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Topology of versal deformations.

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We are going to speak about results obtained with the collaboration of B. Iversen and G.M. Greuel.

As these results will be published in [6] and [7] we only write a summary of our lecture.

Let $\varphi : (\mathbb{C}^n, 0) \longrightarrow (\mathbb{C}^p, 0)$ be a flat holomorphic mapping. Then $(\varphi^{-1}(0), 0)$ is a complete intersection. As G. Tiourina showed in [11], we may define $\Phi : (\mathbb{C}^n \times \mathbb{C}^k, 0) \longrightarrow (\mathbb{C}^p \times \mathbb{C}^k, 0)$ called the miniversal deformation of $(\varphi^{-1}(0), 0)$. We are going to follow the presentation of Φ as it is done in [10].

As it is explained in [8] and [1], we are interested in computing the local fundamental group of the complement of the discriminant of Δ, discriminant of Φ, in the neighbourhood of the origin 0. Using the results of [3] we obtain that there is a generic direction of 2-planes of $\mathbb{C}^p \times \mathbb{C}^k$ and a neighbourhood U of 0 in $\mathbb{C}^p \times \mathbb{C}^k$ such that:

1) the local fundamental group of the complement of Δ at 0 is given by $U - \Delta$;

2) for almost every 2-planes P, parallel to the given one , sufficiently near to it and not passing through 0, this local fundamental group is given by $P \cap (U - \Delta)$.

Using results of D. Vohmann in [12] we obtain that for such a 2-plane P, the curve $P \cap \Delta$ has only nodes and cusps as singularities in U and the number of these nodes ~~of~~ and cusps depends only on the topology of Δ at 0.

In [7] we compute the number κ) of cusps of $P \cap \Delta$), when $(\varphi^{-1}(0), 0)$ is the germ of a hypersurface with an isolated singularity at 0 and [6] we get this number when $(\varphi^{-1}(0), 0)$ is a general complete intersection. We obtain altogether a formula for the number of nodes which is less satisfying because this formula is hardly computable directly from the equations of $(\varphi^{-1}(0), 0)$

without using elimination theory.

Now let P_0 be a sufficiently general plane going through the origin in $\mathbb{C}^p \times \mathbb{C}^k$. Then the set $\Phi^{-1}(P_0 \cap \Delta) \cap C(\Phi)$, where $C(\Phi)$ is the singular locus of Φ is a reduced curve. Let us call $\delta_1 = \dim_{\mathbb{C}} \overline{\mathcal{O}_1}/\mathcal{O}_1$, where $\mathcal{O}_1$ is the local ring of this curve at 0, and r the number of its components. Then we get:

<u>Theorem</u> Let $f_1, \dots, f_p$ be equations of $(\varphi^{-1}(0), 0)$. Then, replacing eventually $f_1, \dots, f_p$ by linear combination of $u_i f_i$ where u_i is a unit at 0, we have:

$$\varkappa = \mu(\varphi^{-1}(0), 0) - \mu(X', 0) + 2\delta_1 - r$$

where X' is the complete intersection determined by $f_1, \dots, f_{p-2}$.

In the case of hypersurface we found (cf [7]) :

<u>Corollary</u> $\quad \varkappa = \mu + 2\delta_1 - r$

But in this case $\Phi^{-1}(P_0 \cap \Delta) \cap C(\Phi)$ is a complete intersection. In this particular case we can show:

$$\mu_1 = \mu(\Phi^{-1}(P_0 \cap \Delta) \cap C(\Phi), 0) = 2\delta_1 - r + 1$$

Thus: $\quad \varkappa = \mu + \mu_1 - 1$

In [7] we show that μ_1 is not a topological invariant and using results of [9] we give a interpretation of the fact that in an analytic family μ and μ_1 are constant.

For instance when $\varphi : (\mathbb{C}, 0) \longrightarrow (\mathbb{C}, 0)$ is $\varphi(z) = z^n$ then $\varkappa = n - 2$.

Such results should be complete if one can compute the number of nodes directly from the equations.

Finally a good result should be to relate these computations to the one of the local fundamental group of the complement of the discriminant Δ at 0. Some recent works of F. Lazzeri [4], [5] and Gabrielov[2] are going in this direction.

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