数理解析研究所講究録
第 258 巻 1975 年 28-29

On the analyticity in time of solutions of initial boundary value problems for semi-linear parabolic differential equations with monotone nonlinearity

By Sunao Ōuchi

In this note we consider the following initial boundary value problems:

$$\text{(I.B.V.P.)} \quad \begin{cases} \frac{\partial u}{\partial t} = \Delta u + f(u) \\ u(t,x)|_{\partial\Omega} = 0 \\ u(0,x) = a(x) \end{cases},$$

where Ω is a bounded domain in R^n with smooth boudary $\partial\Omega$.

The purpose of this note is to report that solutions of (I.B.V.P.) are extensible holomorphically in time t to a sector $\Sigma_\theta = \{ t \in C ; |\arg t| < \theta \}$ in the complex domain which does not depend on initial values, if the nonlinear term f(u) is a monotone decreasing polynomial.

Let us now introduce definitions to state results.

Definition 1. A polynomial with real coefficients f(u) is said to be monotone or to satisfiy condition (M), if $f(0)=0$ and $f'(u) \leqq 0$ for $-\infty < u < +\infty$.

Examples. $f(u) = -u^{2p+1}$, $-u-u^3$, $-u^3-u^5$.

Definition 2. A polynomial with real coefficients f(u) is said to be monotone on $R^+ = [0, \infty)$ or satisfy condition (M_+), if $f(0) = 0$ and $f'(u) \leqq 0$ for $0 \leqq u < \infty$.

Examples. $f(u) = -u^{2p}$, $-u-u^4$, $-u^2-u^6$.

Theorem 1. Suppose that the nonlinear term f(u) in (I.B.V.P.) satisfies condition (M) and the initial value $a=a(x)$ is real-valued and boundedly continuous in Ω . Then there is a sector $\Sigma_{\theta_0} = \{ t ; |\arg t| < \theta_0 \}$ in the complex domain which is independent of a(x) such that the solution u(t,x) of (I.B.V.P.) is analytically extensible in t to the sector .

Theorem 2. Suppose that the nonlinear term f(u) in (I.B.V.P.) satisfies condition (M_+) and the initial value $a = a(x)$ is a nonnegative and boundedly continuous function. Then there is a sector $\Sigma_{\theta_0} = \{t; |\arg t| < \theta_0\}$ in the complex domain which does not depend on a(x) such that the nonnegative solution u(t,x) of (I.B.V.P.) is analitically extensible in t to the sector.

Details are published in

S. Ōuchi, On the analiticity in time of solutions of initial boundary value problems for semi-linear parabolic differential equations with monotone nonlinearity, J. Fac. Sci. Univ. Tokyo, Vol. 21, No.1 (1974) 19-41.

Department of Mathematics
Faculty of Science
Tokyo Metropolitan University
Fukazawa, Setagaya, Tokyo
158 Japan