Cayley-Bacharach Theorem

Exercise 1.

Show the reverse of Pascal's Theorem: If the opposite sides of a plane hexagon meet in 3 collinear points, then the hexagon can be inscribed in a conic.

Exercise 2.

Prove that 6 points in the plane impose independent conditions on cubics (polynomials of degree 3) if and only if any 5 points out of those 6 are not collinear.

Exercise 3.

Show that for any integer k > 0, there exists a subset of $\frac{1}{2}(k+1)(k+2)$ points of the plane which imposes independent conditions on polynomials of degree k.

Exercise 4.

Find a counter example to Cayley's second claim.