

The talk will survey the results in “ *L_2 constructible cohomology and L_2 De Rham Cohomology for coherent D -modules*” Annales Henri Lebesgue (2025) and describe the main problem left open in that direction.

Let X be a complex projective manifold and let $\tilde{X} \rightarrow X$ be an infinite Galois topological covering space with Galois group Γ an infinite finitely generated group. There is a natural locally constant sheaf ℓ^2 of Hilbertian Γ -modules and we may define the L^2 cohomology of a bounded constructible complex of $\mathbb{C}$ -vector spaces F on $\tilde{X}$ by $H^2(X, \ell^2 \otimes_{\mathbb{C}} F)$. This is a Γ module but it has many interesting properties that make it accessible to computations, in particular the Γ -action extends to an action of the Von Neumann algebra of Γ and we have a version of Atiyah’s L_2 -index theorem.

Similarly one can define L^2 cohomology on $\tilde{X}$ with coherent coefficients or L^2 De Rham cohomology with coherent D -module coefficients. When we apply these theories to (Mixed) Hodge Modules, we get Hodge and weight filtrations (that need not form a (mixed) Hodge structure) in an appropriate abelian category of Von Neumann algebra modules.

However, thanks to a nice idea P. Dingoyan used to put a MHS on these L_2 cohomology groups for $F = Rj_*\mathbb{C}$ where $j : U \rightarrow X$ is the inclusion of the complement of a SNC divisor, I expect this to be true after tensoring with the algebra of affiliated operators and I will report on the status of this problem.