A Brief History of Implicit Functions (Answers)

Exercise 1.



Exercise 2.

(a) The steepest slope of the Newton polygon is -2/5. So we put $\mu = 5/2$ and inserting $y = tx^{\mu}$ into the equation $F(x, y) = y^4 - 2x^5y^2 + x^{10}$ gives

$$F(x, tx^{5/2}) = x^{10}(t^4 - 2t^2 + 1) = x^{10}(t^2 - 1)^2$$

with the solutions t = 1 and t = -1. We choose the first t = 1. Thus $y(x) = x^{5/2}$ is a solution to our equation F(x, y) = 0.

(b) We may begin with the solution from (a). So we make the substitution

 $x = x_1^2$ and $y = x_1^5(1+y_1)$.

Inserting into the equation $G(x, y) = y^4 - 2x^5y^2 - 4x^8y + x^{10} - x^{11}$ gives

$$G(x_1^2, x_1^5(1+y_1)) = x_1^{20}G_1(x_1, y_1)$$

where $G_1(x_1, y_1) = y_1^4 + 4y_1^3 + 4y_1^2 - 4x_1y_1 - 4x_1 - x_1^2$ is the polynomial from (c). This can be solved by $y_1 = x_1^{1/2}$ and hence $y = x^{5/2} + x^{11/4}$ solves G(x, y) = 0.