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Some Stability for expansive homeomorphisms with singular points

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Abstruct. Analogues of structural stability are shown for expansive homeomorphisms with POTP or TPOTP of compact metric spaces.

1. Introduction.

Structural stability for differentiable dynamical systems has been studied in many papers including [1,8,9, and 10]. In topological setting, P. walters [12] proved that an expansive homeomorphism with POTP is topologically stable. It is also proved in [12] that a transitive subshift of finite type of order 1 is topologically conjugate to its small perturbations. J. Lewowicz showed in [5] that a pseudo-Anosov map is topologically conjugate to its small perturbations if they coincide with original one on singularities.

The perpose of this paper is to find conditions of perturbations which are topologically conjugate to the original one for expansive homeomorphisms with POTP or TPOTP. We can describe our results as follows;

Theorem 1. *Let X be a compact metric space and f be an expansive self-homeomorphism with POTP on X. Then there exists a compatible metric D^* such that for every $\varepsilon > 0$ there is $\delta > 0$ such that for every self-homeomorphism g of X with $D^*(fx,gx) < \delta$ $(x \in X)$ and $|D^*(fx,fy) - D^*(gx,gy)| < \delta D^*(x,y)$ $(x,y \in X)$ there exists a continuous injective map h of X into X with $D^*(hx,x) < \varepsilon$ $(x \in X)$ such that $h \circ g = f \circ h$.*

Theorem 2. *Let X be a compact metric space and f be an expansive self-homeomorphism with proper TPOTP on X. Then there exists a compatible metric D^{**} such that for every $\varepsilon > 0$ there is $\delta > 0$ such that for every self-homeomorphism g of X with $D^{**}(fx,gx) < \delta D^{**}(x,S_f)$ $(x \in X)$ and $|D^{**}(fx,fy) - D^{**}(gx,gy)| < \delta D^{**}(x,y)$ $(x,y \in X)$ there exists a continuous injective map h of X into X with $D^{**}(hx,x) < \varepsilon$ $(x \in X)$ such that $h \circ g = f \circ h$.*

In the remainder of this section, we shall give some definitions which are used in the proof of our results.

Let X be a compact metric space with metric d, and f be a self-homeomorphism of X. For $x \in X$, $B_\varepsilon(x)$ will denote the closed ε-ball in X centered at x. For $x \in X$ and $\varepsilon > 0$, define subsets $W^s_\varepsilon(x)$ and $W^u_\varepsilon(x)$ of $B_\varepsilon(x)$ by

$$W^s_\varepsilon(x) = \bigcap_{n \geq 0} f^{-n} B_\varepsilon(f^n x) \quad \text{and} \quad W^u_\varepsilon(x) = \bigcap_{n \leq 0} f^{-n} B_\varepsilon(f^n x) \ .$$

<u>Definition 1.</u> (X,f) is said to be <u>expansive</u> if there exists a constant $c^* > 0$ such that

$$\{x\} = \bigcap_{n \in \mathbb{Z}} f^{-n} B_{c^*}(f^n x) \quad (= W^s_{c^*}(x) \cap W^u_{c^*}(x))$$

for all $x \in X$, and such a c^* is said to be an <u>expansive constant</u> for f .

For every $\varepsilon > 0$ definie subsets Y_ε and Z_ε of $X \times X$ by

$$Y_\varepsilon = \{(x,y) \in X \times X \mid W^s_\varepsilon(x) \cap W^u_\varepsilon(y) \neq \emptyset\}$$

and

$$Z_\varepsilon = \{(x,y) \in X \times X \mid (x,y) \in Y_\varepsilon \text{ and } (y,x) \in Y_\varepsilon\} \ .$$

For $x \in X$ and $\varepsilon > 0$, subsets $Y_\varepsilon(x)$ and $Z_\varepsilon(x)$ of X are defined by

$$Y_\varepsilon(x) = \{y \in X \mid (x,y) \in Y_\varepsilon\}$$

and

$$Z_\varepsilon(x) = \{y \in X \mid (x,y) \in Z_\varepsilon\} \ .$$

<u>Definition 2.</u> Let $\mathcal{A}$ be a finite partition of X ,i.e., a finite family of subsets of X whose elements are mutually

disjoint and $\bigcup_{D \in \mathscr{D}} D = X$. A sequence $\{x_i\}_{i \in \mathbb{Z}}$ of points in X is said to be an α-pseudo orbit with respect to $\mathscr{D}$ if $d(fx_i, x_{i+1}) < \alpha$ and $fx_i \underset{\mathscr{D}}{\sim} x_{i+1}$ for all $i \in \mathbb{Z}$ where $x \underset{\mathscr{D}}{\sim} y$ denotes that x and y are in the same element of $\mathscr{D}$. A sequence $\{x_i\}_{i \in \mathbb{Z}}$ of points in X is said to be β-traced if $\bigcap_{i \in \mathbb{Z}} f^{-i} B_\beta(x_i) \neq \emptyset$. (X,f) is said to have Takahashi pseudo orbit tracing property (abbrev. TPOTP) if there exists a finite partition $\mathscr{D}$ such that for every $\alpha > 0$, there is $\alpha > 0$ such that every α-pseudo orbit with respect to $\mathscr{D}$ is β-traced. Especially (X,f) is said to have the pseudo orbit tracing property (abbrev. POTP) if $\mathscr{D}$ can be chosen so that $\mathscr{D} = \{X\}$.

Let (X,f) be expansive and $c > 0$ be a number such that $2c$ is an expansive constant. A point $x \in X$ is said to be regular if there is $\delta > 0$ such that $Y_c(y) \supset B_\delta(x)$ for all $y \in B_\delta(x)$ and singular if x is not regular. We denote by S_f the set of singular points in X . $S_f = \emptyset$ iff (X,f) has POTP. (X,f) is said to have proper TPOTP if (X,f) has TPOTP but not POTP. S_f is an f-invariant nowhere dense closed subset of X if (X,f) has TPOTP.

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