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Some problems in char. $p > 0$

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Let X be a smooth proper scheme over k, k being algebraically closed of char. $p > 0$.

I. Concerning De-Rham-Witt complex :

a) If X is an abelian scheme, try to compute $H^i(X, W_n\Omega^j)$ in terms of $H^1_{crys}(X, W)$.
   If it is not possible, what are the new invariants one has to introduce ?

b) Define a Poincaré duality in terms of D.R.W. (?)
   Probably one will have to lift in char. 0 the Residue calculus.

c) If Y is another smooth scheme, what are the relations between $DRW(X \times Y)$ and $DRW(X)$, $DRW(Y)$?

d) Look for some geometric interpretation of the Cartier modules $H^i(X, W\Omega^j)$/V-Torsion, generalizing the Cartier modules of formal Brauer groups $H^i(X, W\Omega^0)$/V-Torsion.

II. Torsion phenomena in problems of lifting from char. p to char. 0 :

Let R be a complete discrete valuation ring of unequal characteristics, and of ramification index e. Let $\mathfrak{X} \longrightarrow R$ be a smooth proper scheme with closed fibre $\bar{X} \longrightarrow k$.

1) If $e < p - 1$ (or $2e < p - 1$?), can we have non-closed 1-forms on $\bar{X}$?

2) Let L be an ample invertible sheaf on x.

If $e \leq p - 1$, I have proved in my paper at Colloque de Rennes (cf. Asterisque. 64 (1979)) that $H^1(\bar{X}, L^{-1}) = 0$.

If $\mathfrak{X} \longrightarrow R$ is of relative dimension ≥ 3, what can be said about $H^2(\bar{X}, L^{-1})$?

If the dimension of the formal Brauer group does not jump from generic fibre to closed fibre, my proof works also for the H^2. So, can the dimension of the Brauer groups jump ?

III. Problems on surfaces in char. $p > 0$:

X is a proper and smooth surface.

1) (Analog in char. p of the Castelnuovo theorem) Suppose $c_2(X) < 0$ (c_2 = top. Euler characteristic). Does X admit a fibration $f: X \longrightarrow C$ such that genus $C \geq 2$ and the generic fibre of f is of geometric genus 0 ? (A surface with such a fibration is called a false ruled surface, in case it is not a (true) ruled surface.)

2) Let $f: X \longrightarrow C$ be a false ruled surface with genus $C > 2$ and with generic fibre of arithmetic genus ≥ 2.
Is $\chi(\mathcal{O}_X) \geq 0$? (Notice that $12\chi(\mathcal{O}_X) = c_1^2 + c_2$. The interesting case is where $c_1^2 > 0$ and $c_2 < 0$.)

3) Let X be a K3 or an abelian surface, H a general hyperplane section.
Is the difference between jacobian of H and jacobian of X (Picard variety) ordinary ?