Mini-Workshop "Arithmetic Geometry of Covers of Curves and Related Topics"

Abstracts

Keisuke Arai
On uniform lower bound of the Galois images associated to elliptic curves

Let $\rho_{E,p} : G_K \longrightarrow \mathrm{Aut}(T_pE) \cong GL_2(\mathbb{Z}_p)$ be the Galois representation determined by the Galois action on the p-adic Tate module of an elliptic curve E over a number field K. Serre showed that if E has no complex multiplication then $\rho_{E,p}$ has an open image. In this talk, we give an explicit uniform lower bound of the images of $\rho_{E,p}$ except a finite number of j-invariants for fixed K, p and varying E's.

Anna Cadoret
Descent theory for covers and rational points on Hurwitz spaces

Given an integer $r \geq 3$, we prove there exists a bound $d_r \geq 1$ depending only on r such that any k-rational point $\mathbf{p}$ of a reduced inner Hurwitz space can be lifted to a $k(\mathbf{p})$-rational point on the original inner Hurwitz space with $[k(\mathbf{p}) : k] \leq d_r$. This result can also be generalized to towers of Hurwitz spaces with bounded dimension. Introducing a new Galois invariant for G-covers - we call the base invariant - we improve this result for G-covers with a non trivial base invariant. For the sublocus corresponding to such G-covers the bound d_r can be chosen depending only on the base invariant (and no longer on r) and ≤ 6. Refining our method when $r = 4$, we obtain effective criteria to lift k-rational points from reduced inner to inner Hurwitz spaces. This provides in particular a new rigidity criterion and a new genus 0 method to reralize regularly finite group over $\mathbb{Q}$.

Mohamed Saïdi
Galois covers and semi-stable reduction of curves I, II

Let R be a complete discrete valuation ring with fraction field K, and residue field k, which we assume to be algebraically closed of characteristic $p \geq 0$. Let X be a proper and smooth R-curve with generic fibre $X_\eta := X \times_R K$ and special fibre $X_s := X \times_R k$. We denote by $\bar{X}_\eta := X \times_K \bar{K}$ the geometric generic fibre of X, where $\bar{K}$ is an algebraic closure of K. It is well known (by Grothendieck) that there exists a continuous specialization map $\mathrm{Sp} : \pi_1(\bar{X}_\eta) \to \pi_1(X_s)$ between fundamental groups which is surjective, and which induces an isomorphism between the maximal prime-to-p quotients. It is also known that the map Sp is not an isomorphism in the inequal characteristic case, and one conjectures that it is not an isomorphism in the equal characteristic $p > 0$ case, if X is non isotrivial. In terms of étale covers the defect for Sp to be an isomorphism translates as follows: there exists étale Galois covers $f_\eta : Y_\eta \to X_\eta$, with Galois group G, such that Y_η doesn't have good reduction (over any extension of R), in which case the cardinality of G is divisible by p. The basic example is with $G \simeq \mathbb{Z}/p\mathbb{Z}$, in inequal characteristics.

However, by the theorm of semi-stable reduction of curves due to Deligne-Mumford, Grothendieck, and others ..., Y_η always extends to a semi-stable curve over a finite extension R' of R, and such a model is essencially unique. The main problem I am interested in is to describe the geometry of the special fibre of such a model. Among possible applications of such a study: obtain criteria for good reduction of étale Galois covers (hence realise some finite group as a quotient of π_1 in characteristic $p > 0$).

This problem was first studied by Raynaud in his paper in the Grothendieck Festshrift, and then pursued by several authors. In these talks I will report on my work on this problem, which basically solves the following cases: $G \simeq \mathbb{Z}/p\mathbb{Z}$ and R has inequal characteristic, and $G \simeq \mathbb{Z}/p^n\mathbb{Z}$ and R has equal characteristic $p > 0$. Our work includes the case of ramified covers as well. Our approach to this problem is based on the techniques of computation of vanishing cycles and formal patching which I will explain.

Fumiharu Kato
Degeneration of weak ramifications and its application to automorphism group of ordinary curves

A ramification of a discrete valuation ring over an algebraically closed field of positive characteristic is called a weak ramification if the second ramification group is trivial. Ramifications of this kind describe the local structure of group actions around fixed points on ordinary curves. Deformations, moduli spaces, and certain degenerations of weak ramifications will be discussed. As an application, we discuss automorphism groups of ordinary curves.

Makoto Matsumoto
Continuous Malcev completion and Galois action on it

Kinya Kimura
About Fried's modular towers

In this joint talk with Anna Cadoret, we study about certain projective systems of Hurwitz spaces, which are called Modular Towers. Here, we construct them and prove a theorem.

Let G be a finite group (with some properies), p a prime number dividing the order of G, then we obtain a profinite group ${}_p\tilde{G}$ (called universal p-Frattini cover) and projective system $\{{}^k_p\tilde{G}\}$ of finite quotients of ${}_p\tilde{G}$. For $r > 0$ an integer, the projective system $\{H_r({}^k_p\tilde{G})\}$ is one of Modular Towers, where $H_r(G)$ is the (reduced coarse) moduli space of G-cover of $\mathbf{P}^1$ with r branch points. Then, we prove a theorem that $\varprojlim H_r({}^k_p\tilde{G})(K)$ is empty for a number field K. It says also that ${}_p\tilde{G}$ cannot be regularly realized over $K(T)$ with r branch points.

Anna Cadoret
About Fried's conjectures for modular towers

The aim of this talk is to provide an introduction to the diophantine conjectures attached to Fried's modular towers. In particular, we will relate them to the regular inverse Galois problem with a bounded number of branch points and explain how they are related to long standing conjectures in arithmetic geometry, such as the strong torsion conjecture for abelian varieties and Bombieri-Lang conjecture.

Katsutoshi Yamanoi
On solvable quotients of fundamental groups of algebraic varieties in value distribution theory