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ON APPLICATIONS OF INTERVAL ARITHMETIC TO CIRCUIT ANALYSIS

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1.Introduction

When we try to produce any electric or electronic circuit in quantity, we must know how the properties vary when we use circuit elements with specifie tolerance range of values. The elements with values far from the mean values may cause the circuit to occur abnormal phenomena. In our electrical experiments, we alway experience the errors which arise in mesurements of the values of circuit elements. Viewed from this angle, when we analyse the circuits, the values of the elements must essentially be handled not as real numbers but as real interval numbers. One way to attack the problem is to use Monte Carlo Method. Not to mention, this method takes too much time to compute the responses of the circuit as the number of the elements and trials increase. Here, as the other way to attack, we propose to use the interval Gaussian algorithm. First, we show that the interval Gaussian algorithm is applicable to the analysis of linear circuits where parameters are given by interval numbers. Then, we compute the voltage distribution of simple linear circuit by making use of both interval Gaussian algorithm and the Monte Carlo method and compare the results.

Recently, we need to obtain the multiple solutions of the load flow equation in electric power circuits and to test the stablity of them. The load flow equation is a set of quadratic equations written by the node voltages. The problems are as follows; how can we know all the solutions in the given voltage ranges and if there is no solution in the ranges, how can we confirm the nonexistence? As is well known, the Newton and Newton-like method cannot answer these questions. Here, we try to attack the problems by use of Krawczyk-Moore-Jones algorithm(abbreviated as KMJ algorithm) for the solution of nonlinear equation. The results of application of KMJ algorithm to the power circuit with five nodes are described.

2. Application to linear circuit analysis

2.1 Interval equation of linear circuit

We deal with linear passive resistive circuit. Let it have n_t nodes and b branches. We pick the datumn node. Let the branch conductance matrix be $\mathbf{G}=diag(g_1, g_2, \cdots, g_b)$ and the reduced incidence matrix be $\mathbf{A}$ where $\mathbf{A}=(a_{ik})$ is the n x b matrix where $n=n_t-1$. The node equation of the circuit is formulated by

$$\left.\begin{aligned} \mathbf{Y}\mathbf{v} &= \mathbf{J}_s \\ \mathbf{Y} &\triangleq \mathbf{A}\mathbf{G}\mathbf{A}^T \end{aligned}\right\} \qquad (1)$$

where $\mathbf{Y}$ is the node admittance matrix, $\mathbf{J}_s$ is the linear combination of the node current source vectors and $\mathbf{v}$ is the node to datum voltage vector. The superscript T denotes the transpose of the matrix. When the values of the conductances are given by the interval number, we can regard the conductance g_i as the center of the interval values. Now we define the interval branch conductance matrix by

$$\mathbb{G} = \mathrm{diag}\,(G_1, G_2, \cdots, G_b) \tag{2}$$

where G_i is the interval number defined by

$$G_i = \langle g_i, \varepsilon_i \rangle \qquad 1 \le i \le b \tag{3}$$

ε_i is the half width of the interval G_i. Since we deal with the passive resistive circuit, we have

$$g_i > 0, \quad \varepsilon_i > 0, \quad g_i - \varepsilon_i > 0 \qquad 1 \le i \le b \tag{4}$$

Using this interval conductance matrix $\mathbb{G}$, we can define the interval node admittance matrix by

$$\mathbb{Y}_n = \mathbb{A}\,\mathbb{G}\,\mathbb{A}^T \tag{5}$$

The diagonal elements Y_{ii} of $\mathbb{Y}_n$ are given by

$$\left.\begin{aligned} Y_{ii} &\triangleq \langle y_{ii}, r_{ii} \rangle \\ &= \sum_{j=1}^{b} a_{ij}^2 G_j \\ &= \left\langle \sum_{j=1}^{b} (a_{ij})^2 g_j, \sum_{j=1}^{b} (a_{ij})^2 \varepsilon_j \right\rangle \end{aligned}\right\} \tag{6}$$

where a_{ij} is the elements of $\mathbb{A}$ and $(a_{ij})^2$ can only be zero or 1.

Therefore, Y_{ii} is given by the interval addition of G_j. We have

$$y_{ii} > 0 \; , \quad r_{ii} > 0 \qquad 1 \le i \le b \tag{7}$$

The off-diagonal elements $Y_{ik} (i \neq k)$ are given by

$$\left.\begin{aligned} Y_{ik} &\triangleq \langle y_{ik} , r_{ik} \rangle \\ &= \sum_{j=1}^{b} a_{ij} a_{kj} G_j \\ &= \langle \sum_{j=1}^{b} a_{ij} a_{kj} g_j \, , \; \sum_{j=1}^{b} | a_{ij} a_{kj} | \varepsilon_j \rangle \end{aligned}\right\} \tag{8}$$

and we know $a_{ij} a_{kj}$ can only be zero or -1. Therefore, Y_{ik} is also given by the interval addition. Evidently we have

$$y_{ik} < 0 \; , \quad r_{ik} > 0 \qquad \underset{i \neq k}{1 \le i , k \le n} \tag{9}$$

The interval node equation associated with eq.(1) is formulated by

$$\mathbb{Y}_n \mathbb{V} = \mathbb{J}_s \tag{10}$$

where $\mathbb{J}_s$ is the linear combination of the interval node current source vectors and $\mathbb{V}$ is the interval node to datumn voltage vector.

2.2. Condition for feasibility of interval Gaussian algorithm

Here, we try to invert Y_n by using the interval Gaussian algorithm. If the interval matrix Y_n is a strictly diagonally dominant matrix, then the Gaussian algorithm can be carried out for eq.(10)[1]. In our case, from eq.(4),(7),(9) the condition for the strictly diagonal dominancy for Y_n is written by

$$\sum_{j=1}^{n} y_{ij} > \sum_{j=1}^{n} r_{ij} \qquad 1 \le i \le n \qquad (11)$$

When $\varepsilon_i = 0$ $(1 \le i \le b)$, Y_n becomes a strictly diagonally dominant matrix in usual sense. When we solve eq.(10) by the interval Gaussian algorithm, we must check whether inequality of eq.(11) is hold or not.

2.3. Example

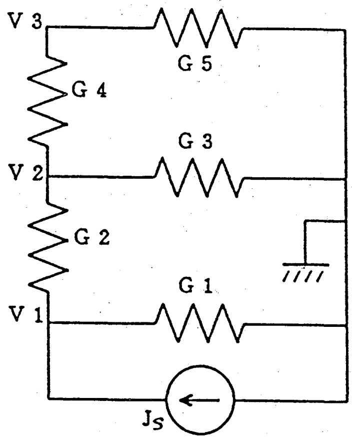
We consider a simple ladder network as shown in Fig.1. The interval conductances are given by $G_i = \langle g_i, \varepsilon \rangle (1<i<5)$ where $g_1 = 20$, $g_2 = 1$, $g_3 = 40$, $g_4 = 2$, $g_5 = 9$ and $J = 100$. We obtain the interval voltages V_i $(1 \le i \le 3)$ for $0 \le \varepsilon \le 0.01$ as shown in Fig.2. Fig.3 shows the results by Monte Carlo method. The both results are fairly in good agreement. In this example, the interval Gaussian algorithm is about 2000 times faster than Monte Carlo method.

3. Application to nonlinear analysis of electric power circuits

3.1 Load flow equation and KMJ algorithm

In this section, we present the results of applications of KMJ algorithm[2] to the load flow equation in electric power

Fig.1 Simple ladder circuit.



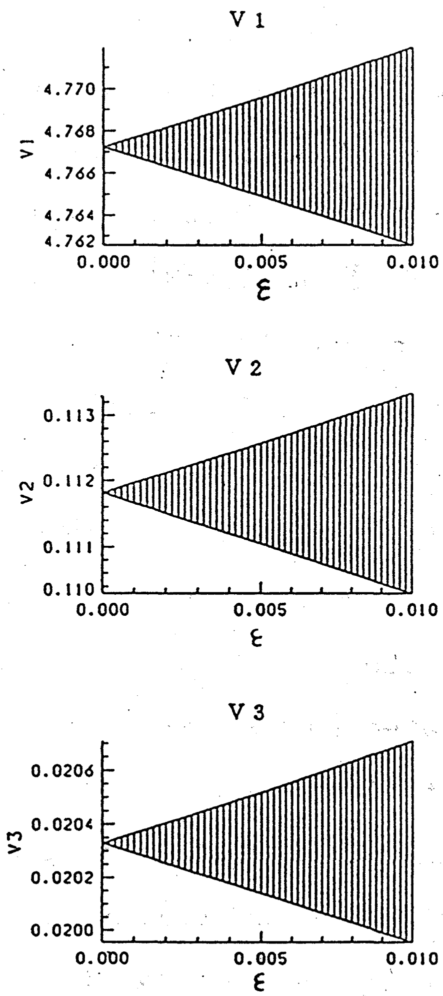


Fig.2 Result by interval Gaussian algorithm.

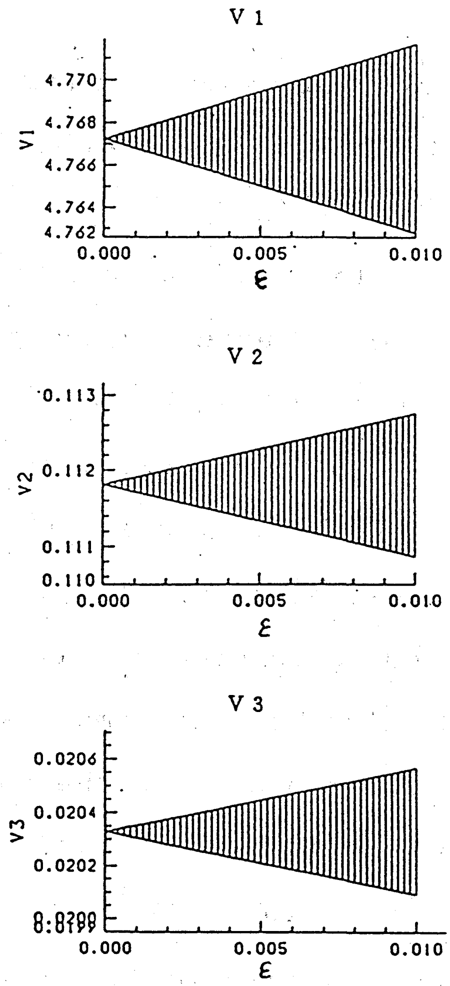


Fig.3 Result by Monte Calro method.

circuits. The load flow equation of N nodes circuit is in real form written as follows:

$$\left.\begin{aligned}
&e_i \sum_{j=1}^{N} (e_j G_{ij} - f_j B_{ij}) + f_i \sum_{j=1}^{N} (e_j B_{ij} + f_j G_{ij}) - P_i^{(s)} = 0 \\
&e_i \sum_{j=1}^{N} (e_j B_{ij} + f_j G_{ij}) - f_i \sum_{j=1}^{N} (e_j G_{ij} - f_j B_{ij}) + Q_i^{(s)} = 0 \\
&\qquad\qquad\qquad\qquad\qquad 2 \le i \le M \\
&e_i \sum_{j=1}^{N} (e_j G_{ij} - f_j B_{ij}) + f_i \sum_{j=1}^{N} (e_j B_{ij} + f_j G_{ij}) - P_i^{(s)} = 0 \\
&e_i^2 + f_i^2 - V_i^{(s)2} = 0 \qquad\qquad M+1 \le i \le N
\end{aligned}\right\} \quad (12)$$

where the reference voltage $E = e_1 + jf_1$ is given where $j = \sqrt{-1}$. $P_i^{(s)}$ and $Q_i^{(s)}$ ($2 \le i \le M$) are PQ-specified effective and reactive power. $P_i^{(s)}$ and $V_i^{(s)}$ ($M+1 \le i \le N$) are PV-specified effective power and node voltage. The coefficients G_{ij} and B_{ij} ($2 \le i,j \le N$) are the elements of nodal conductance and suceptance matrix. The variables e_i and f_i are node voltage components. Eq.(12) is n dimensional quadratic equation where $n=2(N-1)$. We set the left hand sides of eq.(12) as $f(x)$ where $x=(e_2, f_2, \ldots, e_N, f_N)$. We denote the interval extension of $f(x)$ by $F(X)$ where X is the interval vector corresponding to node voltage vector x.

KMJ algorithm is simply stated as follows; Let n dimensional rectangular region B be the initial region. First we set X:=B and compute F(X). If $F(X) \ni 0$, then we compute the Krawczyk interval function K(X). IF $K(X) \subseteq X$, there exists at least one solution of $f(x)=0$ in X. Newton method for $f(x)=0$ is started from the center m(K(X)). If $K(X) \subseteq X$ is not hold, the region X is divided into two regions Xl and Xr. The region Xr is stocked in the list L. We set X=Xl and the same procedures are repeated until all regions are tested.

3.2 Results

We apply KMJ algorithm to a simple load flow equation of the five nodes electric power system as shown in Fig.4. We denote the i-th node voltage by $v_i=e_i+jf_i$ in complex form. The load flow equation is given by a set of quadratic equations with eight variables e_i and f_i $(i=2,...,5)$. The condition is as follows; load $P_i^{(s)}+jQ_i^{(s)}=2.0+j0.501$, $Sc_i=1.0$, $i=3,5$. The regions where we search the solutions are

Region A: $e_i=[0.8,\ 1.2]$, $f_i=[-0.3,\ 0.0]$ $i=2,...,5$

Region B: $e_i=[0.3,\ 1.2]$, $f_i=[-0.3,\ 0.0]$ $i=2,...,5$

Region C: $e_i=[0,0,\ 1.2]$, $f_i=[-0.3,\ 0.0]$ $i=2,...,5$.

In the region A we obtain a unique solution #4 as shown in Table 1. It takes 59.3 seconds to search the solution. In the region B we also obtain the same solution #4 and can not find out the other solution. It takes 233.4 seconds to terminate the algorithm. We obtain four solutions as shown in Table 1 in the region C. It takes 861.4 seconds to terminate the algorithm.

Newton method in KMJ algorithm converges two or three iterations for convergent radius 10^{-9} in each region. The computer used is vector processor VP-200 of FUjitsu Ltd.

Fig.4 The 5 nodes power circuit.

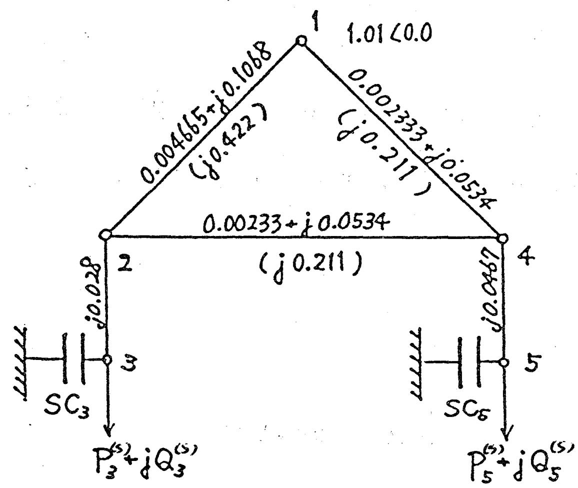


Table 1. Interval solutions and all the solutions in region C.

	#1		#2	
e_2	[0.3374, 0.3468]	0.3450	[0.3750, 0.3843]	0.3782
f_2	[-0.1499, -0.1453]	-0.1463	[-0.1312, -0.1265]	-0.1284
e_3	[0.0937, -0.1011]	0.1011	[0.0562, 0.0656]	0.0642
f_3	[-0.2062, -0.2015]	-0.2052	[-0.1781, -0.1687]	-0.1699
e_4	[0.4687, 0.4781]	0.4766	[0.6562, 0.6656]	0.6582
f_4	[-0.1312, -0.1218]	-0.1247	[-0.1406, -0.1312]	-0.1370
e_5	[0.1218, 0.1312]	0.1253	[0.5906, 0.5999]	0.5947
f_5	[-0.2343, -0.2249]	-0.2287	[-0.2718, -0.2624]	-0.2658

	#3		#4	
e_2	[0.6773, 0.6796]	0.6777	[1.0406, 1.0453]	1.0429
f_2	[-0.1734, -0.1710]	-0.1712	[-0.1640, -0.1593]	-0.1617
e_3	[0.6468, 0.6492]	0.6483	[1.0453, 1.0499]	1.0489
f_3	[-0.2484, -0.2460]	-0.2464	[-0.2203, -0.2156]	-0.2163
e_4	[0.5624, 0.5648]	0.5627	[1.0312, 1.0406]	1.0351
f_4	[-0.1124, -0.11015]	-0.1124	[-0.1406, -0.1312]	-0.1343
e_5	[0.06796, 0.0703]	0.0682	[1.0406, 1.0453]	1.0439
f_5	[-0.1804, -0.1781]	-0.1796	[-0.2343, -0.2249]	-0.2257

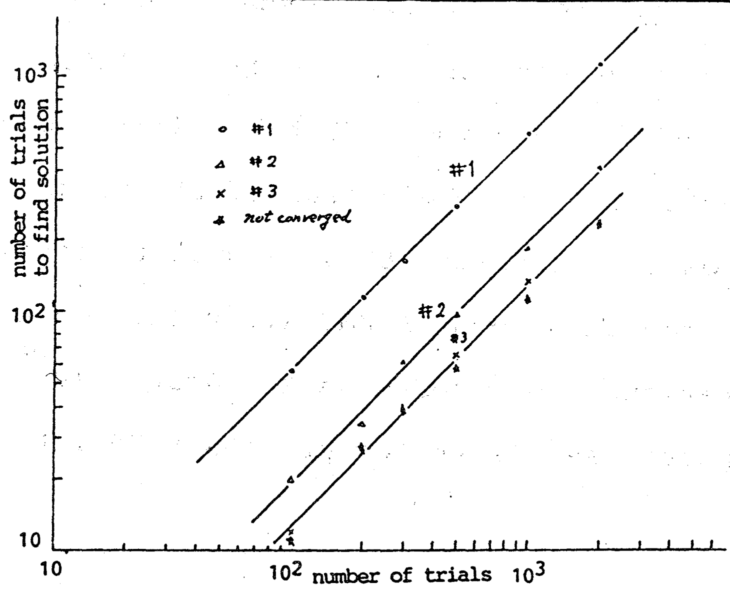


Fig.5 Relation between number of trials and number of times to find solutions.

3.3 Comparison with Results by Newton method

We carry out Newton method for the same load flow equation as in Fig.4 and compare the results obtained by the KMJ algorithm. We choose at random the initial values from the region C. The relation between the number of trials of choosing the initial values and the number of times to find solutions is shown in Fig.5. We obtain three solutions #1,#2 and #3 but can not find out the solution #4. The percetages of times for initial values to converge to the solutions #1,#2 and #3 are about 55%, 20%, 13% respectively. The remainder about 13% corresponds to the case where Newton method is not converged for 100 times iterations. This result means that KMJ-algorithm is useful to find out all the solutions in the specified region. However, the CPU time for carrying out the KMJ algorithm is much longer than for Newton method.

4. Concluding remarks

We try to apply the interval Gaussian algorithm to the tolerance analysis of linear circuit. The node equation might suit the interval Gaussian algorithm. This comes from the fact that the node admittance matrix is diagonally dominant. The more the number of variables becomes, the wider the range of the solution is. This drawback requires any improvement. We also try to apply KMJ algorithm to solution of the load flow equation.

The KMJ algorithm seems to be useful when we need to find out all the solutions with physical meanings. However, it requires some improvement of the algorithm as regards computing time and memory volume when it is applied to the load flow equation of the practical power circuits with many nodes.

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Reference

[1] G.Alefeld and J.Herzberger;"Introduction to Interval Computations", Academic Press, 1983.

[2] R.E.Moore and S.T.Jones;'Safe Starting Regions for Iterative Methods', SIAM J. NUMER. ANAL., 14, 1051-1065, 1977.