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Okishio's Theorem Generalized

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1. Linear case with joint production

1.1. Okishio(1961) showed in a linear model without joint production that when firms adopt cost-reducing new processes the rate of profit will rise provided the real wage rate remain fixed. Let this result be expressed by simple equations. The symbol A denotes the m by n augmented input coefficient matrix. The price equation before technical progress is:

$$p^0=(1+r^0)p^0A^0.$$

Cost-reducing implies

$$p^0\geq(1+r^0)p^0A^*.$$

The superscripts ^{0} and * indicate those symbols before and after technical progress respectively. Given the two equations above, Okishio theorem states that

$$p^*=(1+r^*)p^*A^* \quad \text{with} \quad r^*>r^0$$

if A^* is indecomposable. (The first equation is meaningful only in an economic story.)

1.2. Now let us shift to a linear model with joint production. In such a model we can allow for fixed capital as explained by von Neumann and P.Sraffa. Notation is:

B: output coefficient matrix(m by n) : a process columnwise.

A: input coefficient matrix(m by n). includes workers' feeding stuff.

x: output column n-vector, p: price row m-vector.

r: uniform rate of profit.

Again the superscripts ^{0} and * for old and new processes respectively.

The price equation or the **equilibrium condition**(price side only) is:

$$pB=(1+r)pA.$$

To obtain a generalization of Okishio's theorem we introduce

Quantity Augmenting Property(QAP): A technology (B^*,A^*) is said to have the Quantity Augmenting Property if there exists a column n^*-vector $x^+\geq0$ such that

$$B^*x^+ \gg (1+r^0)A^*x^+,$$

where r^0 is among the possible equilibrium rates of profit with the old technology (B^0,A^0).

Theorem. If the new technology satisfies the QAP, $r^*>r^0$.

Proof. Almost tautological.

1.3. It may be desirable to obtain a condition on the price side because managers are tempted to introduce new processes depending on cost calculation.:

Generalized Profitability Condition(GPC): The technology (B^*,A^*) is said to have the Generalized Profitability Condition over the technology (B^0,A^0), if for no price vector(semi-positive) is it true that $pB^*\leq(1+r^0)pA^*$.

Theorem. QAP and GPC are equivalent.

Proof. (A theorem due to A.Tucker).

A natural economic interpretation of GPC is that there is no price vector under which every process makes losses or breaks even with at least one process making losses. In a sense, to that extent new processes are productive.

2. Nonlinear case

2.1. To allow for economic externalities and variable returns to scale,one may wish to consider nonlinear input-output model. Thus, B and A are now dependent upon the activity level vector x, and written B(x) and A(x). Let us define

$$H(x;r) \equiv B(x)-(1+r)A(x).$$

H(x;r) may be written simply H(x) when r is not relevant. We make the following Assumptions:

(1) $H_i(x)$ is differentiable for every i.

(2) For each i, if $H_i(x)<0$ at some $x\in D\equiv R^n_+-\{0\}$, then $\nabla H_i(x)\cdot x<0$ at the same x.

((2) is satisfied ,e.g.,by functions homogeneous of positive degrees, or pseudoconcave functions such that $H_i(x)\geq 0$. See Mangasarian(1969).)

Theorem. If the system of inequalities $H(x)\geq 0$ has no solution on $S\equiv\{x\in D|\Sigma x_i=1\}$, then there exists a semi-positive vector $p\in R^m_+$ and $x^*\in S$ such that $p\nabla H(x^*)<0$.

Proof. Please refer to Fujimoto(1980).

Now it is not difficult to have a theorem similar to that in section 1.2 above.=

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