全学共通科目講義（１回生～４回生対象）

現代の数学と数理解析
—　基礎概念とその諸科学への広がり

授業のテーマと目的：

数学が発展してきた過程では、自然科学、社会科学などの種々の学問分野で提起される問題を解決するために、既存の数学の枠組みにとらわれない、新しい数理科学的な方法や理論が導入されてきた。また、逆に、そのような新しい流れが、数学の核心的な理論へと発展した例も数知れず存在する。このような数学と数理解析の展開の諸相について、第一線の研究者が、自身の研究を踏まえた入門的・解説的な講義を行う。

数学・数理解析の研究の面白さ・深さを、感性豊かな学生諸君に味わってもらうことを意図して講義し、原則として予備知識は仮定しない。

第７回

日時：　２００８年5月２３日（金）１６：３０－１８：００

場所：　数理解析研究所　４２０号室

講師：　KIRILLOV, Anatol 准教授

題目：　Tales of Catalan numbers

要約：

I will talk about a field of Mathematics called Enumerative Combinatorics. This field of Mathematics concerned with counting the number of elements of a finite set S. This definition, as it stands, is too general, and in a genuine enumerative problem, the elements of S have a specific origin depending on a task have to be solved.

It friquently occurs that a solution to some problem in Topology, Algebraic Geometry, Representation Theory, Mathematical Physics, Biology, ..., can be reduced to a computation of some integer numbers, which of course depend on a task under consideration. Sometimes it happens that the same numbers, but with a big variety of interpretations, appear as answers on completely different problems and questions. It is not always easy to explain why we have a deal with the same numbers in so different problems !

The most famous example of this phenomena in Combinatorics is numerous and unexpected appearances of the Catalan numbers.

Definition *The n-th Catalan number C_n is*

$$\frac{1}{n+1}\binom{2n}{n}, \quad n \geq 0.$$

Nowadays there are more then 140 combinatorial structures enumerated by Catalan numbers, see e.g. Catalan Addendum by R.Stanley.

The fist few Catalan numbers were known from the Ancient times, but probably, the first theorem about Catalan numbers (a theorem about the number of polygon dissections) had been proved by L. Euler in 1760. The Belgian mathematician Eugene Charle Catalan (1814-1894) wrote a several papers about related problems, and later his name was attributed to the numbers C_n.

Combinatorial structures enumerated by Catalan numbers I'm going to review, include
1. Polygon dissections;
2. Parenthesizations;
3. Lattice paths;
4. Trees;
5. Non-crossing partitions;
6. Young tableaux;
7. Jacobi game.

"http://www.kurims.kyoto-u.ac.jp/ja/special-02.html"