数理解析研究所講究録
989 巻 1997 年 180-189

FUZZY OPTIMIZATION FOR CONTROL OF FREE SURFACE

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1 Introduction

This paper deals with control of free surface arising in steady flow, which obeys Navier-Stokes equations. This control problem is transformed into the minimization problem. As a technique to solve the minimization, Fuzzy Optimization Method (FOM) and Mountain Crossing Algorithm (MCA) are applied to obtain a global minimizer, which means the target control. FOM seeks a local minimizer and MCA searchs for all local minimizers to obtain a global minimizer.

Here, as an example, the coating process of magnetic paints on a running film is studied. From a view point of quality control, it is desired that the free surface of coated paints is as flat as possible. On the other hand, from a view point of efficiency, it is desired that the speed of film is as fast as possible.

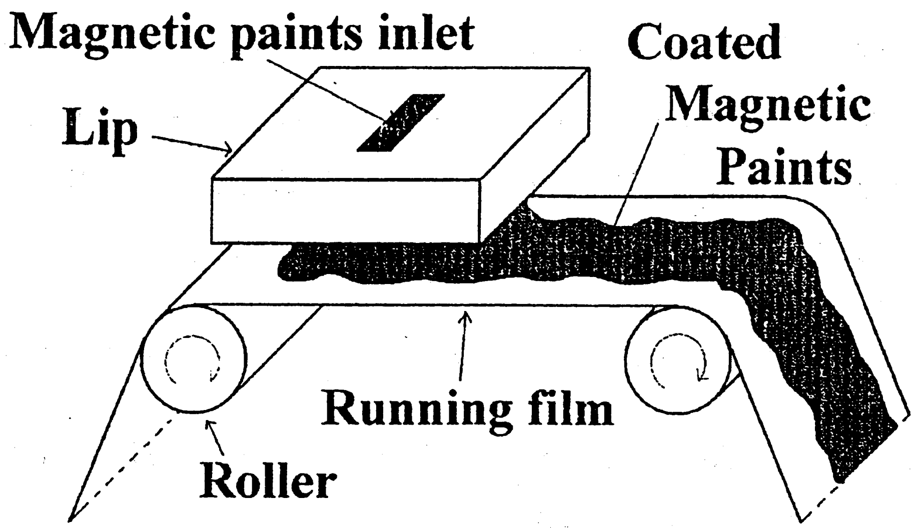

Figure 1: Coating process of magnetic paints

The minimization problem is constructed to describe the engineering desires mentioned above and is solved numerically by means of the unified method equipped with FOM and MCA. The other important skill to treat the minimization problem is how to solve numerically the free surface problem for the flow of magnetic paints. As an algorithm for it, the fictitious domain method is applied by an aid of singular perturbation. Finally, the numerical controls are shown together with the searching paths.

2 Free Surface Flow Problem

2.1 Notations

$u = (u, v, w)$	:	Velocity field of fluid
p	:	Pressure of fluid
$e_{ij}(u) = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$	:	Rate of strain tensor
$\sigma = \{(-p\delta_{ij} + \frac{1}{Re}e_{ij}(u))n_i\}$	:	Stress field
$\sigma_n = \{(-p\delta_{ij} + \frac{1}{Re}e_{ij}(u))\}n_i n_j$	:	Normal component of σ
$\sigma_{\mathcal{T}} = \sigma - \sigma \cdot \mathrm{n}$	:	Tangential component of σ
$\mathrm{n} = (n_1, n_2, n_3)$	:	Outward unit normal vector at the boundary
$\tau = (\tau_1, \tau_2, \tau_3)$	:	Unit tangential vector at the boundary
Re	:	Reynolds number
g	:	Gravitational acceleration
P_∞	:	Atmospheric pressure
P_{in}	:	Pressure at inflow boundary
V_{film}	:	X-component of velocity of film
$\Omega_0 = \Omega_\eta$	:	Domain ($\subset \mathbf{R}^3$) occupied with fluid
Γ_η	:	Free surface of fluid
Γ_{in}	:	Inflow boundary
Γ_{out}	:	Outflow boundary
Γ_{film}	:	Surface of film
Γ_{lip}	:	Wall of lip
Γ_{air}	:	Extra boundary added by use of the fictitious domain

2.2 Free surface Problem

The steady free surface problem defined in $\Omega_0 \subset \mathbf{R}^3$ for the flow of magnetic paints is to find u, p and Γ_η such that

$$
\begin{array}{ll}
-\frac{1}{Re}\Delta u + (u \cdot \nabla)u = -\nabla p + g & \text{in } \Omega_0, \\
\mathrm{div} u = 0 & \text{in } \Omega_0, \\
\sigma_n = (-p\,\delta_{ij} + \frac{1}{Re} e_{ij}) n_i n_j = -P_\infty & \text{on } \Gamma_\eta, \\
\sigma_T = 0 & \text{on } \Gamma_\eta, \\
u_n = 0 & \text{on } \Gamma_\eta, \\
u = 0 & \text{on } \Gamma_{lip}, \\
u = (V_{film}, 0, 0) & \text{on } \Gamma_{film}, \\
\sigma_n = -P_\infty & \text{on } \Gamma_{out} \cup \Gamma_{in}, \\
u_T = 0 & \text{on } \Gamma_{out} \cup \Gamma_{in}.
\end{array} \quad (1)
$$

In this problem, Reynolds number Re based on V_{film} and the gap between lip and film is about 10.

3 Numerical Solution of Free Surface Flow Problem

We apply a fictitious domain method via singular perturbation to solve numerically the free surface flow problem. We leave how to apply concretely it to the problem to the other literatures[10-12]. A lot of numerical experiments show that this method is very efficient to solve free surface flow problems.

In practice, we solve time-dependent distribution Navier-Stokes equations in an analyzed domain which is composed of $\overline{\Omega_0}$ and a fictitious domain by using finite-difference method and MAC algorithm, that makes it possible to adopt a time-independent mesh system. Also distribution evolution equation for free surface is solved similarly in the analyzed domain.

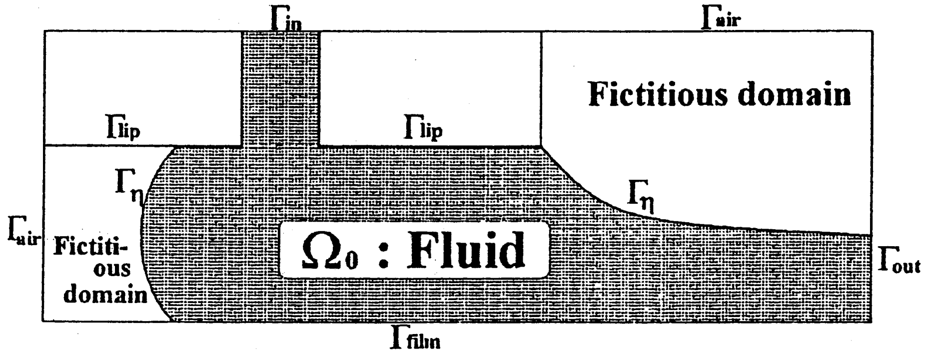

Figure 2: Projection of analyzed domain into $x - z$ plane

4 Description of Minimization Problem

Here, the minimization problem to control free surface arising in the flow of magnetic paints is defined.

As control, let us adopt;

1. $P_{in}(> P_{\infty})$, which is the pressure at the inflow boundary.
2. $V_{film}(> 0)$, which is the x-component of the velocity of the film.

Introduce the cost function;

$$J_h(P_{in}, V_{film}) = \frac{1}{V_{film}} + \frac{1}{\epsilon_r}\int_{x_0}^{x_1}\int_{y_0}^{y_1}(\eta(x,y) - \eta_{ave})^2 dxdy + \frac{1}{\epsilon_h}|\eta_{ave} - h|, \quad (2)$$

where

$$\eta_{ave} = \frac{\displaystyle\int_{x_0}^{x_1}\int_{y_0}^{y_1}\eta(x,y)dxdy}{(x_1 - x_0)(y_1 - y_0)}. \quad (3)$$

$z = \eta(x,y)$	:	The shape of the free surface on the domain $(x_0, x_1) \times (y_0, y_1)$ expected to be flat.
$h\,(> 0)$	:	A given constant, which is the desired thickness of coated paints.
$\epsilon_r, \epsilon_h\,(> 0)$	:	Penalty parameters.
η_{ave}	:	Averaged thickness of coated paints.

$$(P) \qquad Minimize\ \ J_h(P_{in}, V_{film}), \qquad for\ (P_{in}, V_{film}) \in \mathcal{A}$$

where

$$\mathcal{A} = \{(P_{in}, V_{film})\,|\,P_{in} \in (1,4), V_{film} \in (1,4)\}.$$

(P) is solved by use of FOM and MCA.

5 Fuzzy Optimization Method and Mountain Crossing Algorithm

In this section, FOM and MCA are summarized. FOM is an algorithm of the gradient type using stochastic fuzzy averaging to search for a local minimizer. A lot of numerical experiments by means of FOM show its wide convergence domain and low computational cost. MCA is proposed to search for a global minimizer among not so few local minimizers.[12,13] The unified algorithm equipped with FOM and MCA is implemented by repeating up- and down-hill procedures in the following steps;

- Step 1 Search for a local minimizer by FOM, i.e., down-hill algorithm.
- Step 2 Climb a hill of the target manifold up to the top of the hill or the ridge by golden section algorithm(GSA) along straight line started in stochastical direction from a local minimizer already obtained , i.e., up-hill algorithm.
- Step 3 Choose a quasi-local maximizer obtained in step 2 as an initial point for the next down-hill procedure.
- Step 4 Continue to find successively local minimizer by repeating steps 2 and 3.
- Step 5 Sometimes revisits to old minimizers occur. If the number of successive revisits to old minimizers is beyond the preset one, stop steps 2 and 3.
- Step 6 Scan all local minimizers, compare all local minima and set the lowest one as a global minimum.

6 Numerical Results

In figure 3, we see how the sequence of searching vectors to get to the global minimizer through the up- and down-hill procedures mentioned in section 5. Also the contours of cost function are shown here in order to understand the functions of FOM-MCA.

Three local minimizers, ($P_{in} = 1.95, V_{film} = 1.68$), ($P_{in} = 2.43, V_{film} = 2.56$) and ($P_{in} = 1.35, V_{film} = 1.96$), are found. And global minimizer, i.e., optimal control, is the third one.

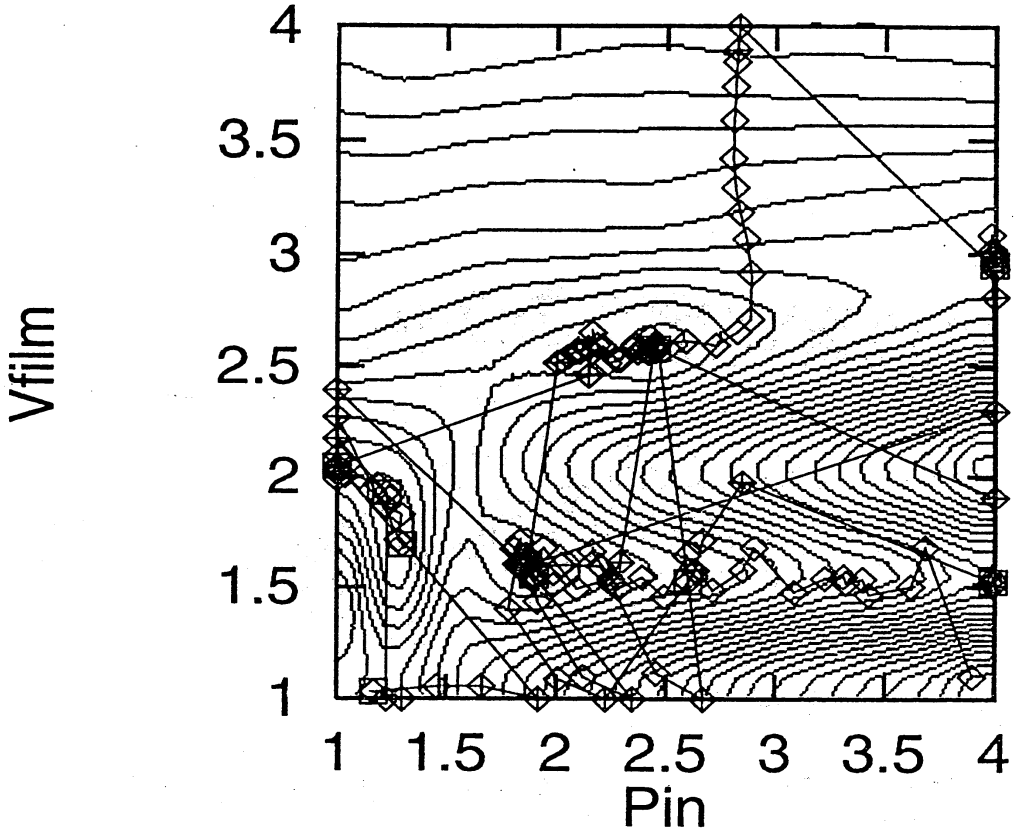

Figure 3: Contours of cost function and searching path by FOM-MCA

The profile of free surface corresponding to optimal control is shown in figure 4. Figures 5, 6 and 7 show the profiles of free surfaces due to the following controls, ($P_{in} = 4.0, V_{film} = 2.5$), ($P_{in} = 4.0, V_{film} = 4.0$) and ($P_{in} = 1.0, V_{film} = 1.0$).

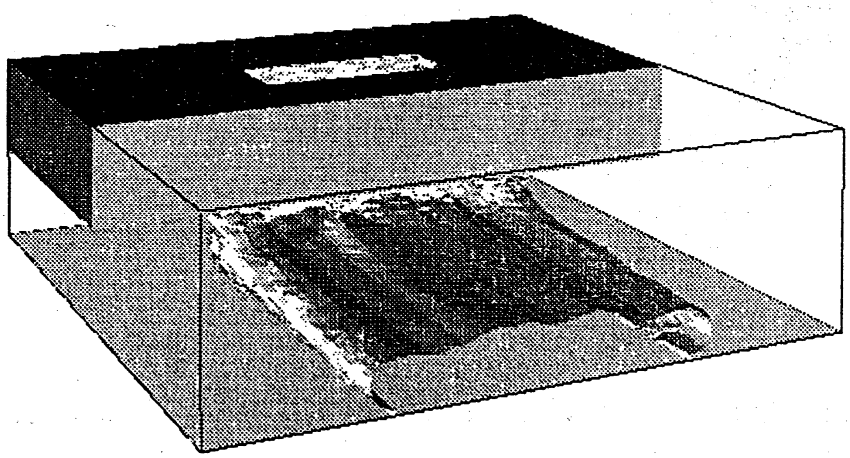

Figure 4: Profile of free surface ($P_{in} = 1.35, V_{film} = 1.96$)

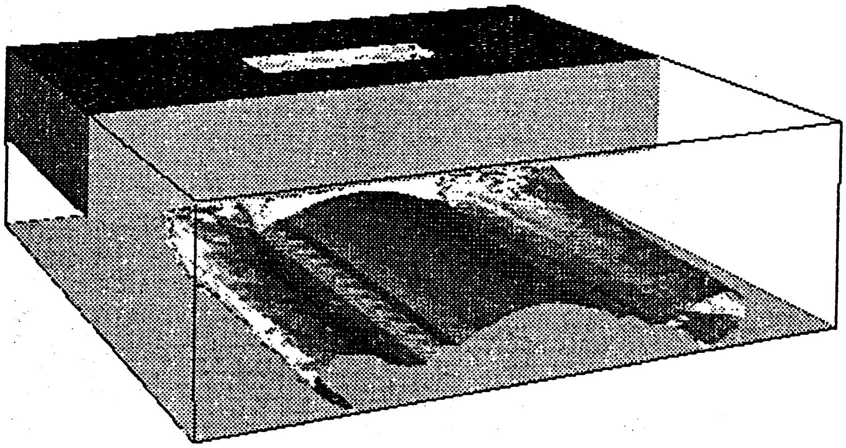

Figure 5: Profile of free surface ($P_{in} = 4.0, V_{film} = 2.5$)

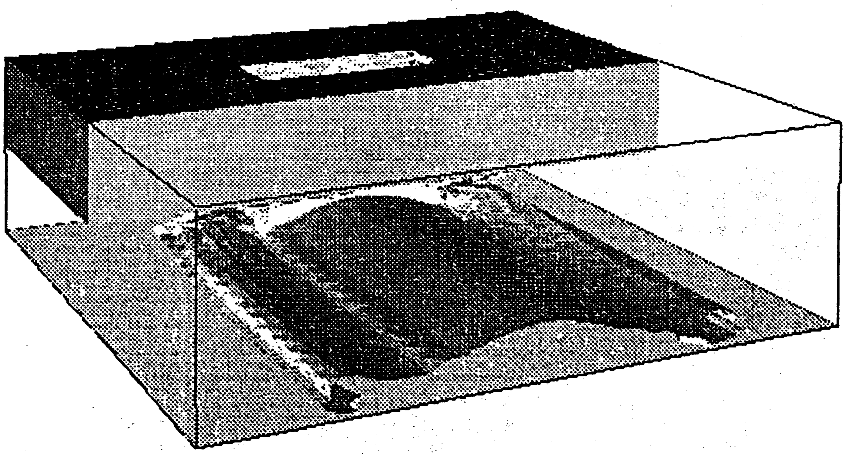

Figure 6: Profile of free surface ($P_{in} = 4.0, V_{film} = 4.0$)

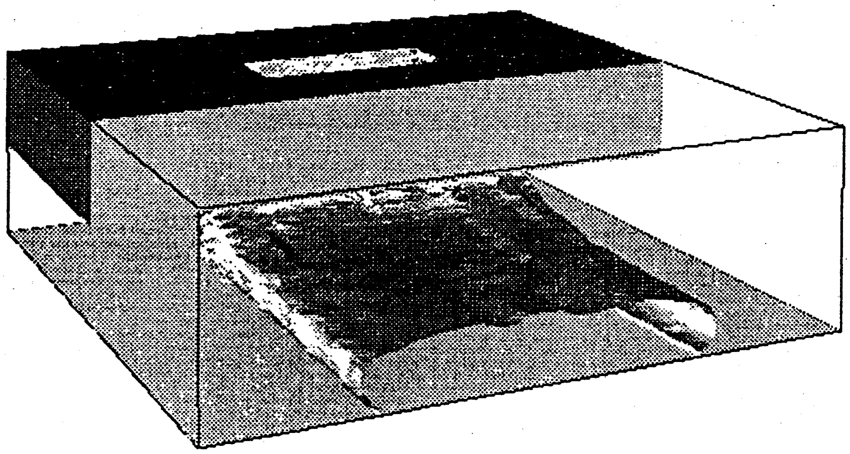

Figure 7: Profile of free surface ($P_{in} = 1.0, V_{film} = 1.0$)

7 Conclusion

After the description of the minimization problem with two parameters, we solved numerically the free surface flow problem at each step of the iteration of FOM by means of fictitious domain method. By application of FOM and MCA, three local minimizers were found. However, the number of these local minimizers are not guaranteed only by this method. Then this should be checked by using another concept. That is our next problem in near future.

It should be noted that the numerical free surface has sometimes the unsteady wavy patterns in spite of treating the steady boundary value problem for Navier-Stokes equations. It may be originated from the numerical errors based on discretization process and the evolution method to solve the steady Navier-Stokes equations. On the other hand, the unsteady wavy patterns are observed in this kind of physical experiments. However, it is unknown what kind of relation exists between them. That is the challenging problem laid between applied science and numerical analysis.

8 Acknowledgment

The authors would like to thank Dr.Yutaka Nakama (KAO corp.) for helpful comments and suggestions.

9 References

1. Berggren, M., Glowinski, R. and Lions, J., L., "Controllability Issues for Flow-related Models: A Computational Approach", Technical Report TR94-47, Department of Computational and Applied Mathematics, Rice University, 1994.

2. Beux, F. and Dervieux, A., "A Hierarchical Approach for Shape Optimization", *Engineering Computations*, 11(1), pp.25-48, Feb. 1994.

3. Fujita, H., Konno, H. and Tanabe, K., "Saitekika-hou", Iwanami-shoten (in Japanese), 1994.

4. Glowinski, R., Kearsley, A., J., Pan, T.,W. and Periaux, J., "Numerical Simulation and Optimal Shape for Viscous Flow by a Fictitious Domain Method", *International Journal for Numerical Methods in Fluids*, 20, pp.695-711, 1995.

5. Glowinski, R. and Lions, J., L., "Exact and Approximate Controllability for Distributed Parameter Systems (Part 1)", *Acta Numerica*, pp.269-378, 1994.

6. Glowinski, R. and Lions, J., L., "Exact and Approximate Controllability for Distributed Parameter Systems (Part 2)", *Acta Numerica*, pp.159-333, 1995.

7. Gunzburger, M., D., Hou, L., S. and Svobodny, T., P., "Boundary Velocity Control of Incompressible Flow with an Application to Viscous Drag Reduction",*SIAM Journal of Control and Optimization*, 30(1), pp.167-181, 1992.

8. Jameson, A., "Computational Aerodynamics for Aircraft Design", *Science*, 245, pp.361-371, 1989.

9. Kawahara, M. and Shimada, T., "Gradient Method of Optimal Control Applied to the Operation of a Dam Water Gate", *International Journal for Numerical Methods in Fluids*, 19, pp.463-477,1994.

10. Kawarada, H., "Application of Fictitious Domain Method to Free Boundary Problems", *Indo-French Conf. on Mathematical Methods for Partial Differential Equations*, Bangalore, India, 1994.

11. Kawarada, H., Fujita, H. and Kawahara, H., "Wave Motion on Sloping Beach", *ENUMATH*, Paris, France, 1995

12. Kawarada, H. and Suito, H., " Fuzzy Optimization for Control of Free Surface", *ECCOMAS* 96, Paris, France, 1996.

13. Kawarada, H. and Suito, H. "Fuzzy Optimization Method", *Computational Science for the 21st Century*, John Wiley & Sons, 1997(to be published).

14. Lions, J., L. *Optimal Control of Systems Governed by Partial Differential Equations*, Springer, 1971.

15. Mohammadi, B., "Optimal Shape Design, Reverse Mode of Automatic Differentiation and Turbulence", *35th AIAA Conference*, Reno, 1996.

16. Neittaanmaki, P. and Tiba, D., *Optimal Control of Nonliner Parabolic Systems. Theory, Algorithms and Applications*, Dekker, New York, 1994.

17. Pironneau, O., *Optimal Shape Design for Elliptic Systems*, Springer-Verlag, New York, 1984.

18. Sritharan, S., S., "Optimal Feedback Control of Hydrodynamics", A progress report. In Max D. Gunzburger, editor, *Flow Control*, pp.257-274, New York, 1995, Springer-Verlag. Vol.68 in *The IMA Volumes in Mathematics and its Applications.*

19. Temam, R., "Remarks on the Control of Turbulent Flows", In Max D. Gunzburger, editor, *Flow Control*, pp.257-274, New York, 1995, Springer-Verlag. Vol.68 in *The IMA Volumes in Mathematics and its Applications.*