

Recent development of
differentiable dynamical systems in Japan

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§1. Introduction

We shall give a brief survey on recent development of differentiable dynamical systems in Japan. The works surveyed in this article were published mostly between 1970 and 1980 by Japanese mathematicians and scientists.

The objects of the works surveyed in this article are mostly limited to the qualitative (topological) theory of differentiable dynamical systems and its closely related topics. We do not intend to be complete, but we would be very grateful if the reader would kindly point out the works left unmentioned in this article.

The general references for this field of research are the followings: D.V. Anosov and Ja.G. Sinai [2], R. Bowen [17], Ja.G. Sinai [122], and S. Smale [124].

§2. Stability and generic properties

The notion of structural stability was first introduced by A. Andronov and L. Pontrjagin in 1937. Study of structural stability was continued by Lefschetz's school. In 1960, M. Peixoto obtained the necessary and sufficient conditions for the smooth flows on 2-dimensional compact differentiable manifolds to be structurally stable generalizing the result of Andronov-Pontrjagin. He also obtained that the set of all structurally stable flows

of class C^1 on a 2-dimensional compact differentiable manifold is dense in the set of all C^1 -flows on it in C^1 -topology. Based on this fact, S. Smale introduced the notion of Morse-Smale system and started to begin the study of differentiable dynamical systems. Morse-Smale systems exist on any compact differentiable manifolds.

In 1962, D.V. Anosov introduced the notion of Anosov system and proved its structural stability. Using this result, he succeeded in proving the structural stability of geodesic flows on negatively curved manifolds. Anosov systems exist only on the very restricted manifolds. In 1970, J. Palis and S. Smale proved that Morse-Smale systems were structurally stable.

These results are unified and generalized as follows.

Theorem (J. Robbin, 1971; R.C. Robinson, 1974) Axiom A systems with strong transversality conditions are structurally stable.

Since Anosov systems and Morse-Smale systems satisfy Axiom A and strong transversality condition, the above theorem is a generalization of Anosov and Palis-Smale. Also, the following is proved.

Theorem (S. Smale, 1970; C. Pugh and M. Shub, 1970) Axiom A systems with no cycle condition are Ω -stable.

Converse of the above two theorems is conjectured and investigated by S. Smale, J. Palis, S. Newhouse, R. Mañé, V.A. Pliss, S.T. Liao, and others. Recently, A. Sannami succeeded in proving the converse problems for C^2 -diffeomorphisms on 2-dimensional closed manifolds.

In 1966, Smale showed that the set of all structurally stable systems was not dense in the set of all dynamical systems in higher dimensions. Many tried to find a suitable class of dynamical systems which is dense in the set of all dynamical systems and is stable in some sense. Toward this problem, G. Ikegami introduced the notion of weak stability and studied it in [33], [34], and [36]. The set of all weakly stable systems properly contains the set of all structurally stable systems in general, but it is not dense in the set of all dynamical systems. Under Axiom A, weak stability implies structural stability.

Generic properties for dynamical systems are very important in the study of dynamical systems. A generic property holds for almost all dynamical systems. The first important and basic generic properties were obtained by I. Kupka and S. Smale in 1963. C. Pugh's closing lemma obtained in 1967 implies that Axiom A(b) is a generic property, where Axiom A(b) states that the set of all periodic points is dense in the nonwandering set.

T. Koike [60] says that the following is generic: If M is a 2-dimensional closed manifold and $f : M \rightarrow M$ is a diffeomorphism, then the interior of the nonwandering set of f is empty or f is an Anosov diffeomorphism. As a corollary to this, the following is generic: If M is a 2-dimensional manifold and is not a torus, then a diffeomorphism $f : M \rightarrow M$ has empty interior for the nonwandering set.

Y. Togawa [128], [129] show that the following properties are generic: (i) An Axiom A diffeomorphism has only trivial centralizers. (ii) A diffeomorphism has no k -root for any integer

$k, k \neq \pm 1$. As a corollary to (ii), we cannot generically imbed a diffeomorphism into a smooth flow.

§3. Anosov systems

D.V. Anosov [1] gives basic results on Anosov systems. Many important results concerning the ergodic properties of Anosov systems were obtained including Ja.G. Sinai's results. Besides these results, J. Moser, J. Franks, S. Newhouse, W.M. Hirsch, A. Manning, and others obtained various important results. The following results are contributions of our colleagues.

K. Shiraiwa [116] gives a necessary condition for the existence of an Anosov diffeomorphism and gives examples of manifolds which do not admit an Anosov diffeomorphism. K. Yano [150] shows that there are no transitive Anosov diffeomorphisms on negatively curved manifolds.

K. Takaki [126] shows that Anosov diffeomorphisms are structurally stable in the space of all lipemorphisms (Lipschitz homeomorphisms). K. Kato and A. Morimoto [53] shows the topological stability of Anosov flows and the triviality of centralizers of Anosov flows. The last property of Kato-Morimoto's result is generalized by M. Oka [93] for expansive flows.

There are other results by N. Otsuki [103], [104] and A. Morimoto [85].

§4. Hyperbolic sets, Axiom A, and related topics

The notion of Axiom A was introduced by Smale to include both Morse-Smale and Anosov systems and to give new examples such as horseshoe systems. Theory of Axiom A systems plays the most important role in Smale's theory of differentiable dynamical

systems. Many important results are obtained by Smale's school and we need a book to describe them. We give here some results obtained by our colleagues.

M. Kurata [69] states that Axiom A(a) does not imply Axiom A(b) for a diffeomorphism $f : M \rightarrow M$ if $\dim M \geq 4$. This gave a counter example of Smale's conjecture. Similar result was given by A. Dankner [21] independently.

Ja.G. Sinai [121] constructed Markov partitions for Anosov diffeomorphisms, and R. Bowen [15] generalized this for Axiom A diffeomorphisms restricted on their basic sets. A Markov partition of a system gives a semi-conjugacy from a suitable symbolic dynamical system to the system. It plays an important role in both ergodic theory and qualitative theory. M. Kurata [66], [67], [68] investigated hyperbolic sets and obtained Markov partitions of hyperbolic sets.

K. Kato and A. Morimoto [54] generalizes their work [53] and shows that an Axiom A flow with no C^0 - Ω -explosion is topologically Ω -stable.

Pseudo-orbit tracing property (sometimes called stochastic stability) is important for the study of dynamical systems (For example, see R. Bowen [16], [17]). K. Sawada [109], A. Morimoto [86] and K. Kato [50] show that an Axiom A diffeomorphism with strong transversality condition has the pseudo-orbit tracing property. Using this result they affirmatively solved Takens' conjecture. Related topics to the above theorem is treated by K. Kato [51], A. Morimoto [87], T. Sasaki [107], K. Yano [147], and N. Aoki [7].

It seems that specification property introduced by R. Bowen and used by R. Bowen, K. Sigmund [120], D.A. Lind [71], and others is very important for both ergodic theory and qualitative theory of dynamical systems. N. Aoki [4], [5], [6], [7], N. Aoki, M. Dateyama, and M. Komuro [11], M. Dateyama [20] investigate the dynamics of the automorphisms of compact metric groups including specification property and obtained many interesting results.

Expansiveness is also important for the study of dynamical systems. Anosov diffeomorphisms and the restriction of Axiom A diffeomorphisms on their basic sets are expansive. The followings are works related to expansive homeomorphisms: N. Aoki and M. Dateyama [10], N. Aoki and C. Saikawa [12], A. Koriyama [62], A. Koriyama and Y. Matsuoka [63], A. Koriyama and T. Nagase [64], M. Kouno [65], and M. Oka [93], [94].

§5. Topological entropy

The notion of topological entropy was introduced by R.L. Adler, A.G. Konheim, and M.H. McAndrew in 1965. It is closely related to the measure theoretic entropy. Many important contributions were done by many mathematicians. The followings are by our colleagues.

S. Ito [45] estimated the topological entropy of a C^1 -diffeomorphism of a compact Riemannian manifold from above. This is generalized by R. Bowen for a C^1 -map on a Riemannian manifold.

K. Sasano [108] investigated the topological entropy of the continuous maps of the circle in detail.

K. Yano [148] shows that the topological entropy of a

homeomorphism of a compact topological manifold of dimension greater than one is generically equal to the infinity. This holds for continuous maps, too.

Other results are as follows: N. Aoki [3], T. Hamachi and H. Totoki [22], M. Hata [23], T. Koike [61], T. Ohno [96], M. Osikawa and T. Hamachi [100], and K. Yano [149].

§6. Chaos and related topics

E.N. Lorenz [72] derived the following system of ordinary differential equation from the convection equation:

$$\begin{cases} x' = -\sigma x + \sigma y \\ y' = rx - y - xz \\ z' = -bz + xy. \end{cases}$$

He studied this equation numerically in the case of $\sigma = 10$, $b = \frac{8}{3}$, $r = 28$ and found that the solutions exhibited chaotic behavior. Later, Y. Ueda and H. Kawakami found the similar result from Duffing's equations. Rössler also found many examples of chaos from systems of ordinary differential equations in dimension three.

In 1974, R.M. May [83] found the similar phenomena for the difference equation of the first order arising from biological populations with nonoverlapping generations. M. Hénon [28] gave a two dimensional mapping exhibiting chaos which is related to both Lorenz model and May model.

Since then many investigations were done by many scientists. We list some of our colleagues' results:

- (1) Concerning Lorenz model: J. Nagashima and I. Shimada [88], [89], I. Shimada [112]; I. Shimada and T. Nagashima [113], [114];

K. Tomita and I. Tsuda [133].

(2) Concerning May model: S. Matsumoto [75], [76]. S. Ushiki, M. Yamaguti, and H. Matano [140].

(3) Others: H. Daido [19], K. Ito [44], [44a], T. Kai and K. Tomita [47], K. Tomita and H. Daido [130], K. Tomita and T. Kai [131], K. Tomita and I. Tsuda [132], S. Ushiki [138].

Mathematical treatment for chaos began with the work of T. Li and J. Yorke [70] on the one dimensional mapping. Their result is generalized by F. Marotto [73] to a higher dimensional case. K. Shiraiwa and M. Kurata [119] generalizes both Marotto's result and S. Smale's work [123]. M. Yamaguti and H. Matano [141], M. Yamaguti and S. Ushiki [143] found chaos discretizing some ordinary differential equations.

Related works are as follows: M. Hata [24], S. Ito, S. Tanaka, and H. Nakada [46], Y. Oono [97], Y. Oono and M. Osikawa [98], Y. Oono and Y. Takahashi [99], M. Osikawa and Y. Oono [101], Y. Oshime [102], M. Yamaguti and S. Ushiki [142], [144].

§7. Dynamical systems from electrical networks

There are many interesting dynamical systems arising from electrical networks. R. Brayton and J. Moser gave a basic treatment on this problem, and S. Smale [125] extended and gave modern formulation to this problem.

The followings are a part of the results obtained by the collaboration of our colleagues: L.O. Chua, T. Matsumoto, and S. Ichiraku [18], S. Ichiraku [30], [31], [32]; T. Matsumoto [77], [78], [79]; T. Matsumoto, L.O. Chua, H. Kawakami, and S. Ichiraku [80]; T. Matsumoto, L.O. Chua, and A. Makino [81];

T. Matsumoto and G. Ikegami [82].

Duffing's equations arising from nonlinear oscillations and electrical networks are very interesting dissipative systems to study.

Followings are some of our results C. Hayashi and Y. Ueda [25]; C. Hayashi, Y. Ueda, and H. Kawakami [26]; N. Kakiuchi [48]; H. Kawakami [55], [56], [57]; F. Nakajima [92]; K. Shiraiwa [117], [118]; Y. Ueda [134], [135], [136], [137].

§8. Miscellany

(1) G. Ikegami [33], [34], [38] investigated the relation between diffeomorphisms and their suspension flows.

I. Ishii [42], [43] investigated minimal flows. Other results related to the differentiable dynamical systems are as follows: K. Hayashi [27], F. Ichikawa [29], G. Ikegami [35], S. Matsumoto [74], K. Sawada [110], S. Ushiki [138].

(2) Concerning the limit cycles of planary flows, K. Yamato's work [145] is interesting. G. Ikegami's work [40] is related to this problem.

M. Oka [95] treated difficult problem of classifications of a certain type of homogeneous differential equations on the plane.

Other results related to the ordinary differential equations are J. Kato and F. Nakajima [49], F. Nakajima [90], [91], and G.R. Sell and F. Nakajima [111].

Bibliography

- [1] D.V. Anosov, Geodesic flows on closed Riemannian manifolds with negative curvature, Proc. Steklov Inst. Math., 90(1967), 103-167.
- [2] D.V. Anosov and Ja.G. Sinai, Some smooth ergodic systems, Russian Math. Surveys, 22 No.5(1967), 103-167.
- [3] N. Aoki, Topological entropy of distal affine transformations on compact abelian groups, J. Math. Soc. Japan, 23(1971), 11-17.
- [4] N. Aoki, Group automorphisms with finite entropy, Monatshefte Math., 88(1979), 275-285.
- [5] N. Aoki, A simple proof of the Bernoullicity of ergodic automorphisms on compact abelian groups, To appear in Israel Math. J..
- [6] N. Aoki, A group automorphism is a factor of a direct product of a zero entropy automorphism and a Bernoulli automorphism, To appear in Fund. Math..
- [7] N. Aoki, Stochastic stability and specification of zero-dimensional automorphisms, Preprint.
- [8] N. Aoki, Zero-dimensional automorphisms having a dense orbit, To appear in Japan. Math. J.,
- [9] N. Aoki, Zero-dimensional automorphisms with specification and Bernoulli measures of group automorphisms, Preprint.
- [10] N. Aoki and M. Dateyama, The relationship between algebraic numbers and expansiveness of automorphisms on compact abelian groups, To appear in Fund. Math..
- [11] N. Aoki, M. Dateyama, and M. Komuro, Solenoidal properties with specification, Preprint.

- [12] N. Aoki and C. Saikawa, Ergodic properties of expansive automorphisms, *Michigan Math. J.*, 22(1975), 337-341.
- [13] N. Aoki and H. Totoki, Ergodic automorphisms of T^∞ are Bernoulli transformations, *Publ. RIMS Kyoto Univ.*, 10(1975), 535-544.
- [14] V.I. Arnold and A. Avez, *Problèmes ergodiques de la mécanique classique*, Gauthier-Villars, 1966.
- [15] R. Bowen, Markov partitions for Axiom A diffeomorphisms, *Amer. J. Math.*, 92(1970), 725-747.
- [16] R. Bowen, Equilibrium states and the ergodic theory of Anosov diffeomorphisms, *Lecture Notes in Math.*, 470(1975), Springer.
- [17] R. Bowen, On Axiom A diffeomorphisms, *Regional Conference Series in Math.*, 35(1977), Amer. Math. Soc..
- [18] L.O. Chua, T. Matsumoto and S. Ichiraku, Geometric properties of resistive nonlinear n-ports: Transversality, structural stability, reciprocity and anti-reciprocity, *IEEE Trans. Circuits and Systems*.
- [19] H. Daido, Analytical contributions for the appearance of homoclinic and heteroclinic points of 2-dimensional mapping -The case of the Hénon mapping, *Progr. Theor. Phys.*, 63(1980), 1190-1201.
- [20] M. Dateyama, Invariant measures for homeomorphisms with weak specification, Preprint.
- [21] A. Dankner, On Smale's Axiom A dynamical systems, *Ann. of Math.*, 107(1978), 517-553.
- [22] T. Hamachi and H. Totoki, A remark on the topological

- entropy, *Memoirs Fac. Sci. Kyushu Univ.*, 25 Ser.A.(1971), 300-303.
- [23] M. Hata, On kneading determinant and topological entropy of quadratic functions, 力学系理論と関連諸分野の総合的研究, 昭和54年度記録集.
- [24] M. Hata, Dynamics of Caianiello's equation, Preprint.
- [25] C. Hayashi and Y. Ueda, Behavior of solutions for certain types of non-linear differential equations of the second order, *Non-linear Vibration Problems, Zagadnienia Drgan Nieliniowych*, 14(1973), 341-351.
- [26] C. Hayashi, Y. Ueda and H. Kawakami, Transformation theory as applied to the solutions of non-linear differential equations of the second order, *Internat. J. Non-linear Mechanics*, 4(1969), 235-255.
- [27] K. Hayashi, Periodic orbits of Lagrangian systems, Preprint.
- [28] M. Hénon, A two-dimensional mapping with a strange attractor, *Commun. Math. Phys.*, 50(1976), 69-77.
- [29] F. Ichikawa, Notes on finitely determined singularities of formal vector fields, Preprint.
- [30] S. Ichiraku, On the transversality conditions in electrical circuits, *Yokohama Math. J.*, 25(1977), 85-89.
- [31] S. Ichiraku, On singular points of electrical circuits, *Yokohama Math. J.*, 26(1978), 151-156.
- [32] S. Ichiraku, Connecting electrical circuits, transversality and well-posedness, *Yokohama Math. J.*, 27(1979), 111-126.
- [33] G. Ikegami, On classification of dynamical systems with cross-sections, *Osaka J. Math.*, 6(1969), 419-433.

- [34] G. Ikegami, Flow equivalence of diffeomorphisms, I, II, Correction, Osaka J. Math., 8(1971), 49-69, 71-76, 9(1972), 335-336.
- [35] G. Ikegami, A note for knots and flows on 3-manifolds, Proc. Japan Acad., 47(1971), 29-30.
- [36] G. Ikegami, On weak concept of stability, Nagoya Math. J., 55(1974), 161-169.
- [37] G. Ikegami, On structural stability and weak stability of dynamical systems, Manifolds-Tokyo 1973, 211-217, Univ. of Tokyo Press, 1975.
- [38] G. Ikegami, Topologically unequivalent diffeomorphisms whose suspensions are C^∞ -equivalent, Proc. Japan Acad., 53(1977), 11-12.
- [39] G. Ikegami, Weak stability implies structural stability under Axiom A, Proc. Japan Acad., 53(1977), 61-63.
- [40] G. Ikegami, On uniformly asymptotically stable orbits,
力学系理論と関連諸分野の総合的研究, 昭和54年度記録集.
- [41] H. Inoue and K. Yano. The Gromov invariant of negatively curved manifolds, Preprint.
- [42] I. Ishii, On a non-homogeneous flow on the 3-dimensional torus, Functional Eq., 17(1974), 231-248.
- [43] I. Ishii, On the first cohomology of a minimal set, Tokyo J. Math., 1(1978), 41-56.
- [44] K. Ito. Periodicity and chaos in great earthquake occurrence, Earth and Planetary Science Letters.
- [44a] K. Ito, Chaos in the Rikitake two-disc dynamo system,

To appear in Earth and Planetary Science Letters.

- [45] S. Ito, On estimate from above for an entropy and the topological entropy of a C^1 -diffeomorphism, Proc. Japan Acad., 46(1970), 226-230.
- [46] S. Ito, S. Tanaka and H. Nakada, On unimodal linear transformations and chaos, I, II, Tokyo J. Math..
- [47] T. Kai and K. Tomita, Statistical mechanics of deterministic chaos -The case of one-dimensional discrete process, Preprint.
- [48] N. Kakiuchi, 3階 Duffing 型方程式の有界性について, 準備中.
- [49] J. Kato and F. Nakajima, On Sacker-Sell's theorem for a linear skew product flow, Tôhoku Math. J., 28(1976), 79-88.
- [50] K. Kato, Stochastic stability of diffeomorphisms, Research Reports of Kôchi Univ., Nat. Sci., 26(1977), 33-43.
- [51] K. Kato. Stochastic stability of Anosov diffeomorphisms, Nagoya Math. J., 69(1978), 121-129.
- [52] K. Kato, On periodic points of stable endomorphisms, Memoirs Fac. Sci., Kôchi Univ., 1 Ser. A(1980), 59-67.
- [53] K. Kato and A. Morimoto, Topological stability of Anosov flows and their centralizers, Topology, 12(1973), 255-273.
- [54] K. Kato and A. Morimoto, Topological Ω -stability of Axiom A flows with no Ω -explosions, J. Diff. Eq., 34(1979), 464-481.
- [55] H. Kawakami, Qualitative study on the solutions of Duffing's equation, Thesis, Kyoto Univ., 1973.
- [56] H. Kawakami, Sur les points fixes des itérés d'un diffeomorphisme dans le voisinage d'un point homocline, J. Diff. Eq., 14(1973), 441-461.
- [57] H. Kawakami, Duffing 方程式の解の概要 (II) - 3階 Duffing

型方程式にみられるカオス, 力学系理論と関連諸分野の総合的研究,
昭和54年度記録集.

- [58] T. Koike, Differentiable dynamical systems on noncompact manifolds, 京大数理解析研究所講究録, 284(1976), 96-112.
- [59] T. Koike, Anosov 微分同相写像と Axiom A の関係について, 数学, 29(1977), 228.
- [60] T. Koike, On nonwandering sets of C^1 -diffeomorphisms of surfaces, Nagoya Math. J., 79(1980), 1-22.
- [61] T. Koike, On endomorphisms of a product space of surfaces, Preprint.
- [62] A. Koriyama, Expansiveness, h-expansiveness and asymptotical h-expansiveness on compact manifolds, Preprint.
- [63] A. Koriyama and Y. Matsuoka, On expansive homeomorphisms on certain compact metric spaces, Preprint.
- [64] A. Koriyama and T. Nagase, On h-expansive homeomorphisms and asymptotically h-expansive homeomorphisms on a compact topological manifolds, Preprint.
- [65] M. Kouno, On expansive homeomorphisms on manifolds, Preprint.
- [66] M. Kurata, Hartman's theorem for hyperbolic sets, Nagoya Math. J., 67(1977), 41-52.
- [67] M. Kurata, On maximal hyperbolic sets, Hokkaido Math. J., 6(1977), 345-349.
- [68] M. Kurata, Markov partitions of hyperbolic sets, J. Math. Soc. Japan, 31(1979), 39-52.
- [69] M. Kurata, Hyperbolic nonwandering sets without dense periodic points, Nagoya Math. J., 74(1979), 77-86.

- [70] T. Li and J. Yorke, Period three implies chaos, Amer. Math. Monthly, 82(1975), 985-992.
- [71] D.A. Lind, Ergodic group automorphisms and specification, Lecture Notes in Math., 729(1979), 93-104, Springer.
- [72] E.N. Lorenz, Deterministic nonperiodic flow, J. Atmospheric Sci., 20(1963), 130-141.
- [73] F.R. Marotto, Snap-back repellers imply chaos in \mathbb{R}^n , J. Math. Anal. Appl., 63(1978), 199-223.
- [74] S. Matsumoto, There are two isotopic Morse-Smale diffeomorphisms which cannot be joined by simple arcs, Invent. Math., 51(1979), 1-7.
- [75] S. Matsumoto, On periodic points of quadratic functions, Preprint.
- [76] S. Matsumoto, On the bifurcation of periodic points of one dimensional dynamical systems of a certain kind, Preprint.
- [77] T. Matsumoto, On the dynamics of electrical networks, J. Diff. Eq., 21(1976), 179-196.
- [78] T. Matsumoto, Dynamical systems arising from electrical networks, Dynamical systems, vol.2 (Ed. Cesari, Hale and La Salle), Academic Press, 1976, 285-290.
- [79] T. Matsumoto, Passivity and eventual passivity of electrical networks, Dynamical Systems (Ed. Bednarck and Cesari), Academic Press, 1977, 459-463.
- [80] T. Matsumoto, L.O. Chua, H. Kawakami and S. Ichiraku, Geometric properties of nonlinear dynamic networks, Preprint.
- [81] T. Matsumoto, L.O. Chua and A. Makino, On the implications of capacitors-only loops in nonlinear networks, IEEE Trans.

Circuits and Systems (1979).

- [82] T. Matsumoto and G. Ikegami, Strong stability of nonlinear resistive n -ports, 力学系理論と関連諸分野の総合的研究, 昭和54年度記録集.
- [83] R.M. May, Biological populations with nonoverlapping generations; Stable points, stable cycles, and chaos, *Science*, 186(1974), 645-647.
- [84] A. Morimoto, On periodic orbits of stable flows, *Hokkaido Math. J.*, 1(1972), 298-304.
- [85] A. Morimoto, Anosov flows on a compact manifold, *Diff. Geom. (in honor of K. Yano)*, Kinokuniya, 1972, 281-290.
- [86] A. Morimoto, Stochastically stable diffeomorphisms and Takens' conjecture, 京大数理解析研究所講究録, 303(1977), 8-24.
- [87] A. Morimoto, Stochastic stability of group automorphisms, 京大数理解析研究所講究録, 313(1977), 148-164.
- [88] T. Nagashima and I. Shimada, On the C-system-like property of the Lorenz system, *Progr. Theor. Phys.*, 58(1977), 1318-1319.
- [89] T. Nagashima and I. Shimada, Strange attractor をめぐって, 統計力学の立場から, 日本物理学会誌, 39(1978), 505-510.
- [90] F. Nakajima, A stability criterion of diagonal dominance type, *SIAM J. Math. Anal.*, 9(1978), 815-824.
- [91] F. Nakajima, Periodic time dependent gross-substitute system, *SIAM J. Appl. Math.*, 36(1979), 421-427.
- [92] F. Nakajima, Duffing 方程式に現われる無限個の周期解について, 力学系理論と関連諸分野の総合的研究, 昭和54年度記録集.
- [93] M. Oka, Expansive flows and their centralizers, *Nagoya Math. J.*, 64(1976), 1-15.

- [94] M. Oka, On stabilities of expanding endomorphisms of compact manifolds, TRU Math., 14(1978), 29-38.
- [95] M. Oka, Classifications of homogeneous differential systems of the form
$$\begin{cases} x' = ax^3 + bx^2y + cxy^2 + dy^3 \\ y' = ex^3 + fx^2y + gxy^2 + hy^3 \end{cases}, \text{ Preprint.}$$
- [96] T. Ohno, A weak equivalence and topological entropy, Publ. RIMS Kyoto Univ., 16(1980), 289-298.
- [97] Y. Oono, Integral equations for absolutely continuous invariant measure for "observable chaos", Preprint.
- [98] Y. Oono and M. Osikawa, Chaos in nonlinear difference equations, I, Qualitative study of formal chaos, Preprint.
- [99] Y. Oono and Y. Takahashi, Chaos, external noise and Fredholm theory, Progr. Theor. Phys. Letters, 63(1980), 1804-1807.
- [100] M. Osikawa and T. Hamachi, Topological entropy of a non-irreducible intrinsic Markov shift, Memoirs Fac. Sci. Kyushu Univ., 25 Ser.A(1971), 296-299.
- [101] M. Osikawa and Y. Oono, Chaos in C^0 -endomorphisms of interval, To appear in Publ. RIMS Kyoto Univ..
- [102] Y. Oshime, Modified-Euler 法による差分法のカオス, 力学系理論と関連諸分野の総合的研究, 昭和54年度記録集.
- [103] N. Otsuki, Geodesic flows and isotopic flows, Proc. Japan Acad., 45(1969), 10-13.
- [104] N. Otsuki, A characterization of Anosov flows for geodesic flows, Hiroshima Math. J., 4(1974), 374-412.
- [105] J. Robbin, A structural stability theorem, Ann. of Math., 94(1971), 447-493.

- [106] A. Sannami, The stability theorem for discrete dynamical systems on 2-dimensional manifolds, In preparation.
- [107] T. Sasaki, Some examples of stochastically stable homeomorphisms, Nagoya Math. J., 71(1978), 97-106.
- [108] K. Sasano, Topological entropy and periodic points of maps of the circle, To appear in J. Fac. Sci. Univ. of Tokyo.
- [109] K. Sawada, Extended f -orbits are approximated by orbits, Nagoya Math. J., 79(1980), 33-45.
- [110] K. Sawada, On the iterations of diffeomorphisms without C^0 - Ω -explosions. An example, Proc. Amer. Math. Soc., 79(1980), 110-112.
- [111] G.R. Sell and F. Nakajima, Almost periodic gross-substitute dynamical systems, Tôhoku Math. J., 32(1980), 255-263.
- [112] I. Shimada, Gibbsian distribution on the Lorenz attractor, Progr. Theor. Phys., 62(1979), 61-69.
- [113] I. Shimada and T. Nagashima, The iterative transition phenomenon between periodic and turbulent states in a dissipative dynamical systems, Progr. Theor. Phys., 59(1978), 1033-1036.
- [114] I. Shimada and T. Nagashima, A numerical approach to ergodic problems of dissipative dynamical systems, Progr. Theor. Phys., 61(1979), 1605-1611.
- [115] K. Shiraiwa, Anosov 微分同相写像について, 数学, 26(1974), 97-108.
- [116] K. Shiraiwa, Some conditions on Anosov diffeomorphisms, Manifolds-Tokyo 1973, 205-209, Univ. of Tokyo Press, 1975.
- [117] K. Shiraiwa, Boundedness and convergence of solutions of Duffing's equation, Nagoya Math. J., 66(1977), 151-166.

- [118] K. Shiraiwa, A generalization of the Levinson-Massera's equalities, Nagoya Math. J., 67(1977), 121-138.
- [119] K. Shiraiwa and M. Kurata, A generalization of a theorem of Marotto, Proc. Japan Acad., 55 Ser.A(1979), 286-289.
- [120] K. Sigmund, On dynamical systems with the specification property, Trans. Amer. Math. Soc., 190(1974), 285-299.
- [121] Ja.G. Sinai, Markovian partitions and Y-diffeomorphisms, Funct. Anal. Appl., 2 No.1(1968), 64-89.
- [122] Ja.G. Sinai, Introduction to ergodic theory, Princeton Univ. Press, 1977.
- [123] S. Smale, Diffeomorphisms with many periodic points, Diff. and Comb. Topology, Princeton Univ. Press, 1964.
- [124] S. Smale, Differentiable dynamical systems, Bull. Amer. Math. Soc., 73(1967), 747-817.
- [125] S. Smale, On the mathematical foundation of electrical circuit theory, J. Diff. Geom., 7(1972), 193-210.
- [126] K. Takaki, Lipeomorphisms close to an Anosov diffeomorphism, Nagoya Math. J., 53(1974), 71-82.
- [127] F. Takens, Tolerance stability, Dynamical Systems-Warwick 1974, Lecture Notes in Math., 468(1975), 293-304, Springer.
- [128] Y. Togawa, Generic Morse-Smale diffeomorphisms have only trivial symmetries, Proc. Amer. Math. Soc., 65(1977), 145-149.
- [129] Y. Togawa, Centralizers of C^1 -diffeomorphisms, Proc. Amer. Math. Soc., 71(1978), 289-293.
- [130] K. Tomita and H. Daido, Possibility of chaotic behavior and multi-basins in forced glycolytic oscillations, To appear in Physics Letters A.

- [131] K. Tomita and T. Kai, Chaotic behavior of deterministic orbits: The problem of turbulent phase, *Progr. Theor. Phys.*, Suppl. 64(1978), 280-294.
- [132] K. Tomita and I. Tsuda, Chaos in the Belousov-Zhabotinsky reaction in a flow system, *Physics Letters*, 71(1979), 489.
- [133] K. Tomita and I. Tsuda, Towards the interpretation of the global bifurcation structure of the Lorenz system - A simple one-dimensional model, Preprint.
- [134] Y. Ueda, Some problems in the theory of nonlinear oscillations, Nippon Printing and Publishing Company, Ltd., 1968.
- [135] Y. Ueda, Randomly transitional phenomena in the system governed by Duffing's equation, *J. Statistical Phys.*, 20(1979), 181-196.
- [136] Y. Ueda, Steady motions exhibited by Duffing's equation - A picture book of regular and chaotic motions, To appear.
- [137] Y. Ueda, Explosion of strange attractors exhibited by Duffing's equation, To appear.
- [138] S. Ushiki, The modulation of sunspot numbers, To appear in *Solar Physics*.
- [139] S. Ushiki, Unstable manifolds of analytic dynamical systems, To appear in *J. of Math. Kyoto Univ.*.
- [140] S. Ushiki, M. Yamaguti and H. Matano, Discrete population models and chaos, *Lecture Notes in Numerical and Applied Math.*, 2(1979), Kinokuniya.
- [141] M. Yamaguti and H. Matano, Euler's finite difference scheme and chaos, *Proc. Japan Aca.*, 55(1979), 78-80.

- [142] M. Yamaguti and S. Ushiki, Discrete competition systems and chaos, Proc. Toulouse Internat. Conf. on Nonlinear Analysis, 1979.
- [143] M. Yamaguti and S. Ushiki, Discrétisation et chaos, C.R. Acad. Sc. Paris, 290(1980), 637-640.
- [144] M. Yamaguti and S. Ushiki, Chaos in numerical analysis of ordinary differential equations, Preprint.
- [145] K. Yamato, An effective method of counting the number of limit cycles, Nagoya Math. J., 76(1979), 35-114.
- [146] K. Yano, Asymptotic cycles on two-dimensional manifolds, 京大数理解析研究所講究録, 331(1978).
- [147] K. Yano. Topologically stable homeomorphisms of the circle, Nagoya Math. J., 79(1980), 145-149.
- [148] K. Yano, A remark on the topological entropy of homeomorphisms, Invent. Math., 59(1980), 215-220.
- [149] K. Yano, Topological entropy of foliation preserving diffeomorphisms, To appear.
- [150] K. Yano, There are no transitive Anosov diffeomorphisms on negatively curved manifolds, Preprint.