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Galois Representations on Profinite Braid Groups on Curves

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ABSTRACT. Let X be an open smooth geometrically connected curve over a field $k \subset \mathbb{C}$, and $B_{0,n}X$ the configuration space of unordered n points on X. The main purpose of this manuscript is to announce that the $\mathrm{Gal}(\bar{k}/k)$-action on the profinite fundamental group of $B_{0,n}X$ can be completely described in terms of only the action on the profinite fundamental group of X and that of $\mathbb{P}^1 - \{0, 1, \infty\}$. This is a generalization of the joint work with Y.Ihara[20], which treats the case $X = \mathbb{A}^1$.

The description tightly relates the $\mathrm{Gal}(\bar{k}/k)$-actions for a positive genus curve and for $\mathbb{P}^1 - \{0, 1, \infty\}$. Using this, we prove a generalization of Belyĭ's Injectivity Theorem: for a number field k, $\mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\pi_1^{alg}(X \otimes_k \bar{k})$ is injective if X is an affine curve over k with non-abelian fundamental group.

Also, we study field towers over $\mathbb{Q}$ introduced by Takayuki Oda, and prove some part of his conjectures.

Introduction

Throughout this manuscript, $k \subset \mathbb{C}$ denotes a subfield of the complex number field $\mathbb{C}$, and $\bar{k}$ denotes the algebraic closure of k in $\mathbb{C}$. For a smooth geometrically irreducible variety V defined over k, we denote by $\overline{V}$ the variety $V \otimes_k \bar{k}$, and denote by $K(V)$ the function field of V. Let x be a scheme-theoretic point of V (not necessarily closed), and let $\bar{x}$ be a geometric point on x. Then, there is a short exact sequence of profinite groups

$$1 \to \pi_1^{alg}(\overline{V}, \bar{x}) \to \pi_1^{alg}(V, \bar{x}) \to \mathrm{Gal}(\bar{k}/k) \to 1. \tag{0.1}$$

By the Comparison Theorem, the left group is (canonically up to an inner automorphism) isomorphic to the profinite completion $\widehat{\pi V} = \widehat{\pi}(V)$ of the topological fundamental group

$$\pi V = \pi(V) := \pi_1(V(\mathbb{C}), *) \tag{0.2}$$

($V(\mathbb{C})$ is the set of $\mathbb{C}$-rational points of V with $\mathbb{C}$-topology, and $*$ is any base point). This exact sequence induces the *outer Galois representation*

$$\rho_{out} : \mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\widehat{\pi}V := \mathrm{Aut}\widehat{\pi}V/\mathrm{Inn}\widehat{\pi}V$$

as follows. We define the image of $\gamma \in \widehat{\pi}V$ by $\sigma \in \mathrm{Gal}(\bar{k}/k)$ to be $\tilde{\sigma}\gamma\tilde{\sigma}^{-1}$, where $\tilde{\sigma}$ is any lift of σ to the middle group of the exact sequence (0.1). The ambiguity of $\tilde{\sigma}$ is absorbed in $\mathrm{Inn}\widehat{\pi}V$, and this provides a well-defined element of $\mathrm{Out}\widehat{\pi}V$.

Let $\pi^l V$ denote the pro-l completion of πV for a fixed prime l. Then, the quotient representation

$$\mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\widehat{\pi}V \to \mathrm{Out}\pi^l V$$

is called the *pro-l outer Galois representation.*

There are three results in this manuscript. The first result (Theorem 1.1) is a generalization of **[20]**: for any open smooth geometrically connected curve X over k with a k-rational puncture specified, there exists a lifting of the outer Galois representation to a true action

$$\mathrm{Gal}(\bar{k}/k) \to \mathrm{Aut}\widehat{\pi}B_{0,n}X$$

which can be completely described by the data for $\mathbb{P}^1 - \{0,1,\infty\}$-case and a specified lifting $\mathrm{Gal}(\bar{k}/k) \to \mathrm{Aut}\widehat{\pi}X$. Here $B_{0,n}X$ is the configuration space of distinct unordered n points on X. This space is the quotient space of $F_{0,n}X$ by the symmetric group S_n, where $F_{0,n}X = X^n - \Delta$ is the configuration space of distinct ordered n points on X.

The theorem asserts that there are nontrivial group homomorphisms

$$\widehat{\pi}F_{0,n}(\mathbb{A}^1 - 0) \to \widehat{\pi}F_{0,n}X, \quad \widehat{\pi}X \to \widehat{\pi}F_{0,n}X$$

compatible with Galois actions. The union of the images of these two homomorphisms generate whole $\widehat{\pi}F_{0,n}X$. These group homomorphisms do not come from any algebraic homomorphisms unless X has genus zero. (See Remark 1.2 for a relation to the Grothendieck's conjecture[**11**].)

The description of Galois action on $\widehat{\pi}B_{0,n}X$ tightly relates the action on $\widehat{\pi}X$ with that on $\widehat{\pi}(\mathbb{P}^1 - \{0,1,\infty\})$. This tight relation comes from the topology of braids. In $\pi B_{0,n}X$, there are intertwining topological relations between πX and $\pi(\mathbb{P}^1 - \{0,1,\infty\})$. For example, the commutator product of some two elements in $\pi B_{0,n}X$ coming from the former group lies in the latter group. (See the relations in Proposition 2.1 and the figure above them.) From this, roughly speaking, we see that if an element of $\mathrm{Gal}(\bar{k}/k)$ acts trivially on $\widehat{\pi}X$, then so does it on $\widehat{\pi}(\mathbb{P}^1 - \{0,1,\infty\})$. As a result, we can prove a conjecture generalizing Belyĭ's Injectivity (see e.g. [**44**]): let X be an affine curve over a number field k. If πX is nonabelian, then the outer Galois representation $\mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\widehat{\pi}X$ is injective (Theorem 2.1). This is the second result of this manuscript. A pro-l analogue is also studied (cf. Theorem 2.2).

The third result is about Oda's field towers. In §3, a Lie version of the above arguments shows a part of his conjecture (Theorem 3.2):

$$\mathbb{Q}[g,r,l;m] \supset \mathbb{Q}[0,3,l;m] \quad \text{for } r \geq 1,\ 2-2g-r<0.$$

The field $\mathbb{Q}[g,r,l;m]$ is, roughly speaking, the smallest field of definition of the moduli stack of r punctured genus g curves with pro-l level m structure on $\pi^l X$. Oda conjectured that this would be independent of $g \geq 2$ and $r \geq 0$, and would coincide with Ihara's tower. From this conjecture, Oda predicted the existence of some obstructions to the surjectivity of the Johnson-Morita homomorphism other than the Morita trace[**26**].

Note that this inclusion has already been proved by H.Nakamura[**30**] in a different manner using [**32**]. A part of his proof was stimulated by a result in this manuscript.

In §4, we prove a special case of Oda's conjecture on the kernel (Theorem 4.1):

$$\mathbb{Q}[g,r,l;\infty] = \mathbb{Q}[0,3,l;\infty] \quad \text{for } r \geq 1,\ 2-2g-r<0,\ l-1|2g.$$

In particular, we show that for $l = 2,3,7$, $r \geq 1$, $2-2g-r<0$, the Oda's conjecture on the kernel is true for any genus. The case $l = 7$ uses a result of Nakamura[**30**].

In the rest, we have no room to state the complete proofs. Please see [**25**] for details.

1. Description of Galois action on Braid groups

1.0. Notation. We denote by k a *subfield* of $\mathbb{C}$. A variety (or a curve) over k is a smooth and geometrically connected scheme (of dimension 1, resp.) of finite type over k, which may not be proper, unless otherwise specified.

For a variety V over k, we denote by πV or $\pi(V)$ the topological fundamental group $\pi_1(V(\mathbb{C}),*)$, with $*$ an arbitrary base point. Its profinite completion is denoted by

$$\widehat{\pi}V = \widehat{\pi}(V) = \widehat{\pi_1}(V(\mathbb{C}),*)$$

and pro-l completion by

$$\pi^l V = \pi^l(V) := \pi_1^l(V(\mathbb{C}),*)$$

for a prime number l.

For $g, r \geq 0$, we call X a (g,r)*-curve* over k if X is a curve over k whose smooth compactification X^* has genus g and the number of $\bar{k}$-rational points on X^* but not on X

is r. We call such a point *a puncture* of X. The term *k-rational puncture* means that the puncture is a k-rational point on X^*.

Following to Birman[6], we denote by $F_{0,n}X$ the configuration space of distinct ordered n points on X. To be precise,

$$F_{0,n}X = X^n - \bigcup_{1 \le i < j \le n} \Delta_{ij}$$

where

$$\Delta_{ij} \hookrightarrow X^n$$

is the divisor $\{(x_1, \ldots, x_n) \in X^n \mid x_i = x_j\}$ of X^n. Thus, $F_{0,n}X$ is an n-dimensional variety over k, and $F_{0,n}X(\mathbb{C})$ is the configuration space defined in [6]. We call $\pi F_{0,n}X$ the *pure braid group of n-strings on* $X(\mathbb{C})$. We also define

$$\pi F_{m,n}X$$

as the fundamental group of $F_{0,n}(X(\mathbb{C}) - S)$, where $S = \{b_1, \ldots, b_m\}$ is a set of m points on $X(\mathbb{C})$. Note that the abstract group $\pi F_{m,n}X$ is independent of the choice of S, but we don't define an algebraic variety like $F_{m,n}X$. Note also that $\pi F_{m,1}X$ is isomorphic to the fundamental group of the open curve $X(\mathbb{C}) - \{b_1, \ldots, b_m\}$.

If X is not $\mathbb{P}^1$, then we have a short exact sequence (e.g.[6])

$$(1.1) \qquad 1 \to \pi F_{n-1,1}X \to \pi F_{0,n}X \to \pi F_{0,n-1}X \to 1.$$

The right morphism comes from the fiber map $F_{0,n}X \to F_{0,n-1}X$ obtained by forgetting the i-th moving point, and the left morphism comes from a fiber of this map at $(b_1, \ldots \overset{i}{\vee} \ldots, b_n) \in F_{0,n-1}X$. The sequence obtained by its profinite completion (also pro-l completion, Lie-algebraization) can be proved to be exact.

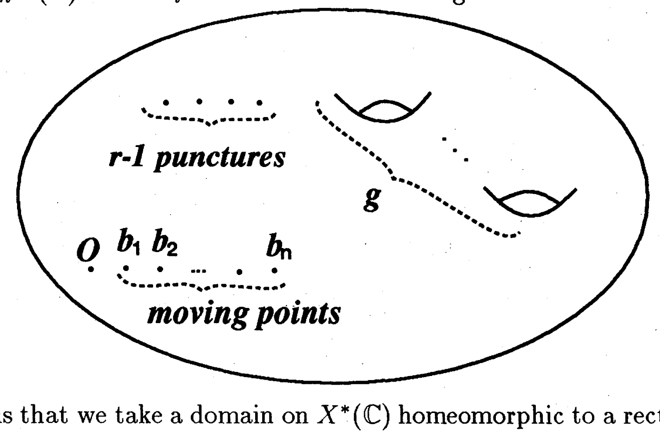
The symmetric group S_n acts on $F_{0,n}X$ without fixed points, so we may consider the quotient variety

$$B_{0,n}X := F_{0,n}X/S_n.$$

Thus, $B_{0,n}X$ is the configuration space of distinct unordered n points on X. Its topological fundamental group is usually called the *braid group of n strings on* $X(\mathbb{C})$.

For a positive real number ϵ, let $(0, \epsilon)$ denote the open interval of the real line $\mathbb{R}$ in $\mathbb{C}$.

1.1. Description of Galois actions. Let X be a (g, r)-curve over k with a k-rational puncture O specified, and let X^* denote its smooth compactification. Let $b = (b_1, \ldots, b_n)$ be a point of $F_{0,n}X(\mathbb{C})$ where b_i's are near O and arranged as below.



This figure means that we take a domain on $X^*(\mathbb{C})$ homeomorphic to a rectangle containing the r punctures, so that $r-1$ punctures are arranged in near the upper edge and O is near the down-left corner. (We now regard X just as a topological space, hence this is possible.)

For the arrangement of b_i's, let us take a uniformizer u of the maximal ideal $\mathfrak{m}_{X^*,O}$ of the local ring $\mathcal{O}_{X^*,O}$. Then, u can be viewed as a meromorphic function $u : X^*(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$,

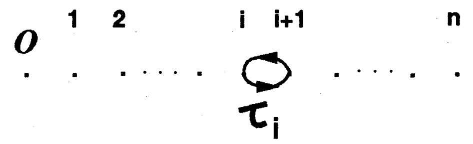
$O \mapsto 0$, giving a homeomorphism of a neighbourhood $\mathcal{N}_O$ of O in $X(\mathbb{C})$ to an open disk centered at 0 with radius ϵ in $\mathbb{P}^1(\mathbb{C})$. Let $\widetilde{(0,\epsilon)}$ be the inverse image of $(0,\epsilon) \subset \mathbb{R} \subset \mathbb{P}^1(\mathbb{C})$ by u restricted to $\mathcal{N}_O$. We may assume that $\widetilde{(0,\epsilon)}$ is parallel to the bottom edge of the rectangle, by a homeomorphic deformation. Now $b_1, \dots, b_n$ are assumed to lie on $\widetilde{(0,\epsilon)}$, so that $0 < u(b_1) < u(b_2) < \cdots < u(b_n) < \epsilon$.

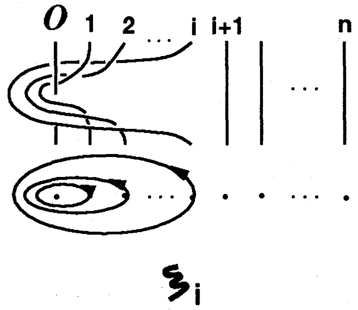
Since

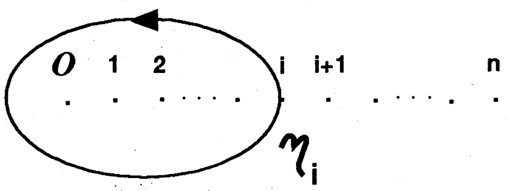
$$\mathcal{B}_n := \mathcal{B} := \{(b_1, \dots, b_n) \in \mathcal{N}_O \mid 0 < u(b_1) < u(b_2) < \cdots < u(b_n) < \epsilon\} \subset F_{0,n}X(\mathbb{C})$$

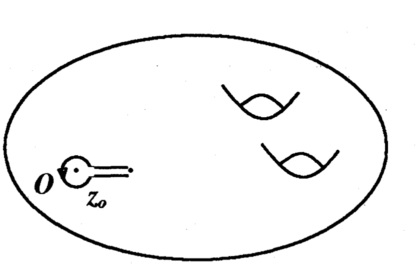
is simply connected (actually contractible), $\pi_1(F_{0,n}X(\mathbb{C}), \mathcal{B})$ makes sense; because the fundamental groups for any two base points in $\mathcal{B}$ are canonically isomorphic via a (homotopically unique) path in $\mathcal{B}$. In the case $n = 1$, we have $\mathcal{B} = \widetilde{(0,\epsilon)}$.

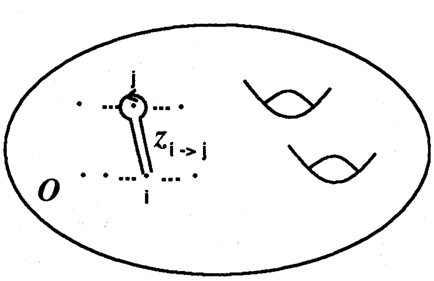
Since the image of $\mathcal{B} \subset F_{0,n}X(\mathbb{C})$ in $B_{0,n}X(\mathbb{C})$, denoted by $\overline{\mathcal{B}}$, is homeomorphic to $\mathcal{B}$, the same arguments apply to $B_{0,n}X$ and $\overline{\mathcal{B}}$. We identify $\pi_1(B_{0,n}X(\mathbb{C}), \overline{\mathcal{B}})$ with $\pi B_{0,n}X$. This means that if we write $\pi B_{0,n}X$ it denotes $\pi_1(B_{0,n}X(\mathbb{C}), \overline{\mathcal{B}})$ from now on. Let τ_i $(1 \le i \le n-1)$, η_i $(1 \le i \le n)$, $\xi_i = \eta_1 \cdots \eta_i$ $(1 \le i \le n)$, and $z_{i \to j}$ $(1 \le i \le n,\ 1 \le j \le r-1)$ be the elements of $\pi B_{0,n}X$ described below. These elements except $z_{i\to j}$'s are defined also in $\pi B_{0,n}(\mathbb{A}^1 - 0)$ in the same manner.











We denote by z_O the element in $\pi X = \pi_1(X(\mathbb{C}), \widetilde{(0,\epsilon)})$ that circles O as drawn above.

DEFINITION 1.1. We define a homomorphism

$$\phi : \pi X = \pi_1(X(\mathbb{C}), \widetilde{(0,\epsilon)}) \to \pi F_{0,n}X = \pi_1(F_{0,n}X(\mathbb{C}), \mathcal{B})$$

as follows. Let us fix a closed disc D of $X^*(\mathbb{C})$ centered at O and containing $\{O, b_1, \dots, b_{n-1}\}$ but $D \not\ni b_n$. Let ϕ be the composite morphism $\pi_1(X(\mathbb{C}) - O, b_n) \xrightarrow{\sim} \pi_1(X(\mathbb{C}) - D, b_n) \to \pi_1(X(\mathbb{C}) - \{b_1, b_2, \dots, b_{n-1}\}, b_n) \to \pi_1(F_{0,n}X(\mathbb{C}), b)$, where the last morphism is the left morphism in the short exact sequence (1.1).

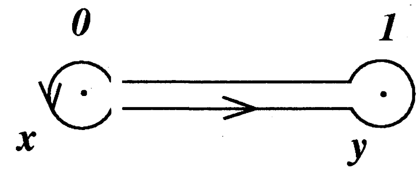
Thus,

$$\phi : \pi_1(X(\mathbb{C}), b_n) \to \pi_1(F_{0,n}X(\mathbb{C}), b); \quad \gamma \mapsto \phi(\gamma)$$

is obtained if we define $\phi(\gamma)$ to be a path in $F_{0,n}X(\mathbb{C})$ such that $b_1, \ldots, b_{n-1}$ are fixed near O, and b_n moves along γ, provided that we chose a representative of γ which does not intersect with D.

Before stating the first result, we need a lifting by Belyĭ[5]. We shall later use a geometric construction of this lifting by Ihara[16].

PROPOSITION 1.1 (BELYĬ). *The group* $\widehat{\pi_1}(\mathbb{P}^1 - \{01\infty\}, (0,1))$ *is the free profinite group* $\widehat{F_2}$ *of two generators* x,y *as below.*



For $\sigma \in \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, *there exists a unique element* $f_\sigma(x,y) \in [\widehat{F_2}, \widehat{F_2}]$ *such that*

$$x \mapsto x^{\chi(\sigma)}, \quad y \mapsto f_\sigma(x,y)^{-1} y^{\chi(\sigma)} f_\sigma(x,y)$$

is an automorphism of $\widehat{F_2}$ ($\chi(\sigma)$ *being the cyclotomic character) and that the image of this automorphism in* $\mathrm{Out}\widehat{F_2}$ *coincides with the image of* σ *by* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{Out}\widehat{F_2}$.

Note that for any two elements ξ and η in any profinite group G, there exists a unique morphism $\widehat{F_2} \to G$ with $x \mapsto \xi$, $y \mapsto \eta$. We denote by $f_\sigma(\xi, \eta)$ the image of $f_\sigma(x,y)$ by this map.

The first result of this manuscript is the following theorem. This theorem generalizes a previous result in the joint work with Y. Ihara [20], which treats the genus zero case. The idea of the proof is an extension of [20].

THEOREM 1.1. *Let* X *be a smooth geometrically connected curve over a field* $k \subset \mathbb{C}$, X^* *its smooth compactification. Assume that there exists a* k*-rational point* O *in* X^* *not on* X. *Then there exist sections*

$$\mathrm{Gal}(\bar{k}/k) \to \pi_1^{alg}(B_{0,n}X, \bar{\eta})$$
$$\mathrm{Gal}(\bar{k}/k) \to \pi_1^{alg}(X, \bar{\eta})$$

to the short exact sequences (0.1) *for* $x = \bar{\eta}$, $V = B_{0,n}X$, *and for* $x = \bar{\eta}$, $V = X$, *respectively, such that the induced morphism*

$$\mathrm{Gal}(\bar{k}/k) \to \mathrm{Aut}\widehat{\pi}B_{0,n}X$$

satisfies the following conditions.

Let $\sigma \in \mathrm{Gal}(\bar{k}/k)$.

(i)

$$\sigma : \xi_i \mapsto \xi_i^{\chi(\sigma)} \ (1 \le i \le n), \quad \tau_i \mapsto f_\sigma(\xi_i, \tau_i^2)^{-1} \tau_i^{\chi(\sigma)} f_\sigma(\xi_i, \tau_i^2) \ (1 \le i \le n-1).$$

In particular, the homomorphism

$$\widehat{\pi}B_{0,n}(\mathbb{A}^1 - 0) \to \widehat{\pi}B_{0,n}X$$

defined (group theoretically) by $\xi_i \mapsto \xi_i$, $\tau_i \mapsto \tau_i$ *is* $\mathrm{Gal}(\bar{k}/k)$*-compatible.*

(ii) *The profinite completion of the above* ϕ *(Definition* 1.1)

$$\widehat{\phi} : \widehat{\pi}X \to \widehat{\pi}B_{0,n}X$$

is compatible with the $\mathrm{Gal}(\bar{k}/k)$*-actions.*

(iii) *If we denote by* X_{+O} *the curve obtained from* X *by filling up the puncture* O, *then the natural map* $\widehat{\pi}B_{0,n}X \to \widehat{\pi}B_{0,n}X_{+O}$ *is* $\mathrm{Gal}(\bar{k}/k)$*-compatible (the action on the right side is given by a suitable section).*

REMARK 1.1. The elements $\xi_1, \tau_1, \ldots, \tau_{n-1}$ and the image of ϕ generates $\pi B_{0,n}X$. Thus, the above description completely determines the action of $\mathrm{Gal}(\bar{k}/k)$ on $\widehat{\pi}B_{0,n}X$.

The key idea in proving this theorem is as follows. We restrict the moving points $b_1, \dots, b_n$ to be near O, and let $u_1, \dots, u_n$ be the coordinates of b_i in terms of the local coordinate u at O. Then, we put $t_i := u_i/u_{i+1}$, $(1 \le i \le n-1)$, and $t_n := u_n$. The parameters t_i give a kind of blow-up of X^n at $(O, O, \dots, O)$, so that the hyper diagonal Δ becomes normal crossing at $t_1 = \cdots = t_n = 0$. We take a tangential base point at this point. This means we take a base point of $F_{0,n}X$ outside $F_{0,n}X$, on which we may consider $t_1, \dots, t_n$ as infinitesimally small. Then, if we move t_i only, then the points $u_1, \dots, u_i$ move in proportion to t_i, but $u_1, \dots, u_{i-1}$ are infinitesimally small and $u_{i+2}, \dots, u_n$ are infinitesimally large compared with u_i. Thus, the branched locus seems to be only $t_i = 0, 1$, the former point giving $u_1 = \cdots = u_i = 0$ and the latter giving $u_i = u_{i+1}$. Thus we have $\mathbb{A}^1 - \{0, \infty\}$. This is the reason why $\pi(\mathbb{P}^1 - \{01\infty\})$ occurs in $\pi B_{0,n}X$.

REMARK 1.2. According to Grothendieck's philosophy [11], any $\mathrm{Gal}(\bar{k}/k)$-compatible map from $\pi_1^{alg}(\overline{V})$ to $\pi_1^{alg}(\overline{V'})$ would come from an algebraic morphism $V \to V'$, if V and V' are "anabelian" varieties over a number field k, under some conditions. The precise formulation of the conjecture is still not clear (cf. [44]).

Theorem 1.1 states that for $V := F_{0,n}(\mathbb{A}^1 - \{0\})$ and $V' := F_{0,n}X$ with a positive genus curve X, there exists a Galois-compatible injective morphism between fundamental groups which does not come from a morphism between varieties. (Note that any morphism from $V = F_{0,n}(\mathbb{A}^1 - \{0\})$ to $F_{0,n}X$ maps whole V to one point. The injectivity of the group homomorphism easily follows by induction on n and the five lemma.

This would be because $F_{0,n}\mathbb{A}^1$ is not anabelian, since it has a nontrivial center $< \xi_n >$.

2. Application to the injectivity of the outer Galois representation

By Belyĭ's uniformization theorem[5], the outer Galois representation

$$G_{\mathbb{Q}} \to \mathrm{Out}\hat{\pi}(\mathbb{P}^1 - \{01\infty\})$$

was proved to be injective. A conjecture generalizing this result (see for example the remark before Theorem 3 in Voevodskiĭ[44] for the affine case) is

CONJECTURE 2.1. *If X is a smooth geometrically connected curve over a number field k with nonabelian fundamental group, then*

$$\mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\hat{\pi}X$$

is injective.

The first application of our Theorem 1.1 is to prove this conjecture for affine curves.

THEOREM 2.1. *Conjecture 2.1 is true if X is affine.*

Voevodskiĭ[44] proved for the case of X being genus 1 with at least one puncture. The author knows no example of proper curves for which the above conjecture is proved or disproved.

To prove Theorem 2.1, we may assume that at least one of the punctures of X is k-rational, by the following reason. Let O be a puncture of X. Then O is k'-rational for some number field k'. Let $\sigma \in G_k$ lie in the kernel. Since $G_{k'}$ is of finite index in G_k, some power of σ lies in $G_{k'}$, then if we could settle the k'-rational puncture case, then this power of σ is identity. It is well-known that G_k has no torsion except the $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$-conjugates of complex conjugation, (see [3]), hence σ must be one of these if $\sigma \neq 1$. Then its cyclotomic character $\chi(\sigma)$ is -1, hence σ acts on the abelianization of $\hat{\pi}X$ nontrivially, leading to a contradiction.

If there exists an algebraic morphism $X \to \mathbb{P}^1 - \{01\infty\}$ over k inducing a surjection on the (topological) fundamental groups, then it is easy to see that X satisfies the Conjecture.

Thus, if X is genus zero and with more than three punctures, then Theorem 2.1 is true. So, we may assume that the genus $g \geq 1$.

This case follows from the following theorem.

THEOREM 2.2. *Let k be any subfield of $\mathbb{C}$. Let X be a smooth geometrically connected curve over k with at least one k-rational puncture and with genus positive. Then the kernel of the outer Galois representation*

$$\mathrm{G}_k \to \mathrm{Out}\widehat{\pi}F_{0,n}X \tag{2.1}$$

is independent of $n \geq 1$, and is contained in the kernel of

$$\mathrm{G}_k \to \mathrm{Out}\widehat{\pi}(\mathbb{P}^1 - \{01\infty\}). \tag{2.2}$$

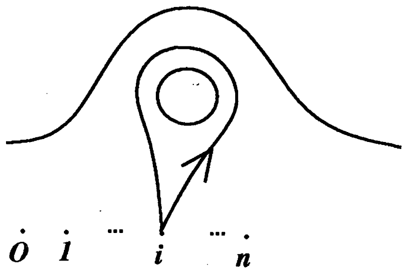
These statements are also correct for the pro-l case, i.e., even if we replace $\widehat{\pi}F_{0,n}X$ with its pro-l completion $\pi^l F_{0,n}X$ and $\widehat{\pi}(\mathbb{P}^1 - \{01\infty\})$ with $\pi^l(\mathbb{P}^1 - \{01\infty\})$.

Note that the kernel from the profinite completion to the pro-l completion is a characteristic subgroup, and hence we have a canonical morphism $\mathrm{Out}\widehat{\pi} \to \mathrm{Out}\pi^l$.

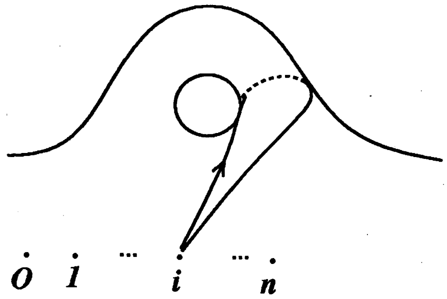
By this theorem and the Belyĭ's result, (2.1) is proved to be injective if k is a number field. Theorem 2.1 follows from this case and the note below Theorem 2.1.

REMARK 2.1. The independence of the kernel of (2.1) for $n \geq 1$ for pro-l case is one of the main results in Ihara-Kaneko[**19**] (a part of Theorem 2 there).

The key in proving this theorem is the following topological relation:



α_i



β_i

PROPOSITION 2.1. (i) $\alpha_i = \tau_i \alpha_{i+1} \tau_i^{-1}$.
(ii) $\alpha_i^{-1} \beta_{i+1}^{-1} \alpha_i \beta_{i+1} = \tau_i^2$.

Let $\sigma \in \mathrm{Gal}(\bar{k}/k)$ be in the kernel of $\mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\widehat{\pi}X$. By Theorem 1.1, we may basically assume that σ acts trivially on α_n and β_n. Hence, the image of the relation

$$[\tau_{n-1}\alpha_n^{-1}\tau_{n-1}^{-1}, \beta_n^{-1}] = \tau_{n-1}^2$$

by σ can be written in terms of only $f_\sigma(x, y)$ and $\chi(\sigma)$. We rewrite the new relation as a nontrivial relation in a free subgroup of $\widehat{\pi}B_{0,n}X$. Then, by some combinatorial group theory, we can show that $f_\sigma = 1$ and $\chi(\sigma) = 1$, and hence by Belyĭ, $\sigma = 1$.

3. Filtrations on Gal($\overline{\mathbb{Q}}/\mathbb{Q}$)

3.1. Induced filtration and a conjecture by Oda. Let Π be a group, and Γ another group with a homomorphism $\varphi : \Gamma \to \mathrm{Out}\Pi$. Suppose Π has a central filtration

$$\Pi = \Pi(1) \supset \Pi(2) \supset \Pi(3) \supset \cdots,$$

i.e., a descending sequence of normal (closed if a topological group) subgroups satisfying $[\Pi(m), \pi(n)] \subset \Pi(m+n)$. Then, we define the *induced filtration*

$$\Gamma = \Gamma[0] \supset \Gamma[1] \supset \Gamma[2] \supset \cdots$$

by

$$\Gamma[m] := \{\sigma \in \Gamma \mid \text{there exists } \tilde{\sigma} \in \mathrm{Aut}\Pi \text{ mapped to } \varphi(\sigma) \text{ in } \mathrm{Out}\Pi, \text{ such that } \tilde{\sigma}(w)w^{-1} \in \Pi(s+m) \text{ for every } w \in \Pi(s) \text{ for every } s \geq 1\}.$$

We can also define the induced filtration if we are given a morphism $\Gamma \to \mathrm{Aut}\Pi$, by replacing $\tilde{\sigma}$ with the image of this morphism in the above definition. For the case $\Gamma = \mathrm{Aut}\Pi$, the latter induced filtration $\mathrm{Aut}\Pi[m]$ is included in the filtration induced from $\mathrm{Aut}\Pi \to \mathrm{Out}\Pi$, but may not coincide. It is known that $\{\Gamma[m]; m \geq 1\}$ gives a central filtration on $\Gamma[1]$ again (see [**7**, Ch.2 §2.4 Exer.3]).

We apply this definition for a variety V over a field k and its outer Galois representation on the pro-l fundamental group

$$\varphi : \mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\hat{\pi}V \to \mathrm{Out}\pi^l V.$$

Now fix a prime number l. Let X be a (g,r)-curve over k. Oda (c.f. [**23**] for no-puncture case) defined the weight filtration on the pro-l group $\pi^l F_{0,n}X$ as the fastest decreasing central filtration with $z_{i\to j}$ (see §§1.0) being in the second filtration; namely:

$$\begin{aligned} \pi^l F_{0,n}X(1) &:= \pi^l F_{0,n}X^l \\ \pi^l F_{0,n}X(2) &:= << [\pi^l F_{0,n}X, \pi^l F_{0,n}X], z_{i\to j} \mid 1 \leq i \leq n, 1 \leq j \leq r >> \\ &\vdots \\ \pi F_{0,n}X(m) &:= << \bigcup_{i+j=m} [\pi^l F_{0,n}X(i), \pi^l F_{0,n}X(j)] >> \quad \text{for } m > 2 \end{aligned}$$

($[A,B]$ denotes the closure of the group generated by the commutator product of A and B, and $<< A >>$ denotes the normally generated closed subgroup by A).

In the case of $\Pi := \pi^l F_{0,n}X$, it is easy to show by induction that

(3.1)

$$\begin{aligned} \mathrm{Out}\Pi[m] := \{\sigma \in \mathrm{Out}\Pi \mid & \text{ there exists a lift } \tilde{\sigma} \in \mathrm{Aut}\Pi \text{ such that} \\ & \tilde{\sigma}(w)w^{-1} \in \Pi(m+1) \text{ for every } w \text{ in a fixed generating set of } \Pi \\ & \text{and } \tilde{\sigma}(z_{i\to j})z_{i\to j}^{-1} \in \Pi(m+2) \text{ for every } 1 \leq i \leq n,\ 1 \leq j \leq r.\ \} \end{aligned}$$

Here note that any element $\tau \in \pi B_{0,n}X$ induces an automorphism $x \mapsto \tau x \tau^{-1}$ on $\pi^l F_{0,n}X$, and this automorphism preserves the filtration, since it only permutes the conjugacy classes of $z_{i\to j}$'s. Similarly, $\mathrm{Gal}(\bar{k}/k)$-action preserves the filtration, since they just permutes the inertia groups.

Since we have a canonical morphism $\varphi : \mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\pi^l F_{0,n}X$, the above filtration provides $\mathrm{G}_k := \mathrm{Gal}(\bar{k}/k)$ an induced filtration, which we shall denote by

$$\mathrm{G}_k = \mathrm{G}_k[F_{0,n}X; 0] \supset \mathrm{G}_k[F_{0,n}X; 1] \supset \mathrm{G}_k[F_{0,n}X; 2] \supset \cdots.$$

For $n = 1$,

$$\mathrm{G}_k[X; m] := \mathrm{G}_k[F_{0,1}X; m]$$

is the induced filtration investigated by many authors (see Asada-Kaneko[**4**] and Kaneko[**21**], and this filtration has a rich application in bounding the Galois centralizer: see Nakamura[**29**], Nakamura-Tsunogai[**33**]). In particular, for $X = \mathbb{P}^1 - \{01\infty\}$, $\mathrm{G}_{\mathbb{Q}}[X; m]$ is the filtration introduced in the pioneering works by Ihara[**13**][**14**] and by Deligne[**8**] (note that the index m here is twice of that in [**13**][**14**] and [**8**]). See [**16**] and a series of Nakamura's works for the significance in studying such filtrations.

Let us define a relative version of this filtration (this was also essentially defined by Oda[**37**][**38**]). Let S be a smooth geometrically connected scheme locally of finite type over k, and let $(\mathcal{C} \to S;\ s_1, \ldots, s_r : S \to \mathcal{C}^*)$ be a smooth family of smooth (g,r)-curves with punctures ordered over S. This means that there exists proper smooth morphism $\mathcal{C}^* \to S$

and its sections $s_1, \dots, s_r$ such that $\mathcal{C} = \mathcal{C}^* - \bigcup_{1 \le i \le n} s_i(S)$ and $\mathcal{C} \to S$ is the restriction of $\mathcal{C}^* \to S$, and each fiber of $\mathcal{C} \to S$ is a (g, r)-curve.

Let η be the generic point of S, $\bar{\eta}$ its geometric point, $\mathcal{C}_\eta, \mathcal{C}_{\bar{\eta}}$ be the fiber on $\eta, \bar{\eta}$, respectively (hence being (g, r)-curves over $k(\eta), k(\bar{\eta})$, respectively). Then, we have an outer representation

$$\mathrm{Gal}(k(\bar{\eta})/k(\eta)) \to \mathrm{Out}\pi_1^{alg}(\mathcal{C}_{\bar{\eta}}). \tag{3.2}$$

REMARK 3.1. By smooth base change theorem in SGA1[**10**, §13],

$$\pi_1^{alg}(\mathcal{C}_{\bar{\eta}}) \cong \pi_1^{alg}(\mathcal{C}_{\bar{x}})$$

holds for any point x on S, and an inertia group in $\mathrm{Gal}(k(\bar{\eta})/k(\eta))$ of x trivially acts on the right hand side. Thus, (3.2) factors through

$$\pi_1^{alg}(S, \bar{\eta}) \to \mathrm{Out}\pi_1^{alg}(\mathcal{C}_{\bar{\eta}}),$$

which is sometimes called the *monodromy representation.*

Now we have an induced filtration

$$\{\mathrm{G}_{k(\eta)}[\mathcal{C}_\eta; m] \mid m = 1, 2, \dots\}$$

on $\mathrm{G}_{k(\eta)} = \mathrm{Gal}(k(\bar{\eta})/k(\eta))$. By taking their image by the surjection $\mathrm{G}_{k(\eta)} \twoheadrightarrow \mathrm{G}_k$, we define the induced filtration on G_k associated with $\mathcal{C}/S$ and denote them by $\mathrm{G}_k[\mathcal{C}/S; m]$. (Note that this notation consistently works for the case $S = \mathrm{Spec}\ k$; that is, $\mathrm{G}_k[C/\mathrm{Spec}k; m] = \mathrm{G}_k[C; m]$.)

Now we can state a conjecture by Oda (an explicit formulation in the punctured case is in [**31**], cf. also [**32**].)

CONJECTURE 3.1. *Let us define*

$$\mathrm{G}_{\mathbb{Q}}[g, r, l; m] := \bigcup_{\mathcal{C}/S/k:(g,r)-family} \mathrm{G}_k[\mathcal{C}/S; m],$$

where the union is taken over all families $\mathcal{C} \to S$ of (g, r)-curves with punctures ordered, with S a smooth scheme over a number field k (hence k moves). Then, if $2 - 2g - r < 0$,

$$\mathrm{G}_{\mathbb{Q}}[g, r, l; m] = \mathrm{G}_{\mathbb{Q}}[\mathbb{P}^1 - \{01\infty\}; m]$$

holds.

REMARK 3.2. If there exists a commutative diagram

$$\begin{array}{ccc} \mathcal{C}' & \to & \mathcal{C} \\ \downarrow & \square & \downarrow \\ S' & \to & S \\ \downarrow & & \downarrow \\ \mathrm{Spec}\ k' & \to & \mathrm{Spec}\ k \end{array}$$

with the upper square being the fiber product and k'/k being an algebraic extension, then

$$\begin{array}{ccccc} \pi_1^{alg}(S', \bar{\eta}') & \to & \pi_1^{alg}(S, \bar{\eta}) & \to & \mathrm{Out}\pi_1^{alg}(\mathcal{C}_{\bar{\eta}}) = \mathrm{Out}\pi_1^{alg}(\mathcal{C}'_{\bar{\eta}'}) \\ \downarrow & & \downarrow & & \\ \mathrm{G}_{k'} & \hookrightarrow & \mathrm{G}_k & & \end{array} \tag{3.3}$$

and hence

$$\mathrm{G}_{k'}[\mathcal{C}'/S'; m] \hookrightarrow \mathrm{G}_k[\mathcal{C}/S; m].$$

Thus, if there exists a solution

$$(\mathcal{C}_{g,r} \to \mathcal{M}_{g,r}; s_1^\mu, \dots s_r^\mu : \mathcal{M}_{g,r} \to \mathcal{C}_{g,r}^*)$$

to the moduli problem:

$$\forall(\mathcal{C} \to S; s_1, \dots, s_r)\text{: } (g,r)\text{-family}$$

$$\exists! S \to \mathcal{M}_{g,r} \text{ such that } \begin{array}{ccc} \mathcal{C} & \to & \mathcal{C}_{g,r} \\ \downarrow & \square & \downarrow \\ S & \to & \mathcal{M}_{g,r}, \end{array} \quad \text{and} \quad \begin{array}{ccc} \mathcal{C}^* & \to & \mathcal{C}^*_{g,r} \\ s_i \uparrow & \circlearrowleft & \uparrow s_i^{\mu} \\ S & \to & \mathcal{M}_{g,r}, \end{array}$$

(thus $\mathcal{M}_{g,r}$ is the moduli scheme of (g,r)-curves with punctures ordered), then the left hand side of Conjecture 3.1 is nothing but

$$\mathrm{G}_{\mathbb{Q}}[g,r,l;m] = \mathrm{G}_{\mathbb{Q}}[\mathcal{C}_{g,r}/\mathcal{M}_{g,r};m].$$

Oda stated his conjecture in this style, namely, $\mathrm{G}_{\mathbb{Q}}[\mathcal{C}g,r/\mathcal{M}_{g,r};m]$ was defined as the image in $\mathrm{G}_{\mathbb{Q}}$ of $\pi_1(\mathcal{M}_{g,r})[m]$ (see Remark 3.1 and **[35][36]**).

Actually, $\mathcal{C}_{g,r}$ and $\mathcal{M}_{g,r}$ are in general not schemes but algebraic stacks for $g \geq 1$. Oda**[37][38]** developed the theory of fundamental groups of stacks**[34]** and stated his conjecture in terms of stacks. We do not want to use the stacks' fundamental groups here, so adopt the above style for stating the conjecture. We only mention the equivalence of the two definitions as below.

(i) If

$$\begin{array}{ccc} \mathcal{C}' & \to & \mathcal{C}_{g,r} \\ \downarrow & \square & \downarrow \\ S' & \to & \mathcal{M}_{g,r} \end{array}$$

is a fiber product of stacks with S' a geometrically connected scheme over $\mathbb{Q}$, and if $\pi_1(S') \to \pi_1(\mathcal{M}_{g,r})$ is surjective, then $\mathrm{G}_{\mathbb{Q}}[\mathcal{C}'/S';m] = \mathrm{G}_{\mathbb{Q}}[\mathcal{C}_{g,r}/\mathcal{M}_{g,r};m]$ follows from (3.3),

(ii) For $g \geq 1$ and $r' > 2g+2$, $\mathcal{M}_{g,r'}$ can be proved to be a scheme**[22]**. Thus, taking $\mathcal{M}_{g,r'}$ for S' and taking fiber product $\mathcal{C}' := \mathcal{M}_{g,r'} \times_{\mathcal{M}_{g,r}} \mathcal{C}_{g,r}$ (this also becomes a scheme), we have the desired surjection between fundamental groups and may define $\mathrm{G}_{\mathbb{Q}}[g,r,l;m] := \mathrm{G}_{\mathbb{Q}}[\mathcal{C}'/\mathcal{M}_{g,r'};m]$.

Note that the right hand side in Conjecture 3.1 can be denoted as $\mathrm{G}_{\mathbb{Q}}[0,3,l;m]$, since the solution to the corresponding moduli problem is $\mathbb{P}^1 - \{01\infty\} \to \mathrm{Spec}\ \mathbb{Q}$.

We use Theorem 1.1 to prove

THEOREM 3.1. *Let X be a smooth geometrically connected curve over k of nonzero genus. Suppose that X is affine and its compactification X^* has a k-rational point outside X. Then,*

$$\mathrm{G}_k[\mathbb{P}^1 - \{01\infty\};m] \supset \mathrm{G}_k[F_{0,n}X;m] = \mathrm{G}_k[X;m]$$

holds for $n \geq 1$ and for $m \geq 0$.

By this theorem, we can prove one inclusion in Oda's conjecture:

THEOREM 3.2. *For $g \geq 0$, $r \geq 1$ with $2-2g-r<0$ and for any $m \geq 0$,*

$$\mathrm{G}_{\mathbb{Q}}[g,r,l;m] := \bigcup_{\mathcal{C}/S/k} \mathrm{G}_k[\mathcal{C}/S;m] \subset \mathrm{G}_{\mathbb{Q}}[\mathbb{P}^1 - \{01\infty\};m].$$

holds.

The idea of proof is same with that of Theorem 2.2, except for that we deal with filtered groups or Lie algebras instead of profinite groups.

REMARK 3.3. This inclusion and some stronger results have been already proved by Nakamura[30], by an independent method using the Deligne-Mumford compactification of moduli stacks (see his paper in this volume). Some part of his results is stimulated by the author's private communication on Theorem 3.1.

The right identity in Theorem 3.1 was essentially proved by Nakamura-Takao-Ueno (see (4.3)Theorem in [32]) generalizing the method of [19]. Also, in [32], $G_{\mathbb{Q}}[g,r,l;m]$ is proved to be independent of r for the case $g \geq 1$ and $r \geq 1$ and for several other cases.

4. The kernel of pro-l Galois Representations

Let $G_k \to \mathrm{Out}\pi^l V$ be the outer Galois representation on a variety V over k. We denote the kernel of this representation by

$$G_k[V;\infty] := \mathrm{Ker}[G_k \to \mathrm{Out}\pi^l V],$$

for if V is a curve X with $2-2g-r<0$, then it is known that

LEMMA 4.1.

$$G_k[X;\infty] = \bigcap_{m\in\mathbb{N}} G_k[X;m].$$

We can define the higher genus version by setting

$$G_{\mathbb{Q}}[g,r,l;\infty] := \bigcap_{m\in\mathbb{N}} G_{\mathbb{Q}}[g,r,l;m].$$

(Actually, we may define $G_{\mathbb{Q}}[g,r,l;\infty]$ as the image of the kernel of $\pi_1^{alg}(\mathcal{M}_{g,r},\bar\eta) \to \mathrm{Out}\pi^l(X)$ by $\pi_1^{alg}(\mathcal{M}_{g,r},\bar\eta) \twoheadrightarrow G_{\mathbb{Q}}$. This can be proved in the same way with Lemma 4.1.)

Then, Oda's Conjecture 3.1 implies its weaker version:

CONJECTURE 4.1. *For any g,r,l with $2-2g-r<0$, we have*

$$G_{\mathbb{Q}}[g,r,l;\infty] = G_{\mathbb{Q}}[\mathbb{P}^1 - \{01\infty\};\infty](= G_{\mathbb{Q}}[0,3,l;\infty]).$$

We prove a case of this conjecture.

THEOREM 4.1. *Let us assume $2-2g-r<0$ and $r \geq 1$. If $l-1$ divides $2g$, then*

$$G_{\mathbb{Q}}[g,r,l;\infty] = G_{\mathbb{Q}}[\mathbb{P}^1 - \{01\infty\};\infty].$$

Thus, the Oda's weaker Conjecture 4.1 for $l=2,3$ and $r \geq 1$ is true. The proof uses a Fermat-like curve which is an l-power cover of $\mathbb{P}^1$. The case $l=7$ and $r \geq 1$ can be proved by using a result of Nakamura[30] (Remark 3.3 above), but we omit it here.

REFERENCES

1. G. Anderson and Y. Ihara, *Pro-l branched coverings of $\mathbb{P}^1$ and higher circular l-units,* Ann. Math. **128** (1988), 271–293.
2. M.P. Anderson, *Exactness properties of profinite completion functors,* Topology **13** (1974), 229–239.
3. E. Artin, *Kennzeichnung des kõrpers der reellen algebraischen Zahlen,* Hamb. Abh. 3 (1924), 319–323
4. M. Asada and M. Kaneko, *On the automorphism groups of some pro-l fundamental groups,* Advanced studies in Pure Math. **12** 1987, 137–159.
5. Belyǐ, G. V; On Galois extensions of a maximal cyclotomic field, Math USSR Izv. **14** (1980), 247-256.
6. J. Birman, "Braids, links, and mapping class groups," Ann. of Math. Studies **82**, Princeton Univ. Press 1975.
7. N. Bourbaki, "Groupes et algèbres de Lie," Ch. 2 et 3, Éléments de Mathématique. 1972
8. P. Deligne, *Le groupe fondamental de la droite projective moins trois points,* in "Galois groups over $\mathbb{Q}$," Publ. MSRI **16** 1989, 79–298.
9. V.G. Drinfel'd, *On quasitriangular quasi-Hopf algebras and a group closely connected with* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, Algebra i Analiz **2** (1990), 114–148; English transl. Leningrad Math. J. **2** (1991), 829–860.
10. A. Grothendieck, "Revêtement Etales et Groupe Fondamental (SGA 1)," Lecture Notes in Math. **224**, Springer-Verlag 1971.
11. A. Grothendieck, *Esquisse d'un programme,* mimeographed note (1984).

12. W. Herfort and L. Ribes, *Torsion elements and centralizers in free products of profinite groups,* J. reine angew. Math. **358** (1985), 155–161.
13. Y. Ihara, *Profinite braid groups, Galois representations and Complex multiplications,* Ann. Math. **123** (1986), 43–106.
14. Y. Ihara, *The Galois representation arising from* $\mathbb{P}^1 - \{0,1,\infty\}$ *and Tate twists of even degree,* in "Galois groups over $\mathbb{Q}$," Publ. MSRI **16** 1989, 299–313.
15. Y. Ihara, *Automorphism of pure sphere braid groups and Galois representations,* in "The Grothendieck Festschrift, vol.2," Progress in Mathematics, vol.87, Birkhäuser, Basel 1991, 353–373.
16. Y. Ihara, *Braids, Galois groups, and some arithmetic functions,* Proceedings of the ICM 90 (I), 1991 99-120.
17. Y. Ihara, *On the stable derivation algebra associated to some braid groups,* Israel J. of Math. **80** (1992), 135–153.
18. Y. Ihara, *On the embedding of* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ *into* $\widehat{\mathrm{GT}}$, Proceedings of the Luminy conference, to appear (1994).
19. Y. Ihara and M. Kaneko, *Pro-l pure braid groups of Riemann surfaces and Galois representations,* Osaka J. Math. **29** (1992), 1–19.
20. Y. Ihara and M. Matsumoto, *On Galois Actions on Profinite Completions of Braid Groups,* RIMS preprint **961** (1994).
21. M. Kaneko, *Certain automorphism groups of pro-l fundamental groups of punctured Riemann surfaces,* J. Fac. Sci. Univ. Tokyo Sect. IA. Math. **36** (1989), 363–372.
22. F.F. Knudsen, *The projectivity of the moduli space of stable curves II: The stacks* $\mathcal{M}_{g,n}$ Math. Scad. **52** (1983), 161-199.
23. T. Kohno and T. Oda, *The lower central series of the pure braid group of an algebraic curve,* Advanced Studies in Pure Math. **12** 1987, 201–219.
24. M. Matsumoto, *On the Galois image in the derivation algebra of* π_1 *of the projective line minus three points,* RIMS preprint **962** (1994).
25. M. Matsumoto, *Galois representations on profinite braid groups on curves,* RIMS preprint **987** (1994).
26. S. Morita, *Abelian quotients of subgroups of the mapping class group of surfaces*, Duke. Math. J. (1993), 699–726.
27. H. Nakamura, *Galois rigidity of the etale fundamental groups of punctured projective lines,* J. reine angew. Math. **411** (1990), 205–216.
28. H. Nakamura, *On galois automorphisms of the fundamental group of the projective line minus three points,* Math. Z. **206** (1991), 617–622.
29. H. Nakamura, *Galois rigidity of pure sphere braid groups and profinite calculus,* To appear in J. Math. Sci. Univ. Tokyo.
30. H. Nakamura, *Coupling of universal monodromy representations of Galois-Teichmüller modular groups,* RIMS preprint **976** (1994).
31. H. Nakamura and N. Takao, *On Oda's field towers associated with moduli of algebraic curves,* unpublished.
32. H. Nakamura, N. Takao and R. Ueno, *Some stability properties of Teichmülcer modular function fields with pro-l weight structures,* RIMS preprint **973** (1994).
33. H. Nakamura and H. Tsunogai, *Some finiteness theorems on Galois centralizers in pro-l mapping class groups,* J. reine angew. Math. **441**, 115–144 (1993).
34. T. Oda, *Etale homotopy type of the moduli spaces of algebraic curves,* preprint (1989).
35. T. Oda, *A note on ramification of the Galois representation on the fundamental group of an algebraic curve,* J. Number Theory **34**, 225–228 (1990).
36. T. Oda, *A note on ramification of the Galois representation on the fundamental group of an algebraic curve II,* To appear in J. Number Theory.
37. T.Oda, *The universal monodromy representations on the pro-nilpotent fundamental groups of algebraic curves,* Mathematische Arbeitstagung (Neue Serie) 9–15 Juni 1993, Max-Planck-Institute preprint MPI/93-57.
38. T. Oda, *Fixed part of the universal monodromy representations on the pro-nilpotent fundamental groups of algebraic curves,* preprint. (1993)
39. J. P. Serre, "Lie algebra and Lie groups," Benjamin 1965.
40. J. P. Serre, "Arbres, amalgames, SL_2," Astérisque **46**, 1977; English transl. "Trees," Springer-Verlag 1980.
41. J. P. Serre, "Topics in Galois Theory," Jones and Bartlett Publishers 1992.
42. A. Tamagawa, *Ramification of the Galois representation on the pro-l fundamental group of an algebraic curve,* A talk in the conference on moduli spaces, Galois representations and L-functions, RIMS, Kyoto, Oct. 1993.
43. R. Ueno, *A stability of the Galois-kernel arising from pro-l monodromy representation* (in Japanese), Master's Thesis in RIMS, Kyoto Univ. 1994.
44. V.A. Voevodskiĭ, *Galois representations connected with hyperbolic curves,* English transl. Math. USSR Izvestiya **39** (1992), 1281–1291.

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