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Numerical examination of applicability of the linearized Boltzmann equation

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Abstract

The validity of the linearized Boltzmann equation in describing the behaviour of rarefied gas flows that deviate only slightly from a uniform equilibrium state is discussed on the basis of several numerical examples. Various examples of the above situation are analyzed numerically by the full nonlinear BKW equation and also by its linearized version, with emphasis on the behaviour for small Knudsen numbers, and the solutions of these equations are compared. When the Knudsen number is comparable to or smaller than the degree of the deviation from the uniform equilibrium state, the solution of the linearized equation generally differs decisively from that of the nonlinear equation, however small the degree of the deviation may be. Situations where the nonlinear effect degenerates are also noted.

1. Introduction

The linearized Boltzmann equation, where the second and higher-order terms of the perturbation from a uniform equilibrium solution are neglected, is widely used in analyzing rarefied gas flow problems where the state of the gas deviates slightly from a uniform equilibrium state at rest. The situation is usually encountered in gas dynamic problems of a small system such as in aerosol science and micromachine engineering, where the Mach number of the flow and the temperature variation, compared with the average temperature, on the boundaries, which are close to each other, are both small. It is, however, known that in some situations the linearized equation does not give a correct answer, however small the deviation from a uniform equilibrium state may be ([Cercignani, 1968]; [Sone, 1978]; [Onishi & Sone, 1983]). They are related to infinite domain problems (e.g., Stokes paradox [C, 1968]). In the analysis of the asymptotic behaviour of steady flows of a rarefied gas for small Knudsen numbers, Sone ([Sone, 1971,1984,1987,1991a,b]; [Sone & Aoki, 1987]) discussed the applicability of the linearized Boltzmann equation and pointed out that the nonlinear effect in the Boltzmann equation is, generally, not negligible for any small deviation from a uniform equilibrium state if the Knudsen number of the system is comparable to or smaller than the degree of the deviation. There are, of course, various situations where the effect of nonlinearity degenerates. The above infinite domain examples are also understood by the general statement if the situation is properly interpreted. Recently Aoki and Masukawa [1994] considered the two surface problem of evaporation and condensation and showed a decisive difference between the numerical solutions of the linear and nonlinear equations for very small temperature difference of the two boundaries.

The example, however, is a special one, where the flow velocity is uniform in the limit of small difference of the surface temperatures. For more general understanding, in the present paper, we consider several different situations whose deviation from a uniform equilibrium state is small and investigate the applicability of the linearized Boltzmann equation numerically. That is, we analyze the problem numerically by two ways, i.e., by the original nonlinear equation and by its linearized version, compare the results, and confirm its applicability with respect to the parameters: the Knudsen number (Kn) and the parameter that represents the deviation from a uniform equilibrium state (say, nonuniformity parameter ε). For simplicity of analysis, we adopt the BKW (or BGK) equation ([Bhatnagar, *et al.*, 1954]; [Welander, 1954]; [Kogan, 1958]) as the basic equation. For the present purpose, this is legitimate from comparison of various results of the BKW equation and the standard Boltzmann equation. In the analytical discussion of the applicability of the linearized Boltzmann equation, which is done in connection with the asymptotic analysis of the Boltzmann system for small Knudsen numbers, the BKW equation shows the same behaviour as the standard Boltzmann equation (e.g., [S & A, 1987]; [S, 1987]; [S, 1991a,b]). The results of the linearized BKW equation are consistent with recent accurate numerical computations by the linearized Boltzmann equation for hard-sphere molecules (e.g., [Sone *et al.*, 1990]; [Ohwada *et al.*, 1989]; [S, 1991b]; [Takata *et al.*, 1993]). On the boundary the Maxwell type condition or the conventional boundary condition of evaporation and condensation (e.g., [Cercignani, 1987]; [S, 1987]) is adopted as the kinetic boundary condition.

2. Basic equation and notations

Let p_0 and T_0 be the pressure and the temperature of the uniform equilibrium state at rest. When we consider problems with evaporation or condensation on a boundary, p_0 is the saturated gas pressure at temperature T_0. The density ρ_0 and the velocity distribution function f_0 of the equilibrium state are given by

(1) $$\rho_0 = p_0/RT_0,$$

(2) $$f_0 = \frac{\rho_0}{(2\pi RT_0)^{3/2}} \exp(-\xi_i^2/2RT_0),$$

where R is the specific gas constant and ξ_i is the molecular velocity. We are interested in the behaviour of the gas for small deviations from this uniform state.

Let L be the characteristic length of the system and let ℓ_0 be the mean free path of the equilibrium state, which is the ratio of the mean molecular speed $(8RT_0/\pi)^{1/2}$ and the mean collision frequency. In the present paper, we use the following nondimensional variables based on the above basic variables of the system: $\mathrm{Kn} = \ell_0/L$; $x_i L$ is the Cartesian coordinate system of the physical space; (r, θ, x_3) is the cylindrical coordinate system in the x_i space with the common x_3 axis; $(2RT_0)^{1/2}\zeta_i$ is the molecular velocity; $\zeta = (\zeta_i^2)^{1/2}$; $f_0(1+\phi)$ is the velocity distribution function; $\rho_0(1+\omega)$ is the density of the gas; $(2RT_0)^{1/2}u_i$ is the flow velocity; $T_0(1+\tau)$ is the temperature; $p_0(1+P)$ is the pressure; n_i is the unit normal vector to the boundary, pointed to the gas; $(2RT_0)^{1/2}u_{wi}$ is the velocity of the boundary with $u_{wi}n_i = 0$ (this is required in steady flow problems); $T_0(1+\tau_w)$ is the temperature of the boundary; $\rho_0(1+\sigma_{ws})$ and $p_0(1+P_{ws})$ are, respectively, the saturation gas density and pressure at temperature $T_0(1+\tau_w)$, determined by the Clausius-Clapeyron relation [Reif, 1965]; and $E(\zeta) = \pi^{-3/2}\exp(-\zeta^2)$. Thus, $f_0 = \rho_0(2RT_0)^{-3/2}E(\zeta)$. The r and θ components of u_i and u_{wi} are denoted by the subscripts r and θ, respectively (e.g., u_r, $u_{w\theta}$).

In these variables, the nondimensional BKW equation for a steady flow is written as

(3) $$\zeta_i \frac{\partial \phi}{\partial x_i} = \frac{2}{\sqrt{\pi}\,\mathrm{Kn}}(1+\omega)(\phi_e - \phi),$$

(4) $$\phi_e E = \frac{1+\omega}{\pi^{3/2}(1+\tau)^{3/2}} \exp\left[-\frac{(\zeta_i - u_i)^2}{1+\tau}\right] - E,$$

where

(5a) $$\omega = \int \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3,$$

(5b) $$(1+\omega)u_i = \int \zeta_i \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3,$$

(5c) $$\frac{3}{2}(1+\omega)\tau = \int (\zeta_j^2 - \frac{3}{2})\phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3 - (1+\omega)u_j^2,$$

(5d) $$P = \omega + \tau + \omega\tau.$$

The Maxwell type boundary condition is written as follows:

(6) $$\phi(x_i, \zeta_i) = (1-\alpha)\phi(x_i, \zeta_i - 2\zeta_j n_j n_i) + \alpha\phi_e(\omega = \sigma_w, u_i = u_{wi}, \tau = \tau_w), \quad (\zeta_i n_i > 0),$$

(7) $$\sigma_w = \frac{1}{(1+\tau_w)^{1/2}}\left(1 - 2\sqrt{\pi}\int_{\zeta_j n_j < 0} \zeta_j n_j \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3\right) - 1,$$

where α $(0 \le \alpha \le 1)$ is the accommodation coefficient of the boundary. The condition is called diffuse reflection when $\alpha = 1$, and specular reflection when $\alpha = 0$. The conventional condition of evaporation and condensation on an interface between a gas and its condensed phase, which is also called a complete condensation condition, is as follows:

(8) $$\phi(x_i, \zeta_i) = \phi_e(\omega = \sigma_{ws}, u_i = u_{wi}, \tau = \tau_w), \qquad (\zeta_i n_i > 0).$$

In the following analysis, P_{ws} is preferred to σ_{ws} as a parameter. It is related to σ_{ws} and τ_w as

(9) $$P_{ws} = \sigma_{ws} + \tau_w + \sigma_{ws}\tau_w.$$

Let ϕE, the deviation from the uniform state given by Eq. (1), be $O(\varepsilon)$. Needless to say, u_{wi} and τ_w should be $O(\varepsilon)$ for such ϕE to be the solution. Then, the macroscopic variables ω, u_i, and τ are also $O(\varepsilon)$. Neglecting the second and higher-order terms of $O(\varepsilon)$ in Eqs. (3)–(8), we obtain the linearized equations. The linearized BKW equation is:

(10) $$\zeta_i \frac{\partial \phi}{\partial x_i} = \frac{2}{\sqrt{\pi}\,\mathrm{Kn}}[\omega + 2\zeta_i u_i + (\zeta^2 - \frac{3}{2})\tau - \phi],$$

where

(11a) $$\omega = \int \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3,$$

(11b) $$u_i = \int \zeta_i \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3,$$

(11c) $$\frac{3}{2}\tau = \int (\zeta^2 - \frac{3}{2})\phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3,$$

(11d) $$P = \omega + \tau.$$

The linearized Maxwell type condition is:

(12) $$\phi(x_i, \zeta_i) = (1-\alpha)\phi(x_i, \zeta_i - 2\zeta_j n_j n_i) + \alpha[\sigma_w + 2\zeta_j u_{wj} + (\zeta^2 - \frac{3}{2})\tau_w], \quad (\zeta_j n_j > 0),$$

$$\sigma_w = -\frac{1}{2}\tau_w - 2\pi^{1/2}\int_{\zeta_j n_j < 0} \zeta_j n_j \phi E \, d\zeta_1 \, d\zeta_2 \, d\zeta_3. \tag{13}$$

The linearized complete condensation condition is:

$$\phi(x_i, \zeta_i) = \sigma_{ws} + 2\zeta_j u_{wj} + (\zeta^2 - \frac{3}{2})\tau_w, \qquad (\zeta_i n_i > 0). \tag{14}$$

The linearized form of Eq. (9) is

$$P_{ws} = \sigma_{ws} + \tau_w. \tag{15}$$

3. Plane Couette flow with evaporation or condensation on the boundaries

In this section we consider the steady behaviour of a gas in the region $0 < x_2 < 1$ bounded by its two parallel plane condensed phases with different temperatures, one of which is moving in its own plane. Let $u_{wi} = 0$, $\tau_w = 0$, and $P_{ws} = 0$ at $x_2 = 0$, and let $u_{wi} = (\varepsilon_1, 0, 0)$, $\tau_w = \varepsilon_2$, and $P_{ws} = \varepsilon_3$ at $x_2 = 1$. We numerically solve the nonlinear system, Eqs. (3)–(5d) subject to boundary condition (8) at $x_2 = 0$ and $x_2 = 1$, and the linear system, Eqs. (10)–(11d) with Eq. (14) at $x_2 = 0$ and $x_2 = 1$ and compare the solutions of the two systems. Our interest is the behaviour of a slightly nonuniform state, i.e., for small values of ε_1, ε_2, and ε_3. The method of numerical computation is a straightforward application of that in [Aoki, *et al.*, 1991]. Thus, it is not repeated here, and only the results of computation are given.

The profiles of ω, τ, u_1, and u_2 of the two systems, linear and nonlinear systems, with $\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.001$, and $\varepsilon_3 = 0.02$ are shown in Fig. 1 for three Knudsen numbers (Kn = 0.002, 0.02, and 0.2). Let $\varepsilon = \max|\varepsilon_m|$, then ε is a measure of deviation from our reference uniform equilibrium state at rest. This notation will also be used in the following examples. When Kn = 0.2, where Kn is fairly larger than $\varepsilon(= 0.02)$, the profiles of the two systems in Fig. 1 are very close to each other, and the difference is bounded by ε^2. Thus, the linear solution is well qualified as the first order approximation of the nonlinear system. When Kn = 0.02, where Kn is comparable to ε, the difference of the two solutions becomes appreciable and it is fairly larger than ε^2 [Fig. 1 (c)]. When Kn = 0.002, where Kn is fairly smaller than ε, the two solutions are markedly different, and the difference is obviously larger than ε^2 by far. The linear solution cannot be considered to be an approximation of the nonlinear solution. Incidentally, the negative temperature-gradient phenomenon ([Pao, 1971]; [Sone & Onishi, 1978]; [Aoki & Cercignani, 1983]; [Hermans & Beenakker, 1986]; [Sone *et al.*, 1991]; [A & M, 1994]) is seen in these examples [Fig. 1 (b)].

When there is neither evaporation nor condensation on the boundaries at $x_2 = 0$ and $x_2 = 1$, where $u_2 \equiv 0$, the situation is different. Figure 2 shows the profiles of ω, τ, and u_1 in the case of diffuse reflection, Eqs. (6) and (7) or Eqs. (12) and (13) with $\alpha = 1$, with $\varepsilon_1 = \varepsilon_2 = 0.02$, i.e., $u_{wi} = 0$ and $\tau_w = 0$ at $x_2 = 0$ and $u_{wi} = (0.02, 0, 0)$ and $\tau_w = 0.02$ at $x_2 = 1$. As in any closed domain problem without evaporation and condensation on the boundary, the solution is not uniquely determined by the diffuse reflection condition; Another condition relating to the mass of the gas in the domain is required for the uniqueness. Here we have chosen the solution with $\sigma_w = 0$ at $x_1 = 0$. For three cases of Knudsen numbers, i.e., Kn = 0.002, 0.02, and 0.2, the deviation of the solution of the linearized equation from that of the nonlinear equation is uniformly very small with respect to the Knudsen number, and the former solution is a good approximation to the latter solution.

4. Cylindrical Couette flow with evaporation or condensation on the boundaries

Here we consider the behaviour of a gas in the region $1 < r < 2$ bounded by its two coaxial circular cylindrical condensed phases with different temperatures, the outer one of which is rotating with a constant angular velocity around its axis. Let $u_{wi} = 0$, $\tau_w = 0$, and $P_{ws} = 0$ at $r = 1$, and let $u_{w\theta} = \varepsilon_1$, $u_{wr} = u_{w3} = 0$, $\tau_w = \varepsilon_2$, and $P_{ws} = \varepsilon_3$ at $r = 2$. We numerically solve the nonlinear system, Eqs. (3)–(5d) with Eq. (8) at $r = 1$ and $r = 2$, and the linear system, Eqs. (10)–(11d) with Eq. (14) at $r = 1$ and $r = 2$ for various sets of the parameters Kn, ε_1, ε_2, and ε_3. The method of computation is a straightforward application of that in [Sugimoto & Sone, 1992], and only the results of computation are given here.

In Fig. 3, the profiles of ω, τ, u_r, and u_θ are shown for Kn = 0.005, 0.02, and 0.2 in the case $\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.001$, and $\varepsilon_3 = 0.02$. The difference of the linear and nonlinear solutions obviously increases as the Knudsen number decreases (Fig. 3). When Kn = 0.2, which is fairly larger than ε ($= \max|\varepsilon_m|$), the two solutions are very close; when Kn = 0.02, which is comparable to ε, the difference of u_θ is fairly larger than ε^2, and when Kn = 0.005, the difference of u_θ is of the order of ε. The relative difference of τ increases in a similar way to that of u_θ, but the absolute difference is much smaller since ε_2 (thus τ itself) is much smaller than ε_1 (or u_θ). Incidentally, the negative temperature-gradient phenomenon is seen in these examples [Fig. 3 (b)].

In Fig. 4, the profiles of ω, τ, u_r, and u_θ are shown for Kn = 0.005, 0.02, and 0.2 in the case $\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.01$, and $\varepsilon_3 = 0.02$. The feature of the difference of the two solutions is similar to that of the previous example, but the difference of τ is comparable to that of u_θ since ε_2 is comparable to ε. Incidentally, the negative temperature-gradient phenomenon is not seen in these examples [Fig. 4 (b)].

An example without evaporation and condensation on the boundaries is shown in Fig. 5, where the case of $\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.01$ and diffusely reflecting boundary are considered. As in the corresponding problem in Sec. 3, we have chosen the solution with $\sigma_w = 0$ at $r = 1$. Again, the linear solution is a good approximation to the nonlinear solution for the three Knudsen numbers Kn = 0.005, 0.02, and 0.2.

5. Flow past an array of flat plates

Here we consider an example of flows past a body without evaporation and condensation. In order to concentrate our interest on the behaviour in a finite region and to avoid the difficulty [Sone & Takata, 1992] of numerical computation owing to the discontinuity of the velocity distribution function around a convex body, we investigate the following somewhat artificial problem in a rectangular domain ($-a < x_1 < a$, $0 < x_2 < b$).

(i) Nonlinear problem: The basic equation is given by Eqs. (3)–(5d). The boundary condition is as follows: $\phi(\zeta_2 > 0)$ on ($x_2 = 0$, $-a < x_1 < a$) is given by Eqs. (6) and (7) where $u_{wi} = 0$, $\tau_w = \varepsilon_2$, and

$$\text{(16a)} \qquad \alpha = \frac{1}{2}[1 + \cos(2\pi x_1)], \qquad (-1/2 \le x_1 \le 1/2),$$

$$\text{(16b)} \qquad \alpha = 0, \ \text{(specular reflection)}, \qquad (-a < x_1 < -1/2 \text{ and } 1/2 < x_1 < a);$$

$\phi(\zeta_2 < 0)$ on ($x_2 = b$, $-a < x_1 < a$) is given by Eqs. (6) and (7) with $\alpha = 0$ (specular reflection); $\phi(\zeta_1 > 0)$ on ($x_1 = -a$, $0 \le x_2 \le b$) and $\phi(\zeta_1 < 0)$ on ($x_1 = a$, $0 \le x_2 \le b$) are given by the corresponding parts ($\zeta_1 \gtrless 0$) of $\phi_e(\omega = 0,\ u_i = (\varepsilon_1, 0, 0),\ \tau = 0)$.

(ii) Linear problem: The basic equation is given by Eqs. (10)–(11d). The boundary condition is as follows: $\phi(\zeta_2 > 0)$ on $(x_2 = 0,\ -a < x_1 < a)$ is given by Eqs. (12) and (13) with $u_{wi} = 0$, $\tau_w = \varepsilon_2$, and Eqs. (16a) and (16b); $\phi(\zeta_2 < 0)$ on $(x_2 = b,\ -a < x_1 < a)$ is given by Eqs. (12) and (13) with $\alpha = 0$; $\phi(\zeta_1 > 0)$ on $(x_1 = -a,\ 0 \leq x_2 \leq b)$ and $\phi(\zeta_1 < 0)$ on $(x_1 = a,\ 0 \leq x_2 \leq b)$ are the corresponding parts $(\zeta_1 \gtrless 0)$ of $2\zeta_1\varepsilon_1$.

The problem is a model of a uniform flow past an array of flat plates without an angle of attack $(-1/2 \leq x_1 \leq 1/2,\ x_2 = 2mb,\ m = 0, \pm 1, \cdots)$. The upstream and downstream regions are limited at $x_1 = -a$ and a, since we want to examine nonlinear effects in a finite-domain problem. According to [S & T, 1992], the discontinuity of a velocity distribution function on a boundary at the tangential velocities propagates into the gas from convex points of the boundary. The present choice, Eq. (16a), of the accommodation coefficient avoids the discontinuity at the leading and trailing edges of the plate, which are the only convex points of the boundary. Thus the velocity distribution function is continuous in the gas.

The flow with the nonuniformity parameters $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.1$ in the domain $a = 1$, $b = 1/2$ is computed for three Knudsen numbers Kn = 0.02, 0.1, and 0.5. The profiles of ω, τ, u_1, and u_2 along the sections $x_1 = 0$, ± 0.4, and ± 0.725 are shown in Figs. 6a–6d. As in the examples with evaporation and condensation in Secs. 3 and 4, the deviation of the linear solution from the nonlinear solution increases as the Knudsen number decreases, and the differences in ω and τ of the two solutions obviously exceed ε^2 ($\varepsilon = \max|\varepsilon_m| = 0.1$) for Kn = 0.1 and 0.02, and are $O(\varepsilon)$ when Kn = 0.02.

6. Discussion

In this paper we considered various rarefied gas flows where the situation is very close to a uniform equilibrium state at rest, and investigated the flows numerically on the basis of two types of basic equations: the (original nonlinear) BKW equation and its linearized version. The results are compared, and the validity of the solution of the linearized equation in describing the flow is examined. The result depends on the Knudsen number of the system.

When the Knudsen number (Kn) is much larger than the nonuniformity parameter (ε), the solution of the linearized equation is a good approximation to that of the nonlinear equation. As the Knudsen number decreases, the deviation of the linear solution from the nonlinear solution generally increases. It is fairly larger than ε^2 when Kn $\sim \varepsilon$, and the two solutions are quite different when Kn $\ll \varepsilon$. Thus for Kn $\lesssim \varepsilon$, the nonlinear solution cannot be obtained by a simple perturbation analysis from the linear solution. In some cases, however, the linear solution is a good approximation to the nonlinear solution irrespective of the Knudsen number (see the second example of Sec. 3 and the last example of Sec. 4). This is discussed below.

General theoretical discussion of the importance of the nonlinear term even in the case where the system deviates slightly from a uniform state was made in [S, 1971] in connection with asymptotic analysis of the Boltzmann equation for small Knudsen numbers (see also [S, 1978, 1984, 1991ab]). The present numerical computations give good examples of the theoretical discussion. The ratio of the Knudsen number and the nonuniformity parameter determines the validity of the linearized Boltzmann equation. Since the case with small nonuniformity parameters is concerned, the ratio takes various values only for small Knudsen numbers. Therefore, the situation can be clarified by the analysis of the case with small Knudsen numbers, which admits a macroscopic description. Thus, the degeneracy of the nonlinear effect is easily surveyed by the macroscopic description. That is, the leading nonlinear term in the macroscopic description is the convection term of the incompressible Navier-Stokes system of equations (continuity, momentum, and energy equations), and therefore the linearized equation gives a good description in the case where the convection term degenerates or is incorporated in the pressure term in the Navier-Stokes system.

The last statement is well exemplified in our numerical computation. In the Couette flow under diffuse reflection in Sec. 3, the convection term vanishes, and in the cylindrical Couette flow under diffuse reflection in Sec. 4, only non vanishing part of the convection term, u_θ^2/r, can be incorporated in the pressure term. In both cases the deviation of the linear solution from the nonlinear solution is at most of the second order of the nonuniformity parameter (Figs. 2 and 5). In the example in Sec. 5, the flow is nearly in the x_1 direction, and therefore the leading convection term of the incompressible Navier Stokes system is $u_1\partial u_1/\partial x_1$ in the x_1-momentum equation and $u_1\partial\tau/\partial x_1$ in the energy equation. The $u_1\partial u_1/\partial x_1$ can be incorporated in the pressure term, but $u_1\partial\tau/\partial x_1$ is left as it is. Since the velocity field can be solved independently from the temperature field in the incompressible Navier-Stokes system, according to the statement the velocity field of the linear equation should be a good approximation, but its temperature field may deviate considerably from that of the nonlinear equation. Figures 6a–6d support this.

In some infinite-domain problems, the discrepancy of the linearized equation such as Stokes paradox [C, 1968] is encountered for arbitrary Knudsen numbers. From the following reason, this is also the same kind of difficulty of the linearized equation as that in the present examples. In these problems, the solution is supposed to approach a uniform state at infinity, and the length scale of the variation of the solution increases with the distance from a body. Then the effective Knudsen number (the mean free path divided by the local length scale of variation), which determines the variation of the variables, decreases to vanish, and it becomes much smaller than the small nonuniformity parameter in the far field, and therefore the criterion on discrepancy of linear solutions applies.

Finally, the computation was carried out by HP 9000 730 and MIPS RS 3230 computers at our laboratory and by FACOM VP-2600 computer at the Data Processing Center of Kyoto University.

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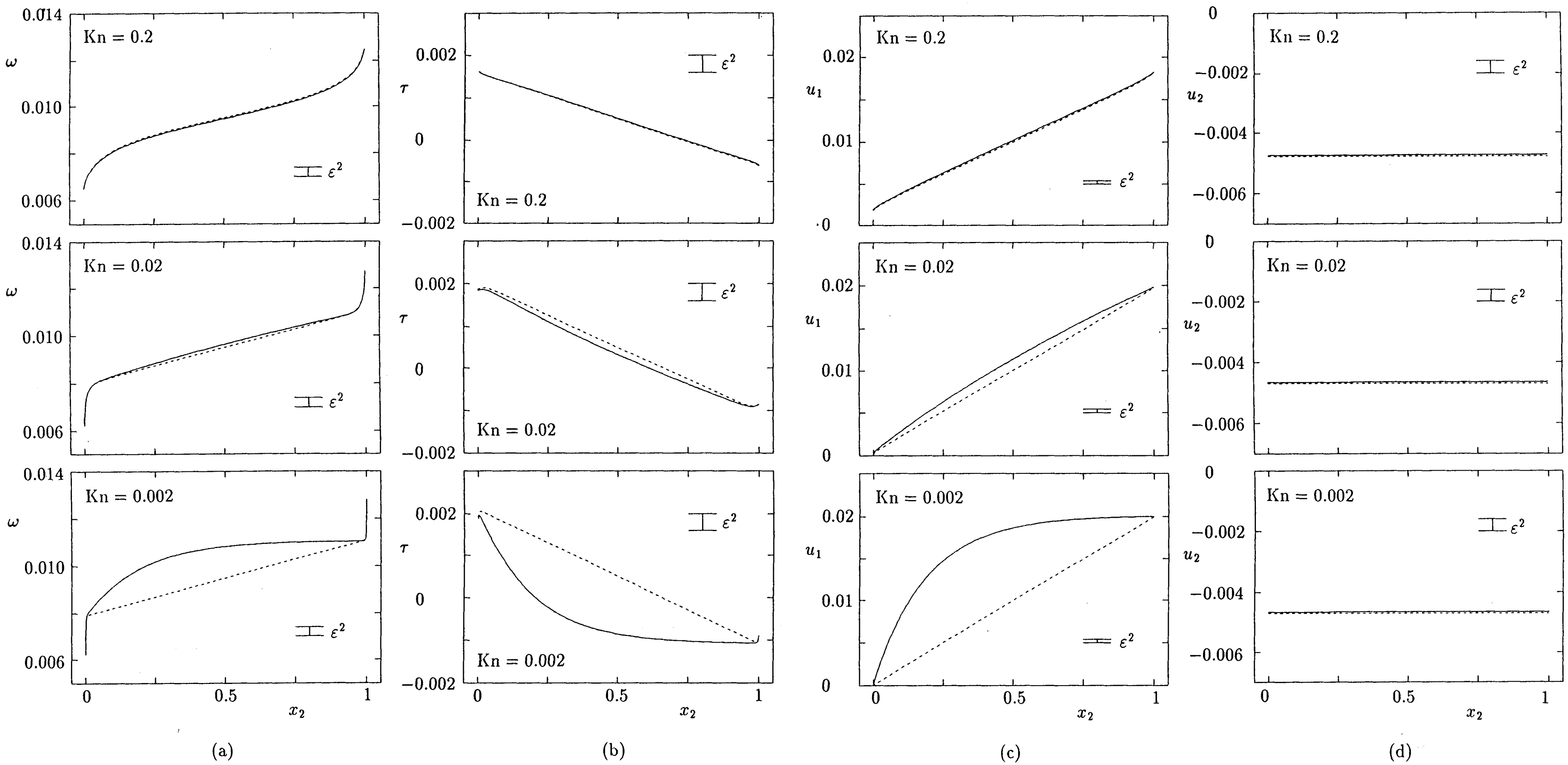

Fig. 1. Plane Couette flow with evaporation or condensation on the boundaries ($\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.001$, and $\epsilon_3 = 0.02$) for three Knudsen numbers (Kn = 0.002, 0.02, 0.2). (a) ω field, (b) τ field, (c) u_1 field, and (d) u_2 field. Here, ——indicates the solution of the nonlinear system, and -----indicates that of the linear system. The size ε^2 ($\varepsilon = \max|\varepsilon_m| = 0.02$) is shown for reference.

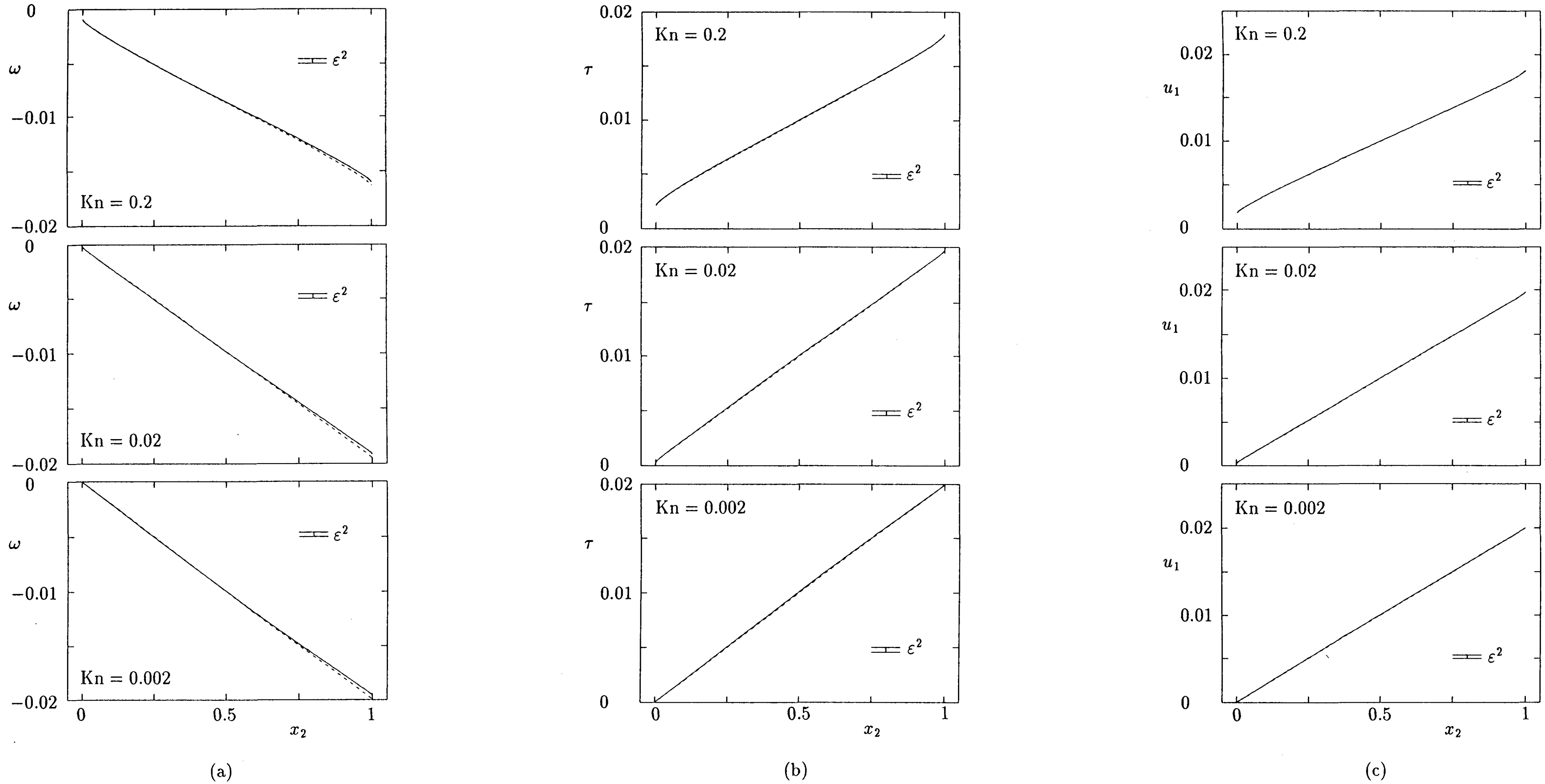

Fig. 2. Plane Couette flow with diffuse reflection on the boundaries ($\varepsilon_1 = \varepsilon_2 = 0.02$, i.e., $u_{wi} = 0$ and $\tau_w = 0$ at $x_2 = 0$, and $u_{wi} = (0.02, 0, 0)$ and $\tau_w = 0.02$ at $x_2 = 1$) for three Knudsen numbers (Kn = 0.002, 0.02, 0.2). (a) ω field, (b) τ field, and (c) u_1 field. Here, ——indicates the solution of the nonlinear system, and -----indicates that of the linear system. The size ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.02$) is shown for reference.

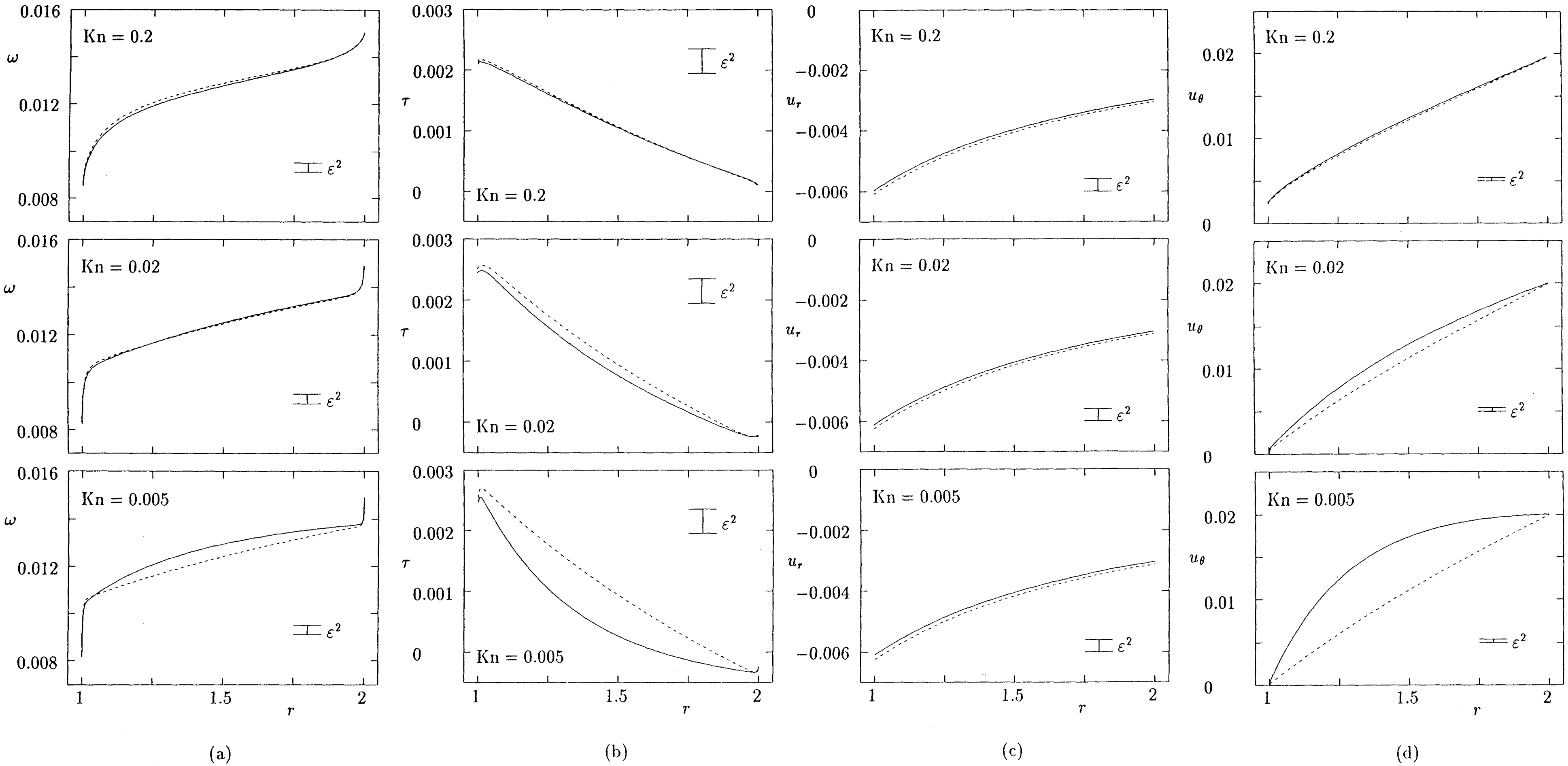

Fig. 3. Cylindrical Couette flow with evaporation or condensation on the boundaries ($\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.001$, and $\varepsilon_3 = 0.02$) for three Knudsen numbers (Kn = 0.005, 0.02, 0.2). (a) ω field, (b) τ field, (c) u_r field, and (d) u_θ field. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The size ε^2 ($\varepsilon = \max|\varepsilon_m| = 0.02$) is shown for reference.

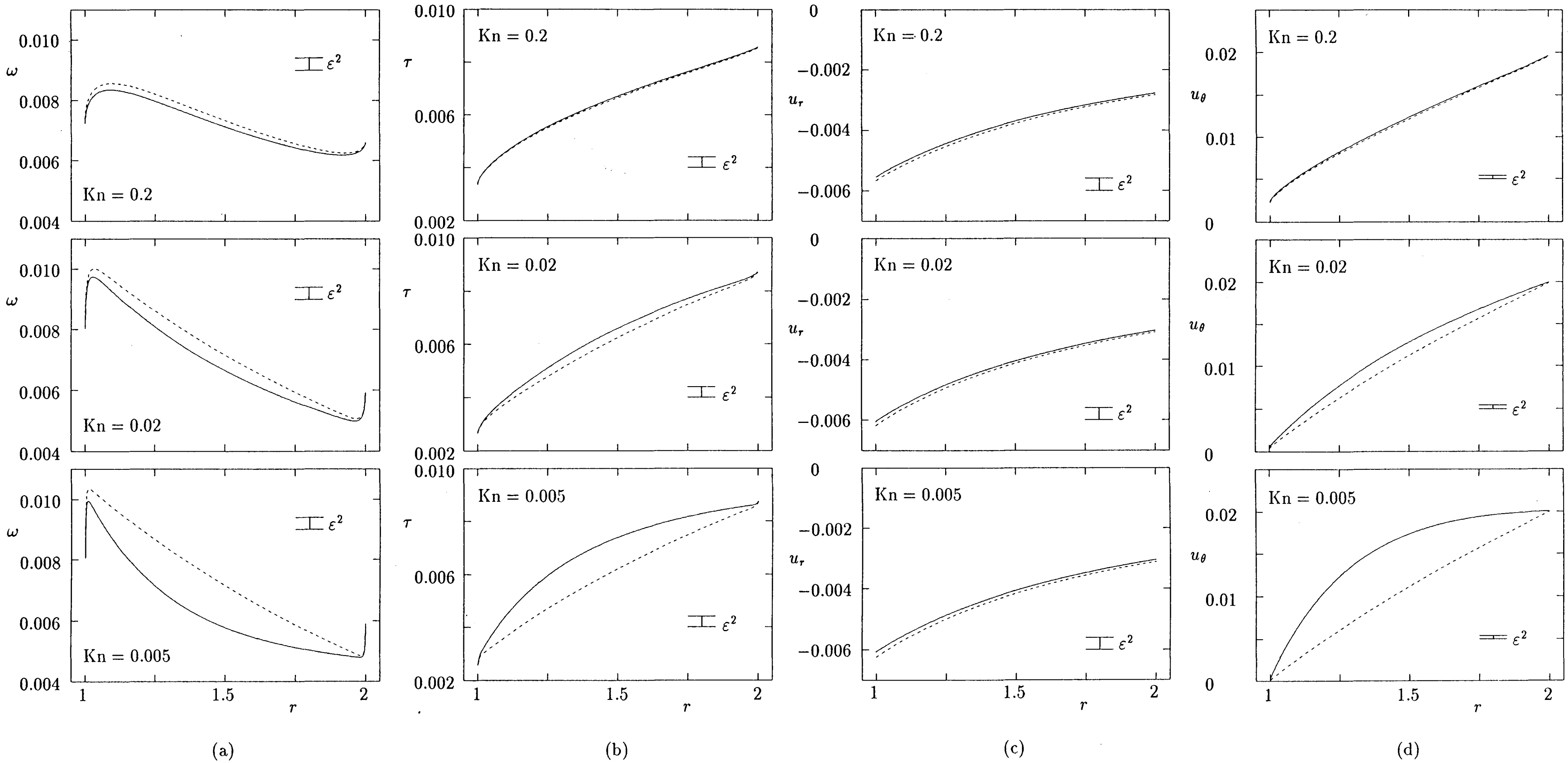

Fig. 4. Cylindrical Couette flow with evaporation or condensation on the boundaries ($\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.01$, and $\varepsilon_3 = 0.02$) for three Knudsen numbers (Kn = 0.005, 0.02, 0.2). (a) ω field, (b) τ field, (c) u_r field, and (d) u_θ field. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The size ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.02$) is shown for reference.

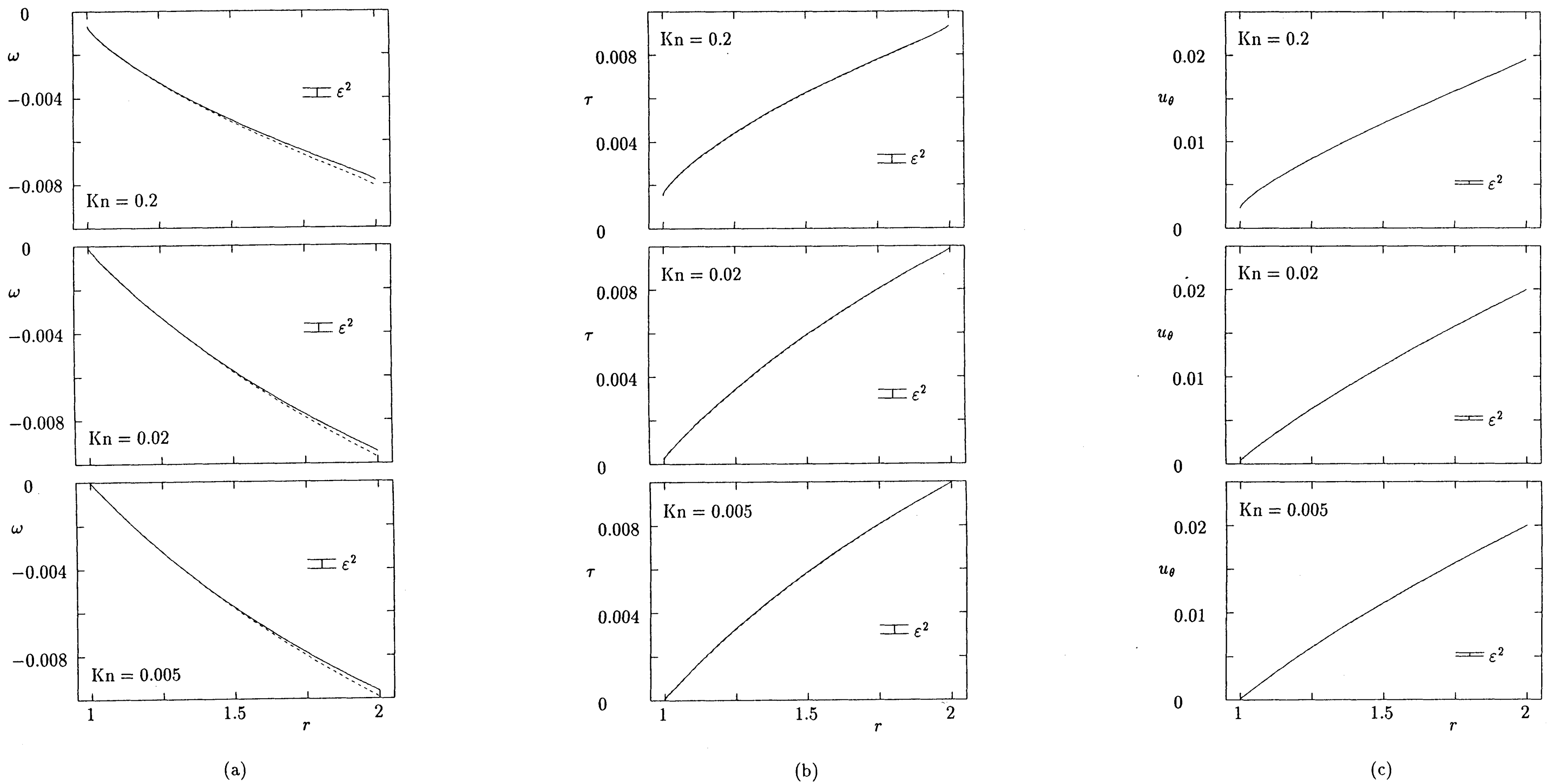

Fig. 5. Cylindrical Couette flow with diffuse reflection on the boundaries ($\varepsilon_1 = 0.02$ and $\varepsilon_2 = 0.01$, i.e., $u_{wi} = 0$ and $\tau_w = 0$ at $r = 1$, and $u_{w\theta} = 0.02$, $u_{wr} = u_{w3} = 0$, and $\tau_w = 0.01$ at $r = 2$) for three Knudsen numbers (Kn = 0.005, 0.02, 0.2). (a) ω field, (b) τ field, (c) u_θ field. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The size ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.02$) is shown for reference.

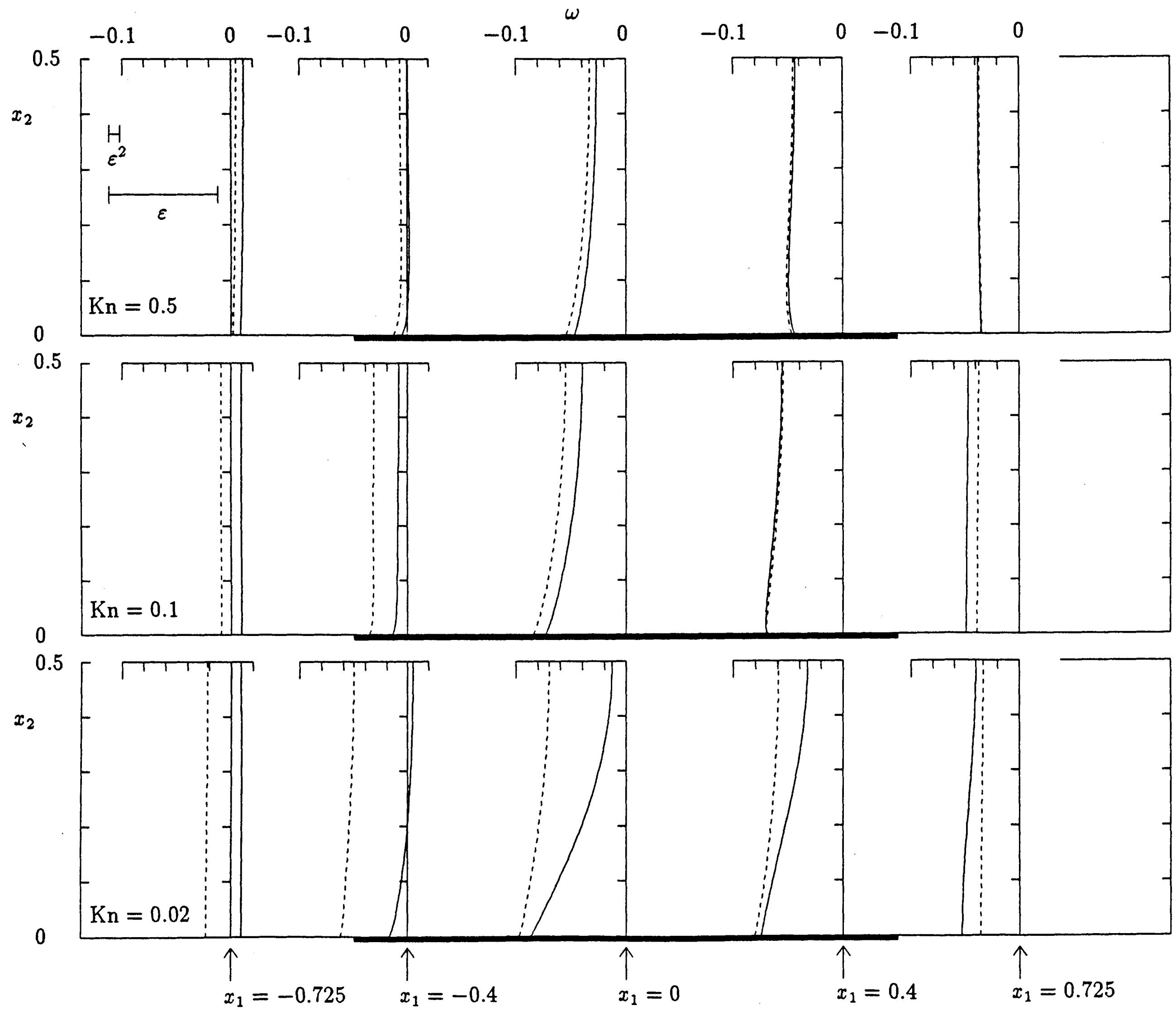

Fig. 6a. Flow past an array of flat plates ($a = 1$, $b = 1/2$, and $\varepsilon_1 = \varepsilon_2 = 0.1$) for three Knudsen numbers (Kn = 0.02, 0.1, 0.5) I: ω field. The profiles along the sections $x_1 = 0$, ± 0.4, and ± 0.725 are shown at the corresponding places in the domain. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The sizes ε and ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.1$) are shown for reference.

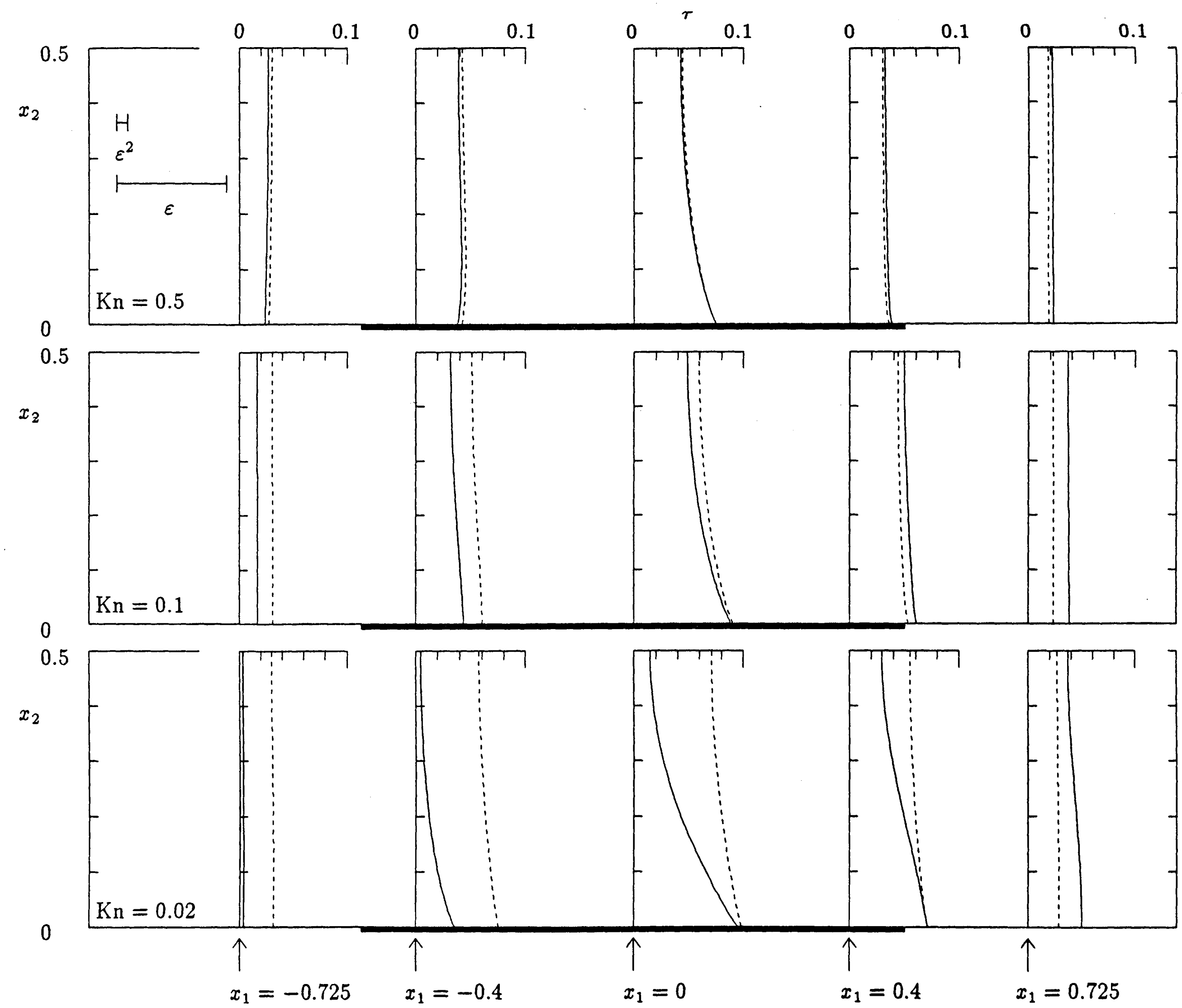

Fig. 6b. Flow past an array of flat plates ($a = 1$, $b = 1/2$, and $\varepsilon_1 = \varepsilon_2 = 0.1$) for three Knudsen numbers (Kn = 0.02, 0.1, 0.5) II: τ field. The profiles along the sections $x_1 = 0$, ± 0.4, and ± 0.725 are shown at the corresponding places in the domain. Here, ——indicates the solution of the nonlinear system, and -----indicates that of the linear system. The sizes ε and ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.1$) are shown for reference.

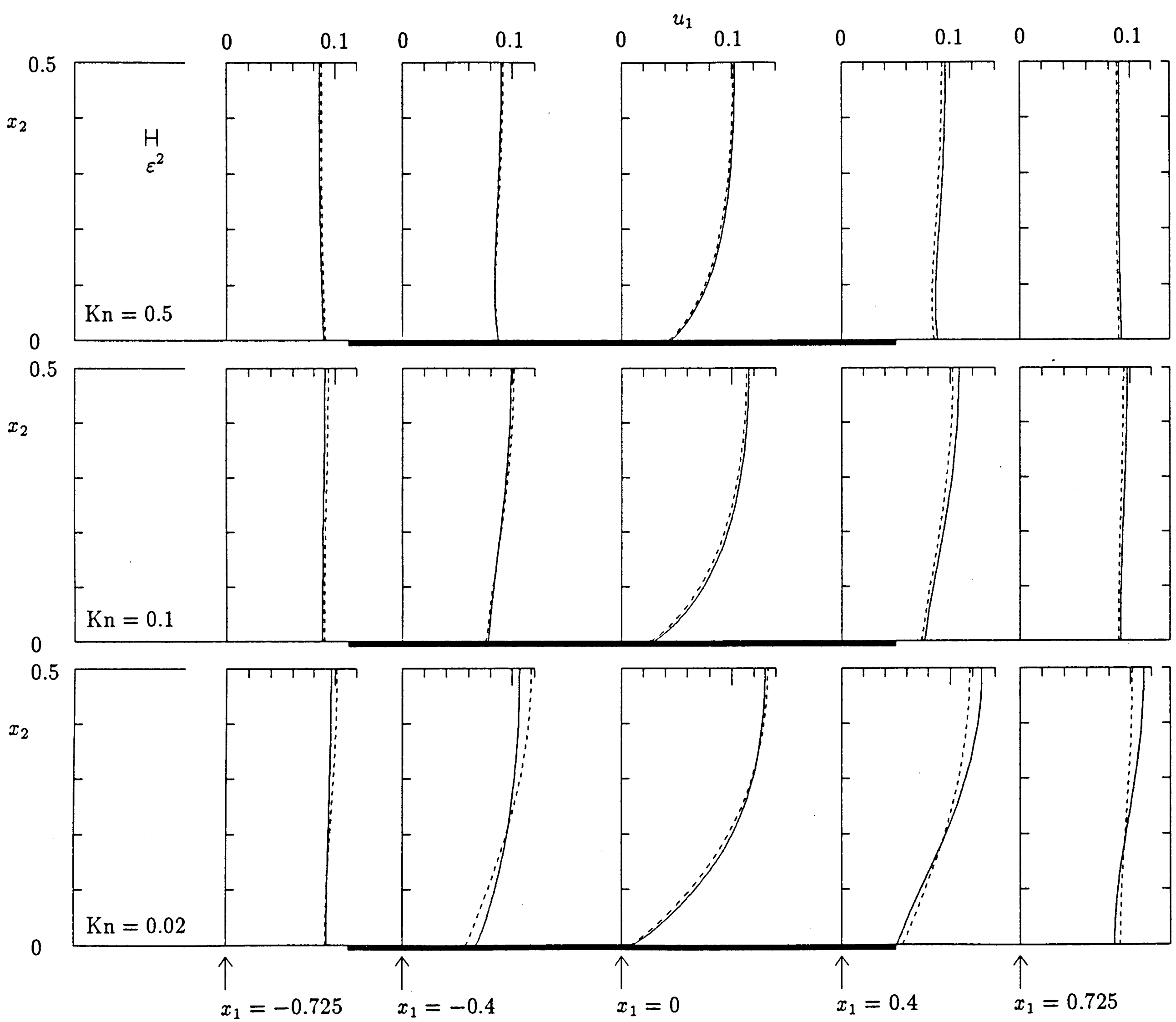

Fig. 6c. Flow past an array of flat plates ($a = 1$, $b = 1/2$, and $\varepsilon_1 = \varepsilon_2 = 0.1$) for three Knudsen numbers (Kn = 0.02, 0.1, 0.5) III: u_1 field. The profiles along the sections $x_1 = 0$, ± 0.4, and ± 0.725 are shown at the corresponding places in the domain. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The size ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.1$) is shown for reference.

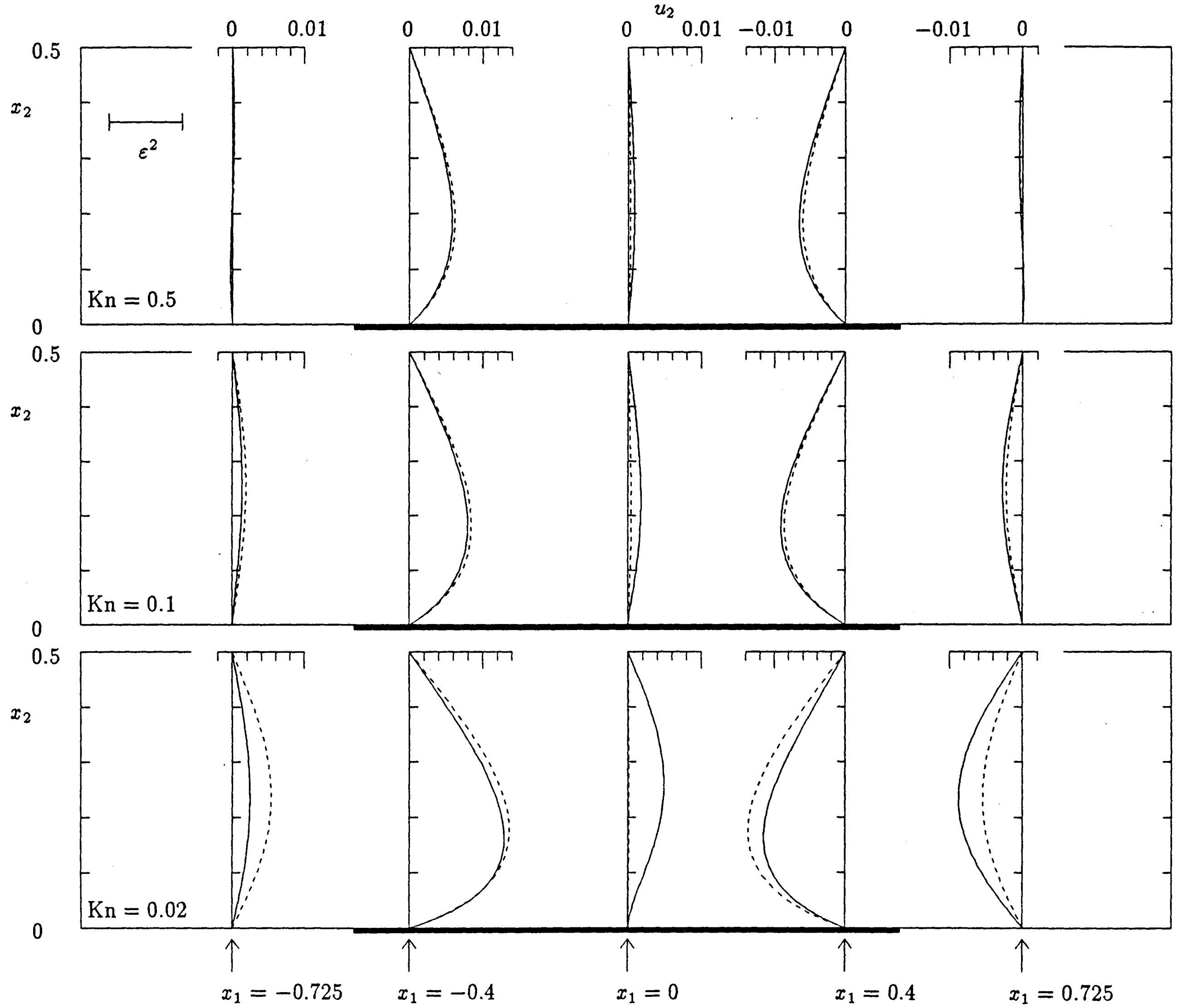

Fig. 6d. Flow past an array of flat plates ($a = 1$, $b = 1/2$, and $\varepsilon_1 = \varepsilon_2 = 0.1$) for three Knudsen numbers (Kn = 0.02, 0.1, 0.5) IV: u_2 field. The profiles along the sections $x_1 = 0$, ± 0.4, and ± 0.725 are shown at the corresponding places in the domain. Here, —— indicates the solution of the nonlinear system, and ----- indicates that of the linear system. The size ε^2 ($\varepsilon = \max |\varepsilon_m| = 0.1$) is shown for reference.