

## Topology from the Differentiable Viewpoint

### Exercise 1.

Suppose that  $X$  is a subset of  $\mathbb{R}^k$  and  $Z$  is a subset of  $X$ . Show that the restriction of any smooth map on  $X$  to  $Z$  is a smooth map on  $Z$ .

### Exercise 2.

The map  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $x \mapsto x^3$  is a smooth bijective map between smooth manifolds and its inverse  $f^{-1}$  is continuous. However, show that  $f$  is not a diffeomorphism.

### Exercise 3.

Suppose that  $f : X \rightarrow Y$  is a diffeomorphism of differentiable manifolds. Show that the derivative  $df_x$  is an isomorphism of the tangent spaces  $T_x X$  and  $T_{f(x)} Y$  for all  $x \in X$ .

### Exercise 4.

Prove that  $\mathbb{R}^k$  and  $\mathbb{R}^l$  are not diffeomorphic if  $k \neq l$ .

Milnor: *Topology from the Differentiable Viewpoint* §1-§2 用語集

boundary	境界.
commutative	可換な.
complement	補集合. 補空間. orthogonal — 直交補空間
constant	定数 (の).
continuous	連続な.
coordinate	座標.
critical	臨界の. — point 臨界点 — value 臨界値
dense	稠密な.
derivative	微分. partial — 偏微分
diffeomorphism	微分同相.
dimension	次元.
euclidean space	ユークリッド空間.
fixed point	不動点.
homogeneous	斉次の.
homeomorphism	同相. 位相同型.
hyperplane	超平面.
image	像.
interior	内包.
manifold	多様体.
map, mapping	写像. 関数. identity — 恒等写像 inclusion — 包含写像 linear — 線形写像
measure	測度. Lebesgue — ルベーグ測度
neighborhood	近傍.
normal vector	法ベクトル.
open set	開集合.
polynomial	多項式. complex — 複素多項式
regular	正則な. — point 正則点 — value 正則値
singular	特異な.
smooth	なめらかな.
tangent space	接空間.
tangent vector	接ベクトル.
topology	位相幾何. トポロジー. differential — 微分位相幾何