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ANALYSIS FOR THE HEAT CONVECTION PROBLEMS BY INTEGRAL EQUATION TECHNIQUE

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This paper is devoted to the study of Heat convection in an enclosed rectangular cavity, the upper and lower walls are maintained at different but uniform temperatures. Two alternate boundary conditions for temperature on side walls namely, a perfect insulation and linear variation, are considered. The Navier-Stokes equations in their vorticity-stream function formulation and Energy equation, are solved by an integral equation technique. Numerical results in the forms of stream lines, equi-vorticity lines and isothermals are presented for a variety of Rayleigh numbers up to 10^5 for fluids with Prandtl number unity, which include transient and steady-state solutions. Steady-state easily exist in 35th for insulated and 25th for that of linear one. At $Ra=10^5$ for linear type boundary conditions, reverse flow in the upper region near the cold wall, appear at early time steps, which decreases in size as time advances.

INTRODUCTION:

Solutions of viscous flow problems which are governed by the Navier-Stokes equations, create such analytical difficulties that numerical methods seems to be the reasonable means to obtain them. Further, since these equations contain non-linear term and higher order derivatives of velocity, it is difficult to obtain stable numerical solutions at a given parameter, and if solutions of the time-dependent forms are to be achieved computing times become very large. Therefore, stable numerical schemes capable of reducing computing times are much in demand. In contrast to the usual finite-difference or finite-element approximation methods, which contain extensive numerical literature in them, we diverted our attentions to develop integral equation scheme. Though this scheme is not new, its use in simulating fluid flows has been limited. In early efforts, it is developed and testified and verified on driven cavity flow problem both for Steady[1] and Unsteady[2] Navier-Stokes equations. Since little is known about its stability with time and its validity in computing flows with various boundary conditions, we extended and applied on Heat Convection problem in a rectangular cavity, heated from below. Both the problems of driven cavity flow and free convection flow in enclosed container, lend themselves to testing and verifying new and improved numerical schemes. Latter is more attractive as it throws light on problems which arise in practical situations such as double-glazed window and a gass filled cavity surrounding a nuclear reactor core.

A considerable attention had been (and is still being) given to the problem of convection in a fluid confined between two horizontal boundaries at different but uniform temperatures, with side walls insulated. The analytical attacks on analogous problem starts with Batchelor's work [3], whose attention was motivated by an interest in the thermal insulation of buildings, in particular by double-glazed windows. Convection in a horizontal cylinder has been treated analytically by several authors. For example, [4] contains a comprehansive bibliography of this kind. Most of the experimental studies are concerned with the problem of convection in a box in which side walls are at different temperatures and upper, lower walls are kept insulated. Elder[5], in a vast experimental study, presented the results for Rayleigh number up to 10^8 and Prandtl number in the range of 10^3 with various aspect ratios. More recent experimental works are supported by Simpkins[6] and Phillip[7]. On the other hand, a numerical study of both two- and three-dimensional motion in a fluid heated from below, which appears to be the first detailed solution of the full equation of motions and energy, has been made by Aziz[8]. He produced transient and steady-state solutions for Rayleigh number up to 3500. A similar type of study have been made by Wilkes[9] and Pepper[10] by employing finite-difference and finite-element methods respectively. The former presented the transient as well as steady-state solutions both for insulated and linear type of boundary conditions. This study was unable to produce stable results for Grashof number greater than 200,000. While the latter, for testing and verifying his time-split finite element recursion relation, reported the steady-state results for $Ra=10^3$, 10^4 and 10^5. He compared and correlated his work with the previous existing finite-difference and experimental solutions. Because of the practical utility concerning this problem, many attempts are still being made and present work is one of them.

§1 PROBLEM STATEMENT:

The physical situation of the Heat convection problem under consideration is illustrated in Fig.1. Consider a rec-angular ecclosure whose lower (y=0) and upper(y=b) walls are at different but uniform temperatures say T_1(hot) and T_0(cold) respectively. The fluid is initially motionless and at a given temperature. Two alternate boundary conditions are considered for temperatures on the side walls, namely 1) perfect insulation and 2) a linear variation $T=T_0+(T_1-T_0)-(T_1-T_0)y/b$. It is assumed that

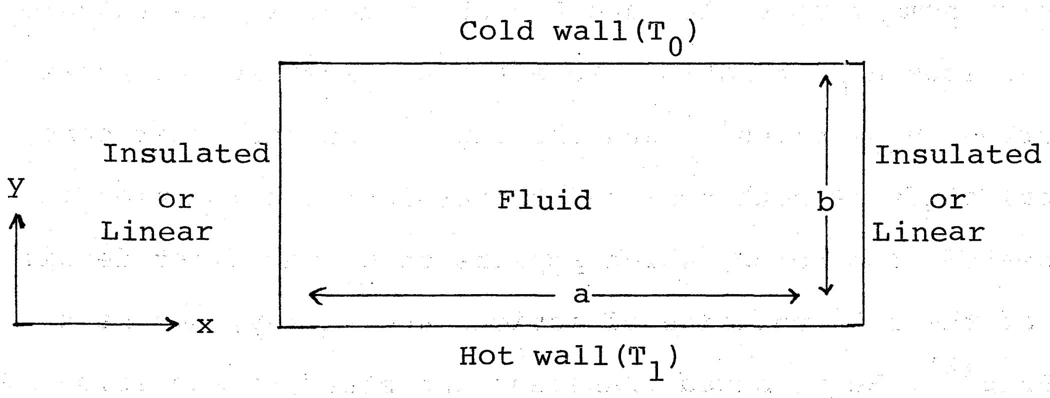


Fig.1. Rectangular enclosure

the fluid properties except density in the buoyancy term of mom-mentum equation , which is essential to the phomenon of Natural Convection, are considered to be constant. It is further assumed that the temperature between the walls(T_1-T_0), is small compared with $1/\alpha$ and also dissipation and compressibility effects are negligible. With these assumptions in mind we start with the 2-dimensional Unsteady equations of mass, momentum and energy in Boussinesq approximation, which are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\,\nabla^2 u \qquad (1.2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + \nu\,\nabla^2 v - \frac{\rho-\rho_0}{\rho_0}g \qquad (1.3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\,\nabla^2 T \qquad (1.4)$$

where $\dfrac{\rho-\rho_0}{\rho_0} = -\alpha(T-T_0)$

The initial and boundary conditions are:

$t=0,\ 0 \leqq x \leqq a,\ 0 \leqq y \leqq b$: $u=v=0,\ T=T_0+(T_1-T_0)(1-y/b)$

$$\left.\begin{array}{l} x=0 \text{ and } x=a:\ u=v=0,\ T=T_0+(T_1-T_0)(1-y/b) \\ \qquad\qquad\qquad\qquad \text{or } \dfrac{\partial T}{\partial x}=0 \\ y=0:\ u=v=0,\ T=T_1 \\ y=b:\ u=v=0,\ T=T_0 \end{array}\right\} t>0 \qquad (1.5)$$

The above equations can be reduced in the following non-dimensional forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1.6)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\nu}{LU}\nabla^2 u \qquad (1.7)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\nu}{LU}\nabla^2 v + \frac{\alpha\Delta T.gL}{U^2}T \qquad (1.8)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{LU}\nabla^2 T \qquad (1.9)$$

with initial and boundary conditions:

$$\begin{aligned} &t=0,\ 0 \leqq x \leqq a,\ 0 \leqq y \leqq 1:\ u=v=0,\ T=1-y \\ &\left.\begin{aligned} &x=0 \text{ and } x=a:\ u=v=0,\ T=1-y \\ &\qquad\qquad\qquad\qquad \text{or } \frac{\partial T}{\partial x}=0 \\ &y=0:\ u=v=0,\ T=1 \\ &y=1:\ u=v=o,\ T=0 \end{aligned}\right\}\ t>0 \end{aligned} \tag{1.10}$$

Introducing stream function ψ by,

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{1.11}$$

and vorticity ω, which is the curl of a velocity vector as given by the following equation:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{1.12}$$

Introduction of stream function automatically satisfies the equation of continuity (1.1). If 't' is defined by $t/\sqrt{Ra} \rightarrow t$, then the above equations can easily be written in the following forms:

$$\nabla^2\psi = -\omega \tag{1.13}$$

$$\left(\nabla^2 - \frac{1}{\sqrt{Pr}} \frac{\partial}{\partial t} \right)\omega = -\sqrt{Ra/pr}\left(J + \frac{\partial T}{\partial x} \right) \tag{1.14}$$

$$\left(\nabla^2 - \sqrt{Pr}\,\frac{\partial}{\partial t} \right)T = -\sqrt{Ra.Pr}\, J_T \tag{1.15}$$

where $J = \frac{\partial(\psi,\omega)}{\partial(x,y)}$, $J_T = \frac{\partial(\psi,T)}{\partial(x,y)}$ are the Jacobians operators

and Ra, Pr are the Rayleigh and Prandtl numbers respectively.

§3 NUMERICAL SCHEME:

The basic idea is to construct a solution in the integral form through the use of Green's theorem, Green's functions, flow equations and imposed boundary conditions. Equations (1.13) to (1.15) which involve stream function, vorticity and temperature as flow variables can conveniently be recasted into integral equations, which in turn contain Green's functions, which are defined below for a given domain.

Green's functions:

Let G_1, G_2 and G_3 are the Green's functions for the operators ∇^2, $(\nabla^2+\frac{1}{\sqrt{Pr}}\frac{\partial}{\partial t})$, $(\nabla^2+\sqrt{Pr}\frac{\partial}{\partial t})$ respectively which are defined uniquely with the following conditions.

$$\nabla^2 G_1(X,X') = \delta^2(X-X')$$

$$(\nabla^2 + \frac{1}{\sqrt{Pr}}\frac{\partial}{\partial t})G_2(X,t;X',t') = \delta^2(X-X')\delta(t-t')$$

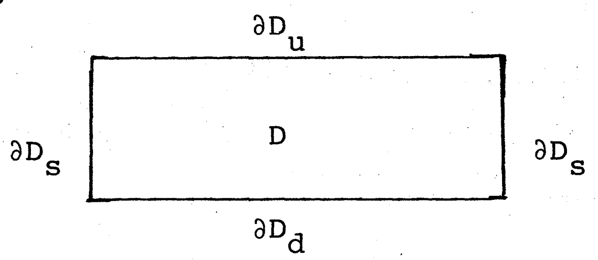
$$(\nabla^2 + \sqrt{Pr}\frac{\partial}{\partial t})G_3(X,t;X',t') = \delta^2(X-X')\delta(t-t') \qquad (3.16)$$

$$G_1(X,X') = 0 \text{ if } X \in D+\partial D,\ X' \in \partial D \text{ and } X \neq X'$$

$$G_2(X,t;X',t') = 0 \text{ if } X \in D+\partial D,\ X' \in \partial D,\ X \neq X' \text{ and } t-t' \longrightarrow +\infty$$

$$G_3(X,t;X',t') = 0 \text{ if } X \in D+\partial D,\ X' \in \partial D_{ud} \text{ and } t-t' \longrightarrow +\infty$$

$$\frac{\partial}{\partial x'}G_3(X,t'X',t') = 0 \text{ if } X \in D+\partial D \text{ and } X' \in \partial D_s$$



where,

$\partial D_{ud} = \partial D_u + \partial D_d$

$\partial D_s = \partial D_{ud} + \partial D_s$

Next we consider the Green's theorem as given below in three different forms. For convenience consider D a closed domain with surface ∂D whose outward normal is denoted by n in Fig.2.

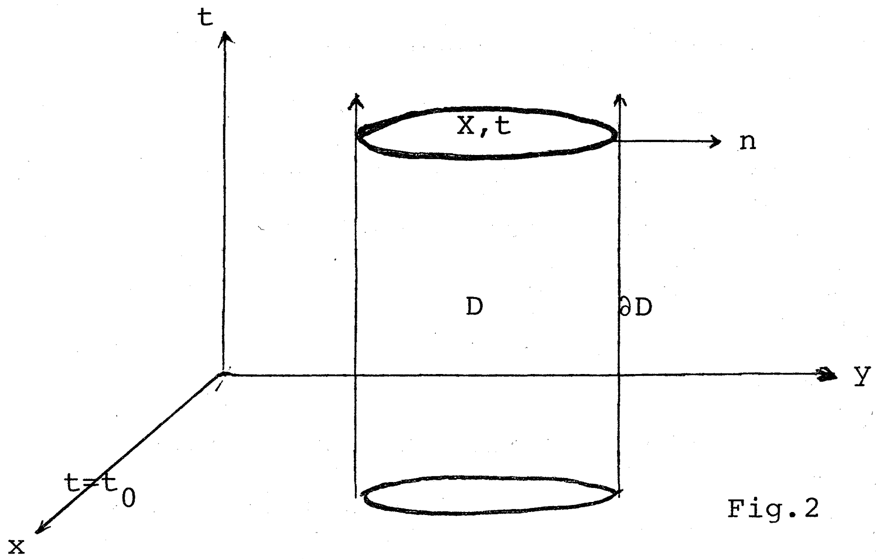


Fig.2

$$\iint_D (u'\nabla'^2 v' - v'\nabla'^2 u')\, d^2X' = \int_{\partial D} (u'\frac{\partial v'}{\partial n'} - v'\frac{\partial u'}{\partial n'})\, dS' \tag{3.17}$$

$$\int_{t_0}^{t}\iint_D (u'(\nabla'^2 - \frac{1}{\sqrt{Pr}}\frac{\partial}{\partial t'})v' - v'(\nabla'^2 + \frac{1}{\sqrt{Pr}}\frac{\partial}{\partial t'})u')\, d^2X'dt' =$$

$$\int_{t_0}^{t}\int_{\partial D} (u'\frac{\partial v'}{\partial n'} - v'\frac{\partial u'}{\partial n'})\, dS'dt' + \frac{1}{\sqrt{Pr}}\iint_D (u'v')_{t_0}\, d^2X' \tag{3.18}$$

$$\int_{t_0}^{t}\iint_D (u'(\nabla'^2 - \sqrt{Pr}\frac{\partial}{\partial t'})v' - v'(\nabla'^2 + \sqrt{Pr}\frac{\partial}{\partial t'})u')\, d^2X'dt' =$$

$$\int_{t_0}^{t}\int_{\partial D} (u'\frac{\partial v'}{\partial n'} - v'\frac{\partial u'}{\partial n'})\, dS'dt' + \sqrt{Pr}\iint_D (u'v')_{t_0}\, d^2X' \tag{3.19}$$

INTEGRAL EQUATIONS OF FLOW VARIABLES:

Stream function:

Substitute $u'=G_1(X',X)$ and $v'=\psi(X',t')$ in (3.17), use (1.13), and the imposed boundary conditions of $\psi=0$ and $G_1=0$ and simplify, we get the following integral equation for ψ.

$$\psi(X,t) = \iint_D \omega(X',t)G_1(X',X)d^2X' \qquad (3.20)$$

(3.20) can be applied to solve stream function values at every point of the domain D if ω is known at that points, which can be obtained in the following way.

Vorticity:

In a similar way associating $u'=G_2(X',t';X,t)$ and $v'=\omega(X',t')$ and putting in (3.18). After using (1.14), (3.16) and simplifying we get the following form.

$$\omega(X,t) = -\sqrt{Ra/Pr}\int_{t_0}^{t}\iint_D (J(X',t') + \frac{\partial T}{\partial x'}))G_2(X',t';X,t)d^2X'dt'$$

$$+ \int_{t_0}^{t}\int_{\partial D}\omega(s',t')\frac{\partial}{\partial n'}G_2(S',t';X,t)dS'dt' - \frac{1}{\sqrt{Pr}}\iint_D \omega(X',t_0)G_2(X',t_0;X,t)d^2X \qquad (3.21)$$

Equation (3.21) can provide vorticity informations provided, the boundary vorticities which appear in the second integral and $\frac{\partial T}{\partial x'}$ are known. From now ownward after constructing the integral equation for temperature, we proceed for the determination of boundary vorticities.

Temperature:

Substituting $u'=G_3(X',t';X,t)$ and $v'=T(X',t')$ in (3.19) and adopting the similar procedure yields the equation,

$$T(X,t)=-\sqrt{Ra.Pr}\int_{t_0}^{t}\iint_D J_T(x',t')G_3(x',t';X,t)d^2X'dt'$$

$$-\int_{t}^{t}\int_{\partial D_s}\frac{\partial T}{\partial n'}(S',t')G_3(S',t';X,t)dS'dt' + \int_{t_0}^{t}\int_{\partial D_{ud}} T(S',t')\frac{\partial}{\partial n'}G_3(S',t';X,t)dS'dt'$$

$$-\sqrt{Pr}\iint_D T(X',t_0)G_3(X',t_0;X,t)d^2X' \qquad (3.22)$$

Boundary vorticity: (Integral expression)

Differentiating (3.20) both sides with respect to n, the outward normal derivative we get,

$$\frac{\partial\psi(S,t)}{\partial n} = -\iint_D \omega(X,t)\frac{\partial G_1}{\partial n}(X',s)\,d^2X' = 0 \quad \text{(Given)}$$

and putting (3.21) in place of ω yields the following integral expression for the boundary vorticity.

$$\int_{t_0}^{t}\int_{\partial D} \omega(S',t')H_1(S',t';S,t)\,dS'dt'$$

$$= \sqrt{Ra}/Pr\int_{t_0}^{t}\int_{D}(J(X',t') + \frac{\partial T}{\partial x'}))H_2(X',t';S.t)\,d^2X'dt'$$

$$+ \frac{1}{\sqrt{Ra}}\iint \omega(X'.t_0)H_2(X',t_0;S,t)\,d^2X' \qquad (3.23)$$

where $H_1(S',t';S,t) = \iint_D \frac{\partial}{\partial n'}G_2(S',t';X'',t)\ \frac{\partial}{\partial n}G_1(X'',S)\,d^2X''$

and $H_2(X',t';S,t) = \iint_D G_2(X',t';X'',t)\frac{\partial}{\partial n}G_1(X'',S)\,d^2X''$

ω(S,t) appearing under the surface integral of left hand side, is an unknown quantity. Its determination in (3.23) requires the evaluations of $H_1(S',t';S,t)$ and $H_2(X',t';S,t)$, which in turn need Green's functions and its derivatives. If ψ, ω at t_0(initial value) and H_1 and H_2 are supposed to be known then (3.23) allows to give boundary vorticities informations directly at each time step with the use of Gauss elimination method. Once these are known, (3.21) gives vorticity informations insde the field.

CONSTRUCTIONS OF GREEN'S FUNCTIONS:

Image method:

Constructions of the Green's functions G_1 and G_2 by image method are given in reference (1) only the outlines and important points for G_3 are discuss here. Consider the illustrations shown in Fig.3. 'abcd' is the physical domain with a positive charge. If the domain has mirrors walls then its amages will be formed in all directions. If we consider the four images as shown, we can satisfy the conditions described in (3.16). Considering only the periodic domain 'ABCD' and use the fundamental solutions, G_3 can easily be constructed in a similar way as described in reference (1).

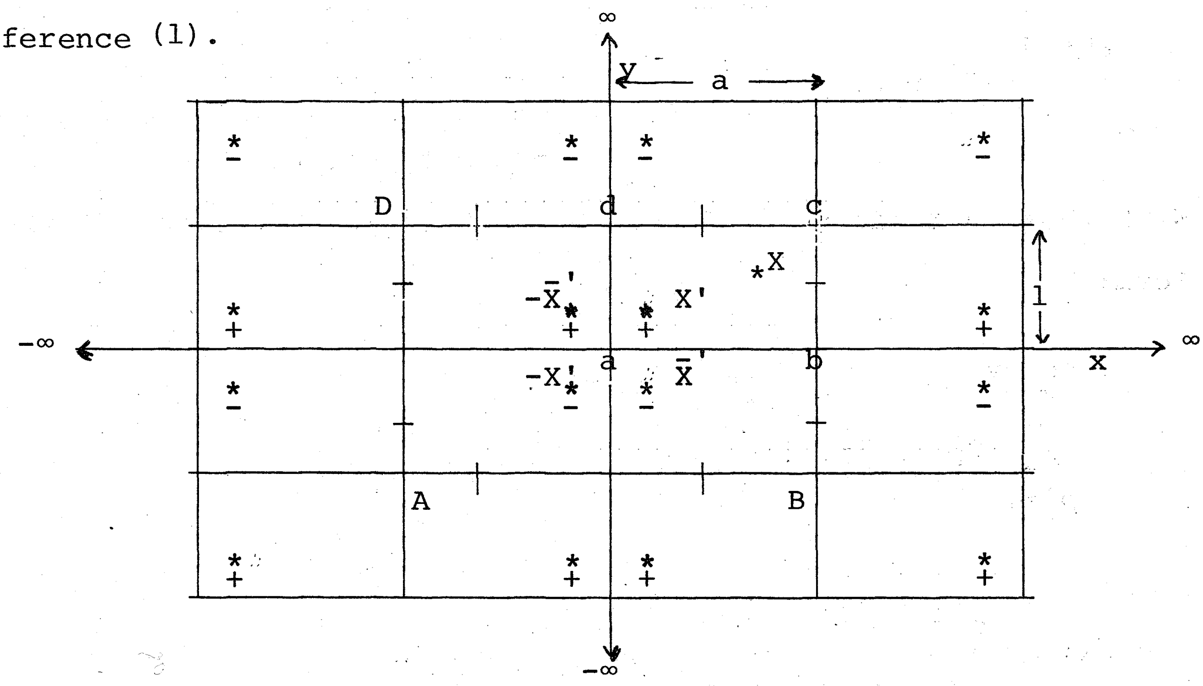
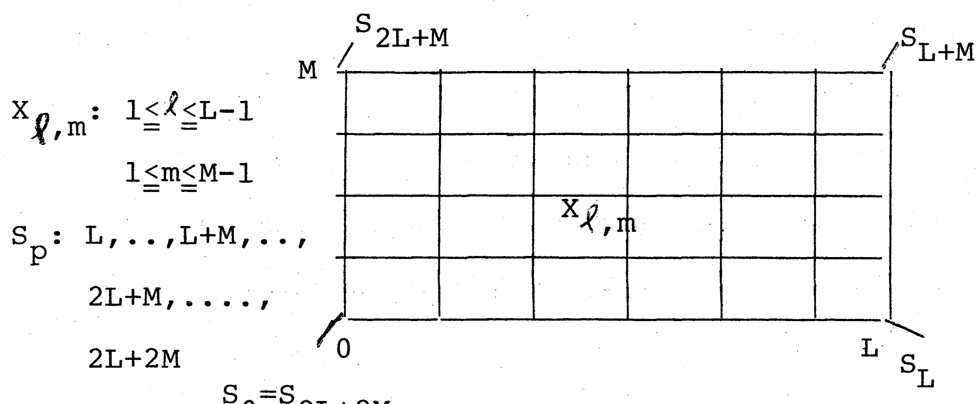


Fig.3. Schematic diagram for the construction of G_3

Derivatives of the Green's functions can conveniently be obtained by simply taking the derivatives with respect to x and y and considering the propoer signs for each wall.

§4. COMPUTATIONAL PROCEDURE:

Integral equations (3.20) to (3.22) are transformed into Algebric equations and Integral expression for the vorticity on the boundary into Sum form by discritization. Physical domain as shown in Fig.4. is devided into meshes, the subscript (ℓ,m) shows a mesh point in the field and that of p a boundary one.



$S_0=S_{2L+2M}$

Fig.4 Configuration of field and boundary points

With these notations (3.23) can be written in the following sum form:

$$\sum_{p'=1}^{2L+2M} I_{p',p}\,\omega^{0}_{p'} + \sum_{n=1}^{N}\sum_{p'=1}^{2L+2M} (H1^{n'}_{p,p'}\;\Delta t^{n}\;\Delta_{p'})\,\omega^{n}_{p}$$

$$= \sqrt{Ra/Pr}\sum_{n=1}^{N-1}\sum_{\ell'=1}^{L-1}\sum_{m'=1}^{M-1} (\,H2^{n}_{(\ell',m')p}\;\Delta t^{n}\,\Delta x\,\Delta y\,)\;J^{n}(\ell',m')$$

$$+\frac{1}{\sqrt{Pr}}\sum_{\ell'=1}^{L-1}\sum_{m'=1}^{M-1} (\,H2^{N}_{(\ell',m')p}\;\Delta x\;\Delta y\,)\;\omega^{N}_{,m'}$$

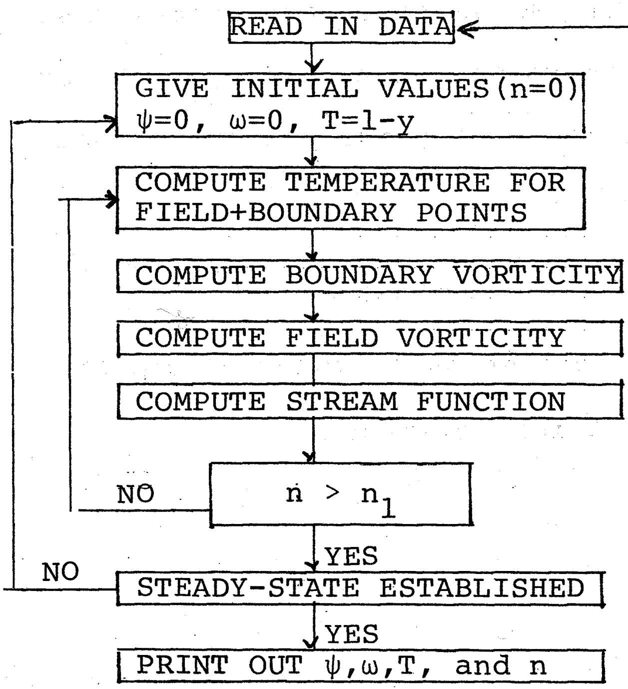
Here $\omega^{0}_{p'}$ is unknown at the p' boundary points, n, the number of time steps. The calculation of $I_{p,p'}$ at P=p' is done by analytical procedure as given below.

$$I_{p,p'} = \frac{2}{\pi}\int_{-\Delta x/2}^{\Delta x/2}\int_{0}^{\Delta y/2}\int_{-\Delta x/2}^{\Delta y/2} \frac{e^{-\frac{1}{2\Delta t}((x''-x')^2+y''^2)}}{(x''-x')^2+y''^2} \frac{\partial}{\partial n} G_1(X'',S)\, d^2X''\,dx' \qquad (\text{if } p\neq p')$$

and

$$\bar{I}_{p,p} = \frac{2}{\pi^2}\int_{-\Delta x/2}^{\Delta x/2}\int_{0}^{\Delta y/2}\int_{-\Delta x/2}^{\Delta x/2} \frac{y''^2\, e^{-\frac{1}{2t}((x''-x')^2+y''^2)}}{(x''+y'')^2((x''-x')^2+y''^2)}\, dx''dy''dx' \qquad (\text{if } p=p')$$

With space mesh 1/10 and time mesh 0.002, Green's functions are computed first and then H_1 and H_2 are computed for t=t', i.e. (n=n') and preserved for actual computations. H_1 for n=n' is calculated from the expressions given above. These computed data are stored for some fixed number of time steps say N_1.



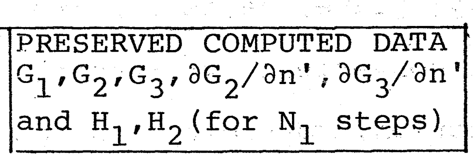


Fig.5 Flow chart

Basic steps of actual computations are as follows. With known values of ψ, ω and T, temperature is computed first from (3.22) for both field and boundary points. For linear boundary conditions G_3 is not used and the values are calculated only for the field.

Boundary vorticities are then calculated from (3.23) by using Gauss elimination method. Later these values are used in (3.21) to evaluate field values of vorticity. Finally (3.20) is employed to give stream function values with the use of boundary and field vorticities. These steps constitute a complete loop of computation for each time step. Since ψ, ω and T are now known for all field and boundary points at a certain time step, the computation can be repeated for subsequent time steps.

The above mentioned procedure is repeated upto N_1 time steps, since preserved data contain N_1 informations only. To proceed the $n=N_1+1$ time step, initial step '0' is shifted to step '1' so that the difference of final and initial step is always N_1 steps. Consequently the informations at steps preceed- to initial step do not enter the computational procedure and are discarded. In this manner we could get transient solutions at each time step directly without iterating the process in (x,y) plane, as is required in Finite-differece approximations. For convergence we repeat the process till the values of stream function, vorticity and temperature satisfy the following conditions.

$$\left|\psi^n - \psi^{n+1}\right| < \varepsilon, \quad \left|\omega^n - \omega^{n+1}\right| < \varepsilon \quad \text{and} \quad \left|T^n - T^{n+1}\right| < \varepsilon$$

where n is the time step and n+1 is the number advanced by one step. Flow chart is given in Fig.5 for reference.

§5 NUMERICAL ILLUSTRATIONS:

Heat convection in a rectangular box with upper cold wall and heated from below is studied, and flow equations of non-steady Navier-Stokes in their stream function-vorticity forms (1.13), (1.14) and Energy (1.15) are solved with the application of an integral equation scheme, modified in the previous section. Results in the forms of stream lines, equi-vorticity lines and isothermals for a square domain (aspect ratio 1:1), with space mesh 1/10, time mesh 0.002 and $\varepsilon=10^{-3}$, are presented as numerical illustrations for Rayleigh number up to 10^5, Pr=1. Two alternate boundary conditions for temperature on side walls namely, perfect insulation and linear variation (T=1-y), are considered. In all the cases initial values of ψ, ω and T are taken to be 0,0 and 1-y respectively.

Fig.6 shows transient solutions at $Ra=10^5$, Pr=1 with perfect insulation on the side walls. At t=0.008(n=4) and 0.014(n=7), a symmetric flow along the vertical center line of the box is obtained. At early times very weak flow in the bottom near the heated wall is obtained which accelerate in the later time steps. Fig.7 show steady-state solutions for $Ra=10^3$ and 10^5. This state easily exists in n=35th step (t=0.07). This tendency is found to be same for the two cases. Isothermals at low Ra are found to be parallel in the upper region near the cold wall. But as Ra increases this pattern is affected by Rayleigh number. Fig.8 and Fig.9 show transient solutions at $Ra=10^4$ and 10^5 respectively for linear type boundary conditions. For $Ra=10^5$ reverse flow in the upper region near cold wall is obtained at early time whose size decreases as time advances and finally at steady-state very weak reverse flow is retained. For linear case steady-state establised at n=25th (t=0.025) step. In this study computation time to reach steady-state does not effect the Rayleigh number.

The principal advantage of this scheme is that it does not require a convergence iterative process in the xy-plane, before advancing a time step. Therefore a considerable reduction of computing time is possible for time-dependent problems. The main analytical difficulty in the present approach is however, the determination of Green's function for a general domain. If the Green's function is known for a certain domain, this scheme can be compared with any of the standared methods in computational efficiency, speed and accuracy.

At present we have tried the problem for a square domain with coarse mesh 1/10 and prandtl number unity. Further study such as effect of Prandtl number and aspect ratios are in progress and successful results are expected.

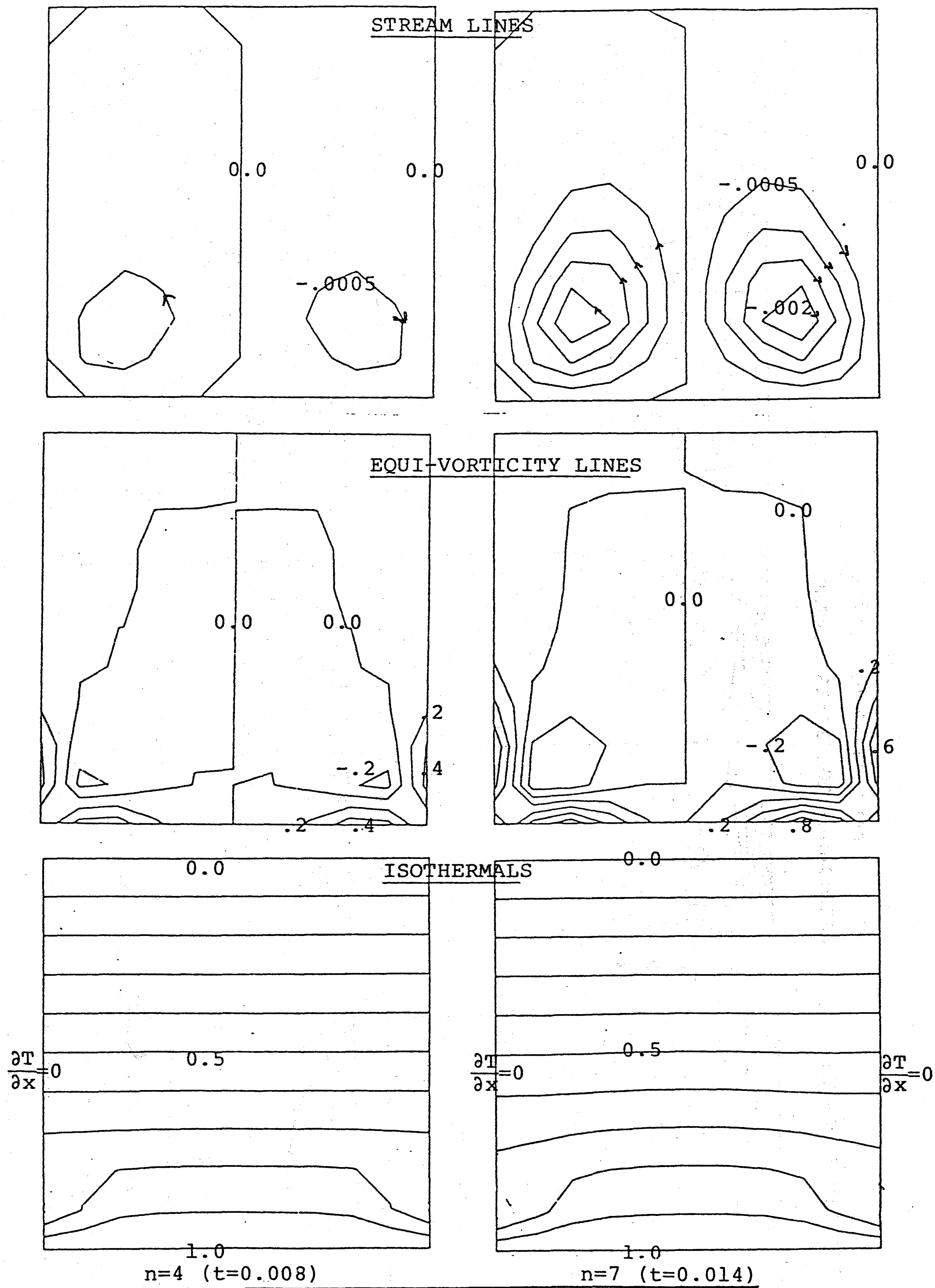
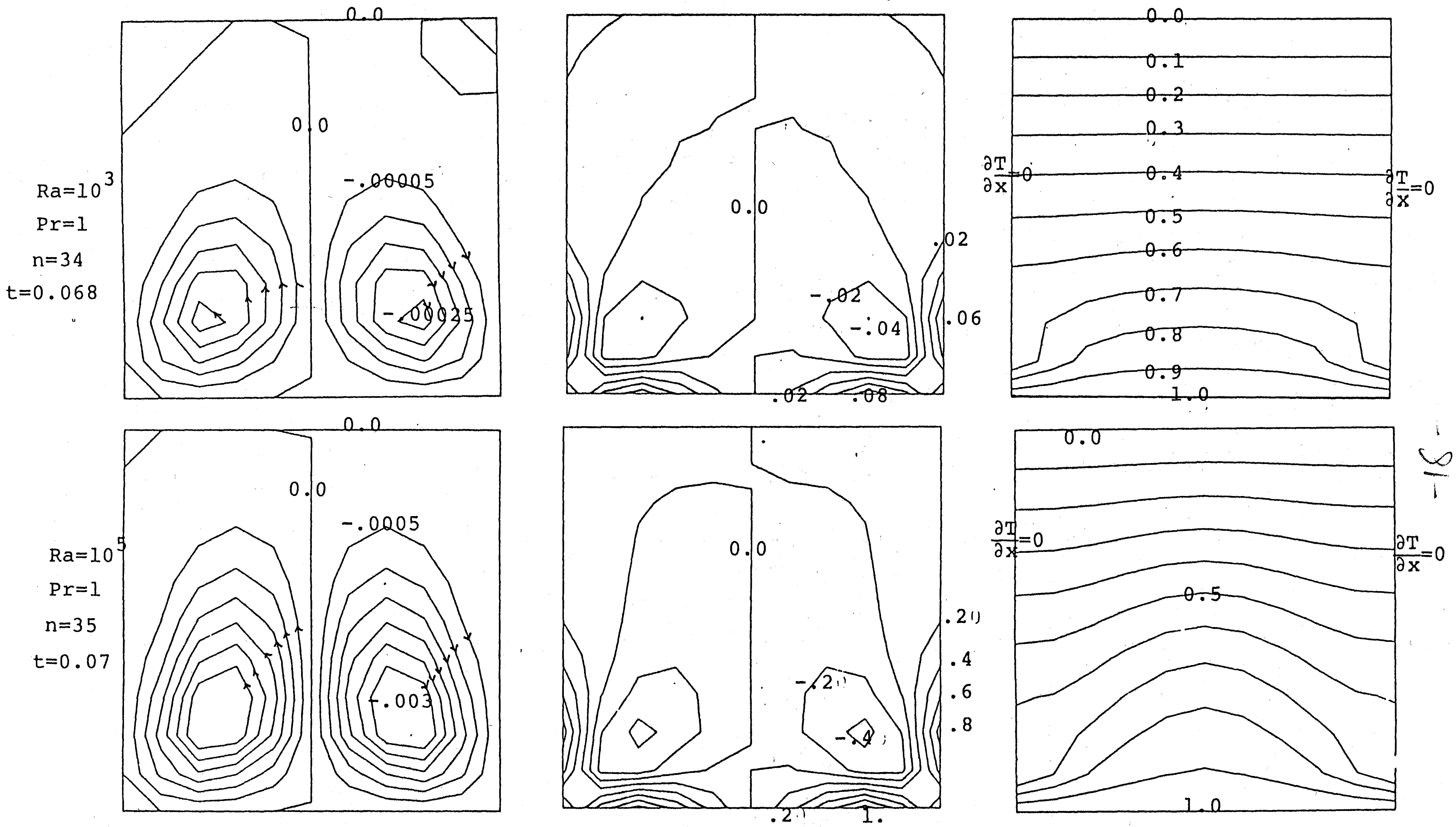


Fig.6 TRANSIENT SOLUTIONS AT $Ra=10^5$, Pr=1
$\Delta X=\Delta Y=1/10$, $\Delta t=0.002$, $\varepsilon=10^{-3}$

INSULATED



STEADY STATE STREAMLINES STEADY STATE EQUI-VORTICITY STEADM STATE ISOTHERMALS

Fig.7 Insulate case

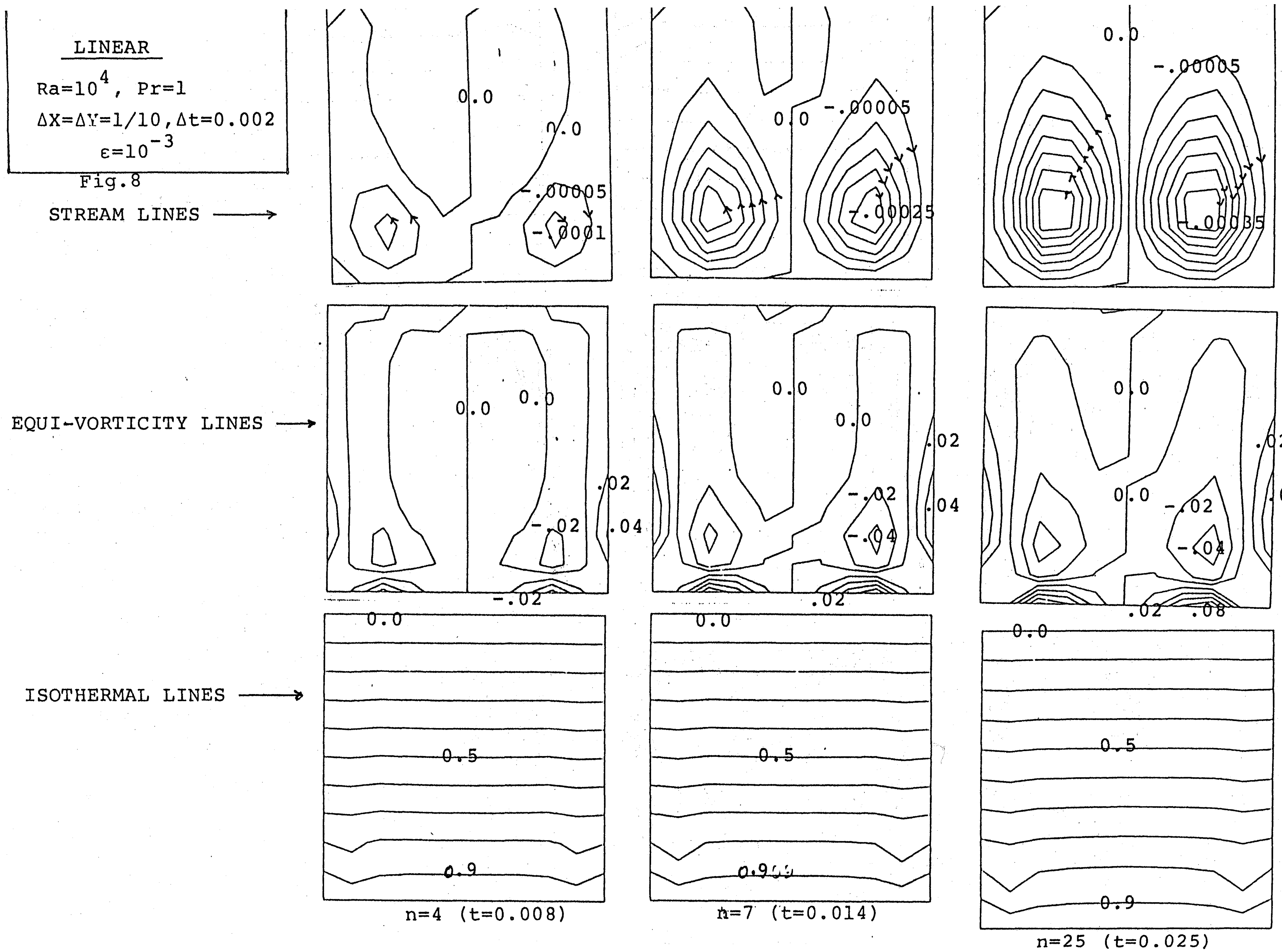

LINEAR
Ra=10^4, Pr=1
ΔX=ΔY=1/10, Δt=0.002
ε=10^-3
STREAM LINES
EQUI-VORTICITY LINES
ISOTHERMAL LINES
0.0
-.00005
-.0001
-.00025
-.00035
.02
.04
.08
-.02
-.04
0.5
0.9
n=4 (t=0.008)
n=7 (t=0.014)
n=25 (t=0.025)

Fig.8

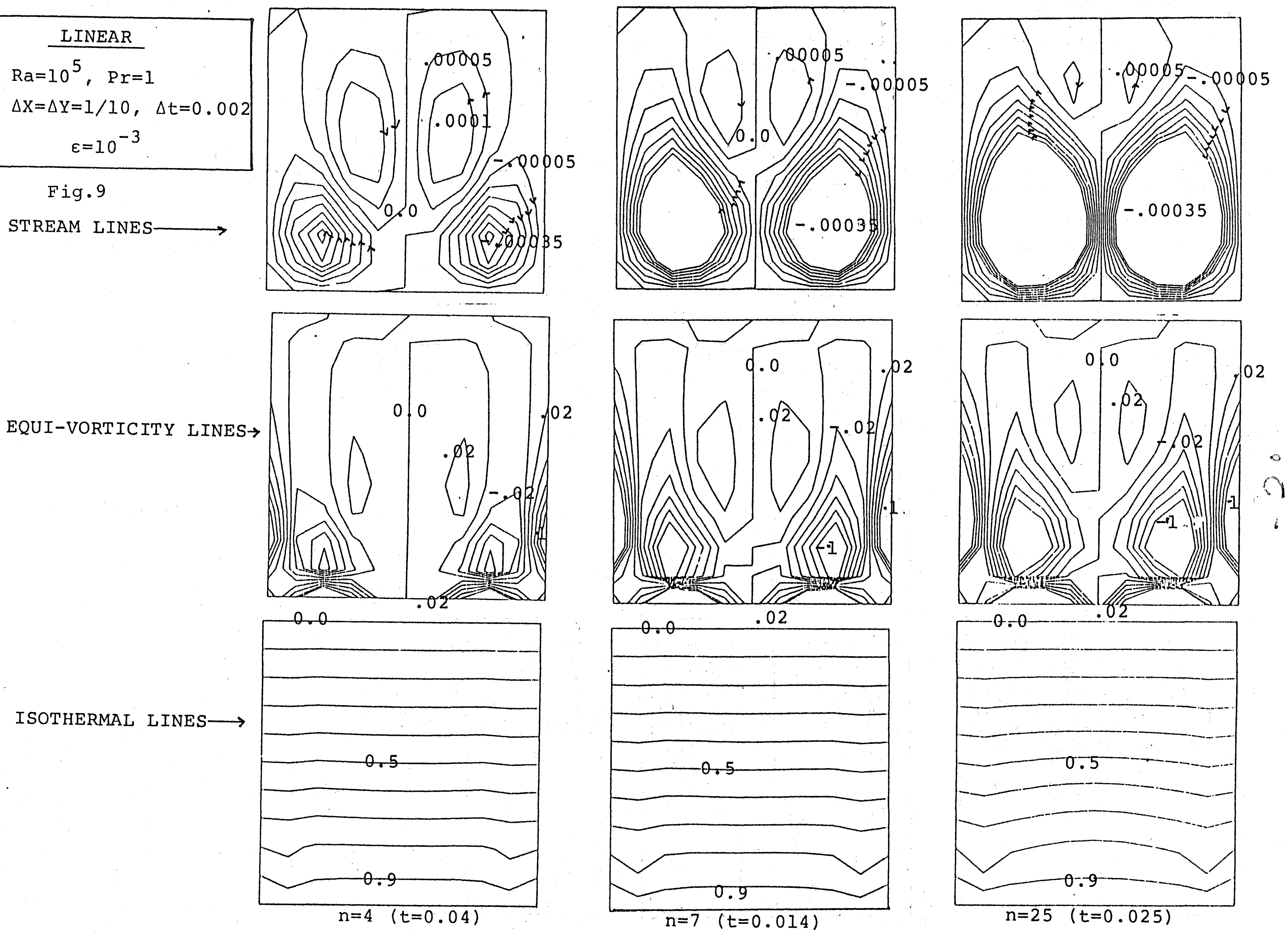
LINEAR

$Ra=10^5$, $Pr=1$

$\Delta X=\Delta Y=1/10$, $\Delta t=0.002$

$\varepsilon=10^{-3}$

Fig.9



NOTATION:

x = Dimensionless horizontal coordinate, = x/L

y = " vertical coordinate, = y/L

T = " temperature, = $T - T_0 / T_1 - T_0$ (T_0 and T_1 are temperatures at cold(upper) and hot(lower) walls respectively.

t = Dimensionless time, = tL/U

p = " pressure, = $p/\rho_0 U^2$

a = Length of the upper wall, =a/b

b = Length of the side wall , L=b

g = Acceleration due to gravity

U = Reference velocity, = $\sqrt{\alpha g \Delta T . b}$

Ra = Rayleigh number, = $\alpha g \Delta T . b^3/\nu\kappa$

Pr = Prandtl number, = ν/κ

X,X' = (x,y), (x',y')

GREEK LETTERS:

Δx = Grid spacing in x direction

Δy = Grid spacing in y direction

Δt = time mesh or increment

∇^2 = Laplacian operator, = $\partial^2/\partial x^2 + \partial^2/\partial y^2$

ω = Dimensionless vorticity, = $-\nabla^2\psi$

ψ = " stream function, such that $u=\partial\psi/\partial x$ and $v=-\partial\psi/\partial y$

ρ = Density

ρ_0 = Constant density

α = Co-efficient of thermal expension of the fluid.

κ = Thermal conductivity

ν = Kinematic viscosity

ε = Convergence parameter

SUBSCRIPTS:

ℓ,m = Space subscripts of mesh points in x and y directions

n = time subscript (step)

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