Abstracts

Aging in mean field spin glasses

Anton Bovier (University of Bonn)

Aging is a common term to describe anomalous slow dynamics of random processes in random environments. A characteristic feature is the emergence of an α-stable subordinator as the universal asymptotic random mechanism that governs the long term behaviour of such systems. I will explain the modelling approaches through trap models and discuss their rigorous justification in some mean field spin glass models.

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An equality on Ginibre random point field and tagged particles of interacting Brownian motions with 2D Coulomb potentials.

Hirofumi Osada (Kyushu University)

In this talk we introduce an equality concerning on a pair of functions of the $2D$ configuration space that holds almost surely with respect to the Ginibre random point field μ. We apply it to prove the tagged particle of the interacting Brownian motions is sub diffusive.

Ginibre random point field μ is a probability measure on the space of configurations over $\mathbb{R}^2$. It is known that μ is translation and rotation invariant. Moreover, μ is so called a determinantal random point field and their n-correlation function is given by

$$\rho^n(x_1, \dots, x_n) = \det[K(x_i.x_j)]_{1\le i,j\le n}, \tag{1}$$

where $K:\mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{C}$ is the kernel defined by

$$K(x,y) = \frac{1}{\pi}\exp(-\frac{|x|^2}{2} - \frac{|y|^2}{2}) \cdot e^{x\bar{y}}. \tag{2}$$

Here we identify $\mathbb{R}^2$ as $\mathbb{C}$ by the obvious correspondence: $\mathbb{R}^2 \ni x = (x_1, x_2) \mapsto x_1 + \sqrt{-1}x_2 \in \mathbb{C}$, and $\bar{y} = y_1 - \sqrt{-1}y_2$ means the complex conjugate under this identification. Intuitively, μ is a measure interacting $2D$ Coulomb potentials Ψ such that

$$\Psi(x) = -2\log|x| \quad (x \in \mathbb{R}^2). \tag{3}$$

We remark that the DLR equation for μ does not make sense. However, we will prove that μ is the reversible measure of the diffusion

$$\mathbb{X}_t = \sum_{i\in\mathbb{N}} \delta_{X_t^i}, \tag{4}$$

where $\mathbf{X}_t = (X_t^i) \in (\mathbb{R}^2)^{\mathbb{N}}$ is the solution of the infinitely dimensional SDE:

$$dX_t^i = dB_t^i + \lim_{r\to\infty} \sum_{|X_t^i - X_t^j|<r} \frac{X_t^i - X_t^j}{|X_t^i - X_t^j|^2} dt \qquad (\mathbf{X}_0 = (x_i)_{i\in\mathbb{N}}). \tag{5}$$

We remark the unlabeled diffusion $\mathbb{X}_t$ is translation and rotation invariant. By (5) one can say μ is a measure with $2D$ Coulomb interaction potentialΨ.

Theorem 1 *There exists a set* $\mathbf{S} \subset (\mathbb{R}^2)^{\mathbb{N}}$ *such that* $\mu(\{\sum_{i\in\mathbb{N}} \delta_{x_i}; \mathbf{x} = (x_i) \in \mathbf{S}\}) = 1$ *and that* (5) *has a solution for all initial points* $\mathbf{x} = (x_i)_{i\in\mathbb{N}} \in \mathbf{S}$*. Moreover, for all initial points* $\mathbf{x} \in \mathbf{S}$*,*

$$P(\mathbf{X}_t \in \mathbf{S} \cap \mathbf{S}_{\text{single}} \textit{ for all } t) = 1.$$

Here $\mathbf{S}_{\text{single}} = \{\mathbf{s} = (s_i) \in (\mathbb{R}^2)^{\mathbb{N}}\,;\, s_i \neq s_j \textit{ if } i \neq j\}$*.*

The key point of Theorem 1 is to calculate the log derivative of the one moment measure of μ and to introduce a coupling of an infinite system of Dirichlet spaces describing (5).

Theorem 2 *Let* α *be the self-diffusion matrix of Ginibre interacting Brownian motion* (5)*. Then* $\alpha = 0$*.*

We remark (5) has only repulsive interaction; there are no center force. If the interaction is of Ruelle class and $d \geq 2$, then α is always non degenerate. This was proved mathematically when the particle have convex hard cores [O. 98, PTRF]. So the result above caused by the strength of the long range part of the interaction potentialΨ.
A key point of the proof is a μ-almost sure equality on functions of the configuration space. We also use the invariant principle and, moreover, the necessary and sufficient condition for the non degeneracy of the limit coefficients under diffusive scaling obtained in [O. 98, IHP].

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Exact value of the resistance exponent for four dimensional random walk trace

Daisuke Shiraishi (Kyoto University)

Let S be a simple random walk starting at the origin in $\mathbb{Z}^4$. We consider $\mathcal{G} = S[0,\infty)$ to be a random subgraph of the integer lattice and assume that a resistance of unit 1 is put on each edge of the graph $\mathcal{G}$. Let R_n be the effective resistance between the origin and S_n. We derive the exact value of the resistance exponent; more precisely, we prove that $n^{-1}E(R_n) \approx (\log n)^{-\frac{1}{2}}$. Furthermore, we derive the precise exponent for the heat kernel of a random walk on $\mathcal{G}$ at the quenched level. These results give the answer to the problem raised by Burdzy and Lawler (1990) in four dimension.

References

[1] Burdzy, K.; Lawler, G. F. Rigorous exponent inequalities for random walks. Journal of Physics A: Mathematical and General, Volume 23, Issue 1, pp. L23-L28 (1990).

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Localization for branching Brownian motions in random environment with extinction

Yuichi Shiozawa (Okayama University)

We consider a model of branching Brownian motions in time-space random environment associated with the Poisson random measure. We prove that, on the survival event, the slow growth of the total population size is equivalent to the localization in terms of the replica overlap. This result is based on an ongoing joint work with Yukio Nagahata and Nobuo Yoshida.

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Hydrodynamic limit for an evolutional model of 2D Young diagrams [1]

Makiko Sasada (University of Tokyo)

We construct dynamics of two-dimensional Young diagrams, which are naturally associated with their grandcanonical ensembles, by allowing the creation and annihilation of unit squares located at the boundary of the diagrams. The grandcanonical ensembles, which were introduced by Vershik [3], are uniform measures under conditioning on their size (or equivalently, area). We then show that, as the averaged size of the diagrams diverges, the corresponding height variable converges to a solution of a certain non-linear partial differential equation under a proper hydrodynamic scaling. Furthermore, the stationary solution of the limit equation is identified with the so-called Vershik curve. We discuss both Bose and Fermi statistics for the Young diagrams.
The asymptotic shapes of two-dimensional random Young diagrams with large size were studied by Vershik [3] under several types of statistics including the so-called Bose and Fermi statistics. To each partition $p = \{p_1 \geq p_2 \geq \cdots \geq \ p_j \geq 1\}$ of a positive integer n by positive integers $\{p_i\}_{i=1}^j$ (i.e., $n = \sum_{i=1}^j p_i$), a Young diagram is associated by piling up j sticks of height 1 and side-length p_i, more precisely, the height function of the Young diagram is defined by

$$\psi_p(u) = \sum_{i=1}^{j} 1_{\{u<p_i\}}, \quad u \geq 0.$$

For each fixed n, Bose statistics μ_B^n assigns an equal probability to each of possible partitions p of n, i.e., to the Young diagrams of area n. Fermi statistics μ_F^n also assigns an

[1] Based on a joint work with T.Funaki

equal probability, but restricting to the partitions satisfying $q = \{q_1 > q_2 > \cdots > q_j \geq 1\}$. These probabilities are called canonical ensembles. Grandcanonical ensembles μ_B^ε and μ_F^ε with parameter $0 < \varepsilon < 1$ are defined by superposing the canonical ensembles in a similar manner known in statistical physics. Vershik [3] proved that, under the canonical Bose and Fermi statistics $\mu_B^{N^2}$ and $\mu_F^{N^2}$ (with $n = N^2$), the law of large numbers holds as $N \to \infty$ for the scaled height variable

$$\tilde{\psi}_p^N(u) := \frac{1}{N}\psi_p(Nu), \quad u \geq 0, \tag{6}$$

of the Young diagrams $\psi_p(u)$ with size (i.e., area) N^2 and for $\tilde{\psi}_q^N(u)$ defined similarly, and the limit shapes ψ_B and ψ_F are given by

$$\psi_B(u) = -\frac{1}{\alpha}\log\left(1 - e^{-\alpha u}\right) \quad \text{and} \quad \psi_F(u) = \frac{1}{\beta}\log\left(1 + e^{-\beta u}\right), \quad u \geq 0, \tag{7}$$

with $\alpha = \pi/\sqrt{6}$ and $\beta = \pi/\sqrt{12}$, respectively. These results can be extended to the corresponding grandcanonical ensembles μ_B^ε and μ_F^ε, if the averaged size of the diagrams is N^2 under these measures. Such types of results are usually called the equivalence of ensembles in the context of statistical physics. The corresponding central limit theorem and large deviation principle were shown by Pittel [2] and Dembo et. al. [1], respectively. All these results are at the static level.

The purpose of our talk is to study and extend these results from a dynamical point of view. We will see that, to the grandcanonical Bose and Fermi statistics, one can associate a weakly asymmetric zero-range process p_t respectively a weakly asymmetric simple exclusion process q_t as natural time evolutions of the Young diagrams, or more precisely, those of the gradients of their height functions. Then, under the diffusive scaling in space and time and choosing the parameter $\varepsilon = \varepsilon(N)$ of the grandcanonical ensembles such that the averaged size of the Young diagrams is N^2, we will derive the hydrodynamic equations in the limit and show that the Vershik curves defined by (7) are actually stationary solutions to the limiting non-linear partial differential equations in both cases.

References

[1] A. Dembo, A. Vershik and O. Zeitouni, *Large deviations for integer partitions*, Markov Processes Relat. Fields, **6** (2000), 147–179.

[2] B. Pittel, *On a likely shape of the random Ferrers diagram*, Adv. Appl. Math., **18** (1997), 432–488.

[3] A. Vershik, *Statistical mechanics of combinatorial partitions and their limit shapes*, Func. Anal. Appl., **30** (1996), 90–105.

Profile convergence of a gradient interface model with non convex potential

Jean-Dominique Deuschel (TU-Berlin)

We consider a model of interface defined on D, a bounded open subset of $\mathbb{R}^2$. The height of the interface is defined inside D at all sites of a grid of width $1/N$ and fixed at O outside D. The height differences of the interface between nearest neighbors are penalized through a non convex potential V. In this setting, we prove the convergence of the profile of the interface as N tends to infinity. Our approch is based on the work of Deuschel, Giacomin and Ioffe (1999) and Cotar, Deuschel (2008) and consists in considering the restriction of the model to half of the sites only. The latter allows to recover a strictly convex potential when the temperature is large enough.

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Branching Random Walks in Random Environment: Survival Probability and Growth Rates

Nobuo Yoshida (Kyoto University)

We study the survival probability and the growth rate for branching random walks in random environment (BRWRE). The particles perform simple symmetric random walks on the d-dimensional integer lattice, while at each time unit, they split into independent copies according to time-space i.i.d. offspring distributions. The BRWRE is naturally associated with the directed polymers in random environment (DPRE), for which the quantity called the free energy is well studied. We discuss the survival probability (both global and local) for BRWRE and give a criterion for its positivity in terms of the free energy of the associated DPRE. We also show that the global growth rate for the number of particles in BRWRE is given by the free energy of the associated DPRE, though the local growth rate is given by the directional free energy. This is a joint work with Francis Comets.

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Central Limit Theorem for Branching Random Walk in Random Environment

Makoto Nakashima (Kyoto University)

We consider the branching random walks in d-dimensional integer lattice with time-space i.i.d. offspring distributions. Then, the normalization of total population is a non-negative martingale and it converges to a certain random variables almost surely. When $d \geq 3$ and the fluctuation of environment satisfies a certain uniform square integrability, then it is non-degenerate and we prove a central limit theorem for the density of the population in terms of almost sure convergence.

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Hydrodynamic limit for the interface model with general potentials

Takao Nishikawa (Nihon University)

We discuss the hydrodynamic limit for the Ginzburg-Landau interface model. Under the assumption that the microscopic interaction potential is strictly convex, the study of the asymptotic behavior for microscopic stochastic dynamics, including the hydrodynamic limit, is highly developed. The aim of this talk is to discuss the behavior of Ginzburg-Landau interface model without the assumption of strict convexity of the potential, and to derive the hydrodynamic limit in this setting.

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Localization for a class of linear systems

Yukio Nagahata (Osaka University)

We consider a class of interacting particle systems with value in $[0, \infty)^{\mathbb{Z}^d}$, of which the binary contact path process is an example. We show a certain relation between growth rate of total number of particles and replica overlap.