数理解析研究所講究録
991 巻 1997 年 101-102

Tight Graphs and Their Primitive Idempotents*

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March 4, 1997

Abstract

In this paper, we prove

Theorem 1. Let Γ denote a distance-regular graph with diameter $d\geq 3$. Suppose E and F are primitive idempotents of Γ, with cosine sequences $\sigma_{0}, \sigma_{1}, \ldots, \sigma_{d}$ and $\rho_{0}, \rho_{1}, \ldots, \rho_{d}$, respectively. Then the following are equivalent.

i) The entry-wise product $E\circ F$ is a scalar multiple of a primitive idempotent of Γ.

ii) There exists a real number ϵ such that

$$\sigma_{i}\rho_{i}-\sigma_{i-1}\rho_{i-1}=\epsilon(\sigma_{i-1}\rho_{i}-\sigma_{i}\rho_{i-1}) \qquad (1\leq i\leq d).$$

Let Γ denote a distance-regular graph with diameter $d\geq 3$ and distinct eigenvalues $\theta_{0}>\theta_{1}>\cdots>\theta_{d}$. In [1], Jurišić, Koolen and Terwilliger proved that the valency k and the intersection numbers a_{1}, b_{1} satisfy

$$\left(\theta_{1}+\frac{k}{a_{1}+1}\right)\left(\theta_{d}+\frac{k}{a_{1}+1}\right)\geq\frac{-ka_{1}b_{1}}{(a_{1}+1)^{2}}.$$

They called the graph *tight* whenever Γ is not bipartite, and equality holds above. Combining Theorem 1 with some of their results, we obtain

Corollary 2. Let Γ denote a nonbipartite distance-regular graph with diameter $d\geq 3$ and distinct eigenvalues $\theta_{0}>\theta_{1}>\cdots>\theta_{d}$. The following are equivalent.

i) There exist nontrivial primitive idempotents E, F of Γ such that (i), (ii) hold in Theorem 1.

ii) Γ is tight.

Moreover, if (i), (ii) hold then the eigenvalues of Γ associated with E, F are a permutation of θ_{1}, θ_{d}.

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*This work was done when the author was an Honorary Fellow at the University of Wisconsin-Madison (September 1996 – September 1997) supported by the Department of Science and Techonology, Philippines.

Reference

[1] A. Jurišić, J. Koolen and P. Terwilliger, 1-Homogeneous Graphs (in preparation).

Acknowledgement

The author wishes to thank Professor Paul Terwilliger for his many valuable suggestions.