even cycle decomposition. Later, Zhang [On even circuit decompositions of Eulerian graphs. J. Graph Theory, 18(1): 51–57, 1994] generalized this to graphs with no K5-minor. We propose a conjecture involving signed graphs which contains all of these results. Our main result is a weakened form of this conjecture. Namely, we prove that every 2-connected loopless Eulerian odd-K4-minor free signed graph with an even number of odd edges has an even cycle decomposition.

I'll mention another result motivated by the above conjecture. A graph is strongly even cycle decomposable if every subdivision with even number of edges is even cycle decomposable. We prove several composition operations that preserve the property of being strongly even cycle decomposable.

The first part is a joint work with Tony Huynh and Maryam Verdian-Rizi. The second part is a joint work with Tony Huynh, Andrew D. King, and Maryam Verdian-Rizi.

Minimally Non-ideal Clutters and Set Functions

Tamás Király (Eövös Loránd University, Budapest)

A fundamental theorem about clutters and covering problems is Lehman's Theorem on the properties of minimally non-ideal clutters. In the talk I discuss possible extensions of the notion of idealness to set functions, and give some partial results, examples and conjectures, as well as a characterization in terms of convex extensions. Joint work with Júlia Pap.

Iterative Rounding Approximation Algorithms for Degree-Bounded Node-Connectivity Network Design

Takuro Fukunaga (Kyoto University)

We consider the problem of finding a minimum edge cost subgraph of an undirected or a directed graph satisfying given connectivity requirements and degree bounds $b(v)$ on nodes v. We present an iterative rounding algorithm of the set-pair LP relaxation for this problem. When the graph is undirected and the connectivity requirements are on the element-connectivity with maximum value k, our algorithm computes a solution that is an $O(k)$-approximation for the edge cost in which the degree of each node v is at most $O(k)b(v)$. We also consider the no edge cost case where the objective is to find a subgraph satisfying connectivity requirements and degree bounds. Our algorithm for this case outputs a solution in which the degree of each node is at most $O(1)b(v) + O(k^2)$. These algorithms can be extended to other well-studied undirected node-connectivity requirements such as uniform, subset, and rooted k-connectivity. When the graph is directed and the connectivity requirement is k-out-connectivity from a root, our algorithm computes a solution that is a 2-approximation for the edge cost in which the degree of each node v is at most $2b(v) + O(k)$. This is a joint work with R. Ravi (Carnegie Mellon University).