A Brief History of Implicit Functions

Exercise 1 (easy). Draw the Newton polygon of the following polynomials.

(a) $y^4 - 2x^5y^2 + x^{10}$

(b) $y^4 - 2x^5y^2 - 4x^8y + x^{10} - x^{11}$

(c) $y^4 + 4y^3 + 4y^2 - 4xy - 4x - x^2$

(d) $y^5 + 12xy^3 - 3x^2y^4 + 2x^3y^2 + 5x^7$

(e) $y^6 + 5x^2y^3 + x^3y^2 + 2x^8y$

Exercise 2 (not so easy). Compute a Puiseux expansion

$$y(x) \;=\; \sum a_i x^{i/n}$$

for the polynomials (a) and (b) from Exercise 1.

Hint. The Equation (a) can be solved exactly in the form $y(x) = tx^\mu$, where μ is an appropriate rational number and there are two choices for t, one is positive and the other is negative. The lowest order term of the polynomial (b) is the polynomial (a) from Exercise 1. So as a first approximation for (b), we use the solution $y = tx^\mu$ for (a). Here, only use the positive solution for t. Then, the Puiseux expansion for (b) will have the simple form $y(x) = tx^\mu + sx^\sigma$.