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Boundary values of hyperfunction solutions of linear partial differential equations (*)

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Let $P(x, D)$ be a linear partial differential operator with real analytic coefficients in a domain V in $\mathbb{R}^{n+1}$ and let $S \subset V$ be a real analytic hypersurface non-characteristic with respect to $P(x, D)$. The purpose of this paper is to show that every hyperfunction solution u of $P(x, D)u = 0$ on one side of $V \setminus S$ has boundary values on S which are hyperfunctions of n variables on S.

This fact has been proved by H. Komatsu [6] and P. Schapira [8] in the case where $P(x, D)$ is elliptic. Their method applies with minor modifications to the general operators.

In §1 we show that the Cauchy-Kowalevsky theorem for the dual equation with the initial values on S is equivalent to a theorem of division of hyperfunctions with supports in S by the differential operator $P(x, D)$.

We define the boundary values in §2 and prove the uniqueness of hyperfunction solutions of the Cauchy problems.

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1. Division of hyperfunctions with supports in S.

Let $P(x, D)$ be a linear differential operator of order m with real analytic coefficients defined on a domain V in $\mathbb{R}^{n+1}$ and let S be an oriented real analytic hypersurface in V non-characteristic with respect to $P(x, D)$.

We denote by $\mathcal{A}$ and $\mathcal{B}$ ($'\mathcal{A}$ and $'\mathcal{B}$) the sheaf of real analytic functions and that of hyperfunctions on V (on S respectively). When K is a compact set in V (in S), the space $\mathcal{A}(K)$ ($'\mathcal{A}(K)$) has a natural (DFS)-topology and its dual is identified with the space $\mathcal{B}_K(V)$ ($'\mathcal{B}_K(S)$) of hyperfunctions with supports in K under the inner product

$$<\varphi, f> = \int_V \varphi(x)f(x)dx, \quad \varphi \in \mathcal{A}(K), \quad f \in \mathcal{B}_K(V)$$

$$(<\varphi, f> = \int_S \varphi(x')f(x')d\omega, \quad \varphi \in {'\mathcal{A}}(K), \quad f \in {'\mathcal{B}}_K(S)),$$

where dx ($d\omega$) denotes the Lebesgue measure on V (on S).

Let $P'(x, D)$ be the formal dual of $P(x, D)$. Then, $P(x, D)$ and $P'(x, D)$ induce sheaf homomorphisms $P(x, D) : \mathcal{B} \longrightarrow \mathcal{B}$ and $P'(x, D) : \mathcal{A} \longrightarrow \mathcal{A}$ respectively. We denote by $\mathcal{B}^P$ and $\mathcal{A}^{P'}$ the kernel sheaves, i.e., the sheaf of solutions of

(1) $$P(x, D)f = 0, \quad f \in \mathcal{B},$$

and that of solutions of

(2) $$P'(x, D)\varphi = 0, \quad \varphi \in \mathcal{A}$$

respectively.

Theorem 1. *Let K be a compact set in S. Then, there is no non-trivial solution of (1) over V with support in K:*

(3) $$\mathcal{B}_K^P(V) = 0.$$

The quotient space $\mathcal{B}_K(V)/P\mathcal{B}_K(V)$ *is identified with the dual of the* (DFS)-*space* $\mathcal{A}^{P'}(K)$.

Proof. Consider the complexes:

(4) $$0 \longrightarrow \mathcal{A}(K) \xrightarrow{P'(x,D)} \mathcal{A}(K) \longrightarrow 0$$

$$\updownarrow \qquad\qquad \updownarrow$$

(5) $$0 \longleftarrow \mathcal{B}_K(V) \xleftarrow{P(x,D)} \mathcal{B}_K(V) \longleftarrow 0,$$

which are dual to each other in the sense that $\mathcal{A}(K)$ and $\mathcal{B}_K(V)$ with their natural (DFS)- and (FS)-topologies are the strong dual spaces of each other and that $P'(x, D)$ and $P(x, D)$ are continuous linear operators dual to each other.

The 0-th cohomology group of (4) is $\mathcal{A}^{P'}(K)$ and the 1-st cohomology group of (4) vanishes by the Cauchy-Kowalevsky theorem. In particular, $P'(x, D)$ has a closed range. Thus it follows from Serre's lemma (see e.g. [5] Theorem 19) that $P(x, D)$ has a closed range and that the cohomology groups of (4) and (5) are the strong dual spaces of each other. Therefore, $\mathcal{B}_K(V)/P\mathcal{B}_K(V)$ is the dual of $\mathcal{A}^{P'}(K)$ and $\ker P(x, D) = \mathcal{B}_K^P(V)$ vanishes.

Let $C_j(x, D)$, $j = 1, 2, \cdots, m$, be linear differential operators of order $m-j$ with real analytic coefficients on a neighborhood of S for which S is non-characteristic (e.g. $C_j(x, D) = (\partial/\partial n)^{m-j}$). Then the Cauchy-Kowalevsky theorem yields the topological isomorphism

(6) $$\rho : \mathcal{A}^{P'}(K) \approx {}'\mathcal{A}(K)^m$$

defined by

(7) $$\rho(\varphi) = (C_j(x, D)\varphi|_S), \quad \varphi \in \mathcal{A}^{P'}(K).$$

We have, therefore, the dual isomorphism

(8) $$\rho' : {}'\mathcal{B}_K(S)^m \approx \mathcal{B}_K(V)/P\mathcal{B}_K(V).$$

Obviously ρ can be extended by (7) to a continuous linear operator $\tilde{\rho}: \mathcal{A}(K) \longrightarrow {}'\mathcal{A}(K)^m$. Since the open mapping theorem holds for (DFS)-spaces, the exact sequence

(9) $$0 \longrightarrow {}'\mathcal{A}(K)^m \xrightarrow{\rho^{-1}} \mathcal{A}(K) \xrightarrow{P'(x,D)} \mathcal{A}(K) \longrightarrow 0$$

splits topologically and we have the topological isomorphism:

(10) $$\mathcal{A}(K) \approx {}'\mathcal{A}(K)^m \oplus \mathcal{A}(K)$$

defined by

(11) $$\varphi \longmapsto (C_j(x, D)\varphi|_S) \oplus P'(x, D)\varphi.$$

Correspondingly the dual exact sequence

(12) $$0 \longleftarrow {}'\mathcal{B}_K(S)^m \xleftarrow{(\rho^{-1})'} \mathcal{B}_K(V) \xleftarrow{P(x,D)} \mathcal{B}_K(V) \longleftarrow 0$$

splits topologically.

Since $\tilde{\rho}$ is the composite of the differential operators $(C_j(x, D))$ and the restriction to S, the dual $\tilde{\rho}' : {}'\mathcal{B}_K(S)^m \longrightarrow \mathcal{B}_K(V)$ is the mapping $(f_j) \longmapsto \sum_{j=1}^{m} C'_j(x, D)(f_j \otimes \delta_S)$, where $C'_j(x, D)$ is the formal dual of $C_j(x, D)$ and $f_j \otimes \delta_S$ is the hyperfunction on V defined by

(13) $$\langle f_j \otimes \delta_S, \varphi \rangle = \int_S f_j(x')\varphi(x')d\omega, \quad \varphi \in \mathcal{A}(K).$$

Consequently, each $f \in \mathcal{B}_K(V)$ is uniquely decomposed as

(14) $$f = \sum_{j=1}^{m} C'_j(x, D)(f_j \otimes \delta_S) + P(x, D)g,$$

where $f_j \in {'\mathcal{B}}_K(S)$ and $g \in \mathcal{B}_K(V)$. Under this correspondence we have a topological isomorphism

(15) $$\mathcal{B}_K(V) \approx {'\mathcal{B}}_K(S)^m \oplus \mathcal{B}_K(V).$$

In particular, the inverse $(\rho')^{-1}$: $\mathcal{B}_K(V)/P\mathcal{B}_K(V) \approx {'\mathcal{B}}_K(S)^m$ of isomorphism (8) is the mapping which takes the class of f to (f_j) in the decomposition (14). Obviously f_j depend on the choice of $C_j(x, D)$. However, the sum $\sum C'_j(x, D)(f_j \otimes \delta_S)$ and $P(x, D)g$ do not depend on $C_j(x, D)$ because neither $\operatorname{im} \rho^{-1} = \mathcal{A}^{P'}(K)$ nor $\ker \tilde{\rho}$ depends on $C_j(x, D)$.

The uniqueness of the decomposition shows that the components f_j and g are independent of the compact set K which contains the support of f. Namely we have an isomorphism

(16) $$\Gamma_*(\mathcal{H}^0_S(\mathcal{B})|_S, S) \approx \Gamma_*({'\mathcal{B}}, S)^m \oplus \Gamma_*(\mathcal{H}^0_S(\mathcal{B})|_S, S)$$

which preserves the support, where Γ_* denotes the space of sections with compact supports and $\mathcal{H}^0_S(\mathcal{B})|_S$ the restriction to S of the sheaf of sections of $\mathcal{B}$ with supports in S.

Let us denote $\mathcal{H}^0_S(\mathcal{B})|_S$ by $\mathcal{B}_S$ for short. Since $\mathcal{B}_S$ and $'\mathcal{B}$ are flabby, it follows that the isomorphism is extended to a sheaf isomorphism (see e.g. [4] Lemma 2.3). Thus we have proved the following theorem.

Theorem 2. If $C'_j(x, D)$, $j = 1, \cdots, m$, are linear differential operators of order $m - j$ with real analytic coefficients on a neighborhood of S for which S is non-characteristic, then we have a sheaf isomorphism

(17) $$\mathcal{B}_S \approx {}'\mathcal{B}^m \oplus \mathcal{B}_S$$

defined by

(18) $$f = \sum_{j=1}^{m} C'_j(x, D)(f_j \otimes \delta_S) + P(x, D)g,$$

where $f \in \mathcal{B}_S$, $f_j \in {}'\mathcal{B}$ *and* $g \in \mathcal{B}_S$. *The last component* g *does not depend on the choice of* $C'_j(x, D)$.

In particular, there is no non-trivial solution $g \in \mathcal{B}^P(V)$ *with support in* S:

(19) $$\mathcal{B}^P_S(V) = 0.$$

This theorem means that on division by $P(x, D)$ each $f \in \mathcal{B}_S$ has a unique quotient $g \in \mathcal{B}_S$ and a remainder $\sum C'_j(x, D)(f_j \otimes \delta_S)$ with $f_j \in {}'\mathcal{B}$. We have derived this from the Cauchy-Kowalevsky theorem via the duality of $\mathcal{A}(K)$ and $\mathcal{B}_K(V)$ and that of ${}'\mathcal{A}(K)$ and ${}'\mathcal{B}_K(S)$. Conversely Theorem 2 implies the exactness of (12) and hence that of (9). Thus Theorem 2 of division is equivalent to the Cauchy-Kowalevsky theorem.

2. Boundary values of hyperfunction solutions.

Let W be an open subset of V. We have the following commutative diagram:

$$
(20) \qquad \begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & 0 & \longrightarrow & \mathcal{B}^P(W) & \longrightarrow & \mathcal{B}^P(W\setminus S) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \mathcal{B}_{S\cap W}(W) & \longrightarrow & \mathcal{B}(W) & \longrightarrow & \mathcal{B}(W\setminus S) & \longrightarrow & 0 \\
 & & \downarrow P(x, D) & & \downarrow P(x, D) & & \downarrow P(x, D) & & \\
0 & \longrightarrow & \mathcal{B}_{S\cap W}(W) & \longrightarrow & \mathcal{B}(W) & \longrightarrow & \mathcal{B}(W\setminus S) & \longrightarrow & 0 , \\
 & & \downarrow & & & & & & \\
 & & 0 & & & & & &
\end{array}
$$

where $\mathcal{B}_{S\cap W}(W)$ denotes the space of hyperfunctions on W with supports in $S \cap W$. Since $\mathcal{B}$ is flabby, the last two rows are exact; the last two columns are exact by the definition; the 0-th cohomology group of the first row and that of the first column vanish since there is no non-trivial solution with support in $S \cap W$.

For the remaining cohomology groups we have a natural homomorphism

$$b : \mathcal{B}^P(W\setminus S)/\mathcal{B}^P(W) \longrightarrow \mathcal{B}_{S\cap W}(W)/P\mathcal{B}_{S\cap W}(W).$$

Let $u \in \mathcal{B}^P(W\setminus S)$ and let $\tilde{u}$ be an extension in $\mathcal{B}(W)$. Since $P(x, D)\tilde{u} = 0$ on $W\setminus S$, $P\tilde{u}$ belongs to $\mathcal{B}_{S\cap W}(W)$. If $\tilde{u}_1$ is another extension of u, $\tilde{u} - \tilde{u}_1$ belongs to $\mathcal{B}_{S\cap W}(W)$. Therefore the class of $P\tilde{u}$ in $\mathcal{B}_{S\cap W}(W)/P\mathcal{B}_{S\cap W}(W)$ is determined uniquely by u. If u is the restriction to $W\setminus S$ of a $\tilde{u} \in \mathcal{B}^P(W)$, we have $P\tilde{u} = 0$. Thus we can define a homomorphism b which assigns for the class of $u \in \mathcal{B}^P(W\setminus S)$ the class of $P\tilde{u} \in \mathcal{B}_{S\cap W}(W)$.

Theorem 3. The homomorphism

(21) $b : \mathcal{B}^P(W\setminus S)/\mathcal{B}^P(W) \longrightarrow \mathcal{B}_{S\cap W}(W)/P\mathcal{B}_{S\cap W}(W)$.

is injective for any open set W in V and commutes with restrictions. b is surjective if and only if

(22) $P(x, D)\mathcal{B}(W) \supset \mathcal{B}_{S\cap W}(W)$.

Proof. By the definition it is clear that b commutes with restrictions. To prove the injectivity, let $P\tilde{u} = Pu_1$ for a $u_1 \in \mathcal{B}_{S\cap W}(W)$. Since $\tilde{u} - u_1 \in \mathcal{B}^P(W)$ and its restriction to $W\setminus S$ is equal to u, the class of u is zero.

Let b be surjective. Then, for each $g \in \mathcal{B}_{S\cap W}(W)$ there exist $h \in \mathcal{B}_{S\cap W}(W)$ and $\tilde{u} \in \mathcal{B}(W)$ such that $g + Ph = P\tilde{u}$. Thus $\mathcal{B}_{S\cap W}(W) \subset P\mathcal{B}(W)$.

Conversely suppose that for each $g \in \mathcal{B}_{S\cap W}(W)$ there is a $\tilde{u} \in \mathcal{B}(W)$ such that $g = P\tilde{u}$. Then, the restriction u of $\tilde{u}$ belongs to $\mathcal{B}^P(W\setminus S)$. Therefore b is surjective.

It is known that (22) holds if the coefficients of $P(x, D)$ are constants or if $P(x, D)$ is elliptic.

Now, let ω be an open set of S and let $W \supset W'$ be two open sets in V with $S \cap W = S \cap W' = \omega$. The restriction $\mathcal{B}^P(W\setminus S) \longrightarrow \mathcal{B}^P(W'\setminus S)$ induces a homomorphism

(23) $r : \mathcal{B}^P(W\setminus S)/\mathcal{B}^P(W) \longrightarrow \mathcal{B}^P(W'\setminus S)/\mathcal{B}^P(W')$.

Since $\mathcal{B}_{S\cap W}(W)/P\mathcal{B}_{S\cap W}(W) = \mathcal{B}_{S\cap W'}(W')/P\mathcal{B}_{S\cap W'}(W')$ and since the injections b_W and $b_{W'}$ commute with r, it follows that r is injective.

r is surjective if and only if $\mathcal{B}^P(W'\setminus S) =$

$\mathcal{B}^P(W \setminus S)|_{W'\setminus S} + \mathcal{B}^P(W')|_{W'\setminus S}$ and this holds if

(24) $$H^1(W, \mathcal{B}^P) = 0$$

by the Mayer-Vietoris theorem.

It is also known that (24) holds for any open set W if the coefficients of $P(x, D)$ are constants or if $P(x, D)$ is elliptic.

Taking the inductive limit with respect to the open neighborhoods of ω, we have the injection

(25) $$b : (\mathcal{B}^P_+(\omega) \oplus \mathcal{B}^P_-(\omega))/\mathcal{B}^P(\omega) \longrightarrow \mathcal{B}_S(\omega)/P\mathcal{B}_S(\omega),$$

where $\mathcal{B}^P_+(\omega)$ $(\mathcal{B}^P_-(\omega))$ denotes the space of germs of solutions on $W \setminus S$ which vanish on the negative (positive) side of S. $\mathcal{B}^P_\pm$ are sheaves over S which describe the boundary behavior of solutions outside S.

It follows from Theorem 3 that b in (25) is surjective if and only if

(26) $$P(x, D)\mathcal{B}(\omega) \supset \mathcal{B}_S(\omega).$$

Furthermore, noticing that the sheaf associated with the presheaf $(\mathcal{B}^P_+(\omega) \oplus \mathcal{B}^P_-(\omega))/\mathcal{B}^P(\omega)$ is the restriction $\mathcal{H}^1_S(\mathcal{B}^P)|_S$ to S of the first derived sheaf with support in S (see [4]), we have the injection

(27) $$b : \mathcal{H}^1_S(\mathcal{B}^P)|_S \longrightarrow \mathcal{B}_S/P\mathcal{B}_S$$

which is surjective if and only if

(28) $$P(x, D)\mathcal{B}|_S \supset \mathcal{B}_S.$$

Obviously (28) is satisfied if $P(x, D)$ is locally solvable on S i.e. if

(29) $$P(x, D) : \mathcal{B}(x) \longrightarrow \mathcal{B}(x) \text{ is surjective for } x \in S.$$

This is known for operators with constant coefficients or of elliptic type. Moreover, T. Kawai [3] proves the existence of local elementary solutions and hence the local solvability of operators $P(x, D)$ of simple characteristics with real principal parts. Thus (27) is an isomorphism for such operators. Combining this with the isomorphism $\mathcal{B}_S/P\mathcal{B}_S \approx {}'\mathcal{B}^m$ given in Theorem 2, we have an isomorphism

$$(30) \qquad \mathcal{H}^1_S(\mathcal{B}^P)|_S \approx {}'\mathcal{B}^m .$$

Definition. Let W be an open set in V, let $\omega = S \cap W$ and let W_+ be the positive part of $W \setminus S$. For each solution $u \in \mathcal{B}^P(W_+)$ we define its boundary values $(f_j) \in {}'\mathcal{B}(\omega)^m$ on S to be the image of u under the composite of mappings $\mathcal{B}^P(W_+) \longrightarrow \mathcal{B}^P_+(\omega) \longrightarrow (\mathcal{B}^P_+(\omega) \oplus \mathcal{B}^P_-(\omega))/\mathcal{B}^P(\omega) \xrightarrow{b} \mathcal{B}_S(\omega)/P\mathcal{B}_S(\omega) \longrightarrow {}'\mathcal{B}(\omega)^m$, where the last mapping is the isomorphism obtained in Theorem 2 as an extension of $(\rho')^{-1}$ in (8). In other words, $(f_j) \in {}'\mathcal{B}(\omega)^m$ is the unique m-tuple of hyperfunctions on ω which satisfy

$$(31) \qquad P(x, D)\tilde{u} = \sum_{j=1}^{m} C'_j(x, D)(f_j \otimes \delta_S)$$

for an extension $\tilde{u} \in \mathcal{B}(W)$ vanishing on the negative side of $W \setminus S$.

As we remarked earlier, the extension $\tilde{u}$ which satisfies (31) is uniquely determined by u and does not depend on the choice of $C_j(x, D)$, so that we call $\tilde{u}$ the canonical extension of u.

Let θ_S be the characteristic function of W_+ in W. Then, there are unique linear differential operators $B_j(x, D)$, $j = 1, \cdots, m$, of order $j-1$ with real analytic coefficients in a neighborhood of S such that S is non-characteristic and that

$$P(x, D)(\theta_S(x)u(x)) - \theta_S(x)(P(x, D)u(x))$$

(32) $$= \sum_{j=1}^{m} C'_j(x, D)(B_j(x, D)u(x)(1 \otimes \delta_S))$$

$$= \sum_{j=1}^{m} C'_j(x, D)((B_j(x, D)u(x))|_S \otimes \delta_S)$$

for any $u \in \mathcal{A}(W)$ or more generally for any $u \in \mathcal{B}(W)$ which is real analytic in the normal direction on S (see [1] for the real analyticity in parameter and the restrictions of hyperfunctions to submanifolds).

Conversely if $B_j(x, D)$, $j = 1, \cdots, m$, are linear differential operators of order $j-1$ with real analytic coefficients for which S is non-characteristic, we can find linear differential operators $C_j(x, D)$ of order $m-j$ such that S is non-characteristic and that (32) holds. This is only a local formulation of Green's formula.

Therefore, if u is the restriction to W_+ of a solution $u_1 \in \mathcal{A}^P(W)$ we have

(33) $$f_j = B_j(x, D)u_1|_S, \quad j = 1, \cdots, m.$$

This holds also for the restriction u of a solution $u_1 \in \mathcal{B}^P(W)$, because u_1 is real analytic in the normal direction on S by Sato's fundamental theorem of analyticity (see [1]).

Taking this into account we will write the boundary values

(34) $$f_j = B_j(x, D)u|_{S_+}, \quad j = 1, \cdots, m.$$

Similarly we can define the boundary values $B_j(x, D)u|_{S_-}$ of solutions u on the negative side of $W \setminus S$. The following is clear from the definition.

Theorem 4. A solution $u \in \mathcal{B}^P(W \setminus S)$ is extended to a solution $u \in \mathcal{B}^P(W)$ if and only if

(35) $$B_j(x, D)u\big|_{S_+} = B_j(x, D)u\big|_{S_-}, \quad j = 1, \cdots, m.$$

This may be regarded as a generalization of the classical Painlevé theorem.

If the operator $P(x, D)$ is locally solvable on S or if (28) holds, then the isomorphism (30) shows that the Plemelj problem

(36) $$B_j(x, D)u\big|_{S_+} - B_j(x, D)u\big|_{S_-} = f_j, \quad j = 1, \ldots, m$$

has a local solution $u \in \mathcal{B}_+^P(x) \oplus \mathcal{B}_-^P(x)$ for any $f_j \in {}'\mathcal{B}(x)$ on S.

Lastly the Holmgren theorem by T. Kawai [21] and P. Schapira [8] asserts that

(37) $$\mathcal{B}_+^P(\omega) \cap \mathcal{B}^P(\omega) = \{0\} \quad \text{and} \quad \mathcal{B}_-^P(\omega) \cap \mathcal{B}^P(\omega) = \{0\}.$$

Therefore the mapping $\mathcal{B}_+^P(\omega) \longrightarrow {}'\mathcal{B}(\omega)^m$ is injective. Thus we have

Theorem 5. A solution $u \in \mathcal{B}^P(W_+)$ on the positive side of $W \setminus S$ vanishes in a neighborhood of $\omega = W \cap S$ if and only if the boundary values $B_j(x, D)u\big|_S$ vanish for all $j = 1, \cdots, m$.

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