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1-COCYCLES ON INFINITE DIMENSIONAL SPACES

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1. Introduction

Let us consider a σ-finite measure space $(X, \mathfrak{B}, \mu)$ on which G acts as a measurable transformation group. We assume that μ is G-quasi-invariant. That is, μ_g is equivalent to μ $(\mu_g \simeq \mu)$, for all $g \in G$, where μ_g is the image measure of μ by the map $x \longmapsto gx$. It follows that a unitary representation $(R_\theta, \mathrm{L}^2_\mu(X))$ of G is defined as follows,

$$R_\theta(g) : f(x) \in \mathrm{L}^2_\mu(X) \longmapsto \theta(x, g)\sqrt{\frac{d\mu_g}{d\mu}}(x) f(g^{-1}x) \in \mathrm{L}^2_\mu(X), \tag{1.1}$$

where θ, so called 1-cocycle*, is a S^1-valued function on $G \times X$ such that

(1) for each fixed $g \in G, \theta(x, g)$ is a measurable function of x, and

(2) for all $g_1, g_2 \in G$, $\theta(x, g_1)\theta(g_1^{-1}x, g_2) = \theta(x, g_1 g_2)$ for μ-a.e.x.

Moreover if a group topology is induced to G and the following condition (3) is satisfied, we say that θ is continuous.

(3) $\theta(x, g) \longrightarrow 1$ in μ, if $g \longrightarrow e$ in τ.

A simple example of 1-cocycles is the one described below which is so called 1-coboundary,

$$\theta(x, g) = \frac{\phi(g^{-1}x)}{\phi(x)},$$

where ϕ is a S^1-valued measurable function. In this report we will pick up infinite dimensional one linear space and two groups as G and will discuss on the 1-cocycles, especially its characterization, connecting with canonical representations defined by (1.1).

(*) The names, 1-cocycle and 1-coboundary, come from group cohomological theory (cf.[10]), which is privately communicated by Y.Yamasaki. Let us explain it briefly. Let G be a group, A be an Abelian group and assume that G acts on A from the left. Further let $\mathcal{F}_m$ be a set of all maps from $\prod_{i=1}^m G_i$ to A, where G_i is the same copy of G for all $1 \leq i \leq m$. Put ∂_m be a map from $\mathcal{F}_m$ to $\mathcal{F}_{m+1}$ such that

$$(\partial_m \varphi)(g_1, \cdots, g_{m+1}) := \sum_{i=0}^{m+1} (-1)^i \varphi_i(g_1, \cdots, g_{m+1}),$$

where

$$\varphi_0(g_1, \cdots, g_{m+1}) := g_1\varphi(g_2, g_3, \cdots, g_{m+1})$$
$$\varphi_1(g_1, \cdots, g_{m+1}) := \varphi(g_1 g_2, g_3, \cdots, g_{m+1})$$
$$\cdots$$
$$\varphi_i(g_1, \cdots, g_{m+1}) := \varphi(g_1, g_2, \cdots, g_i g_{i+1}, \cdots, g_{m+1})$$
$$\cdots$$
$$\varphi_{m+1}(g_1, \cdots, g_{m+1}) := \varphi(g_1, g_2 \cdot \cdots, g_m).$$

Then we have $\partial_{m+1} \circ \partial_m = 0$, and mth cohomology group $H^m(G, A) := \ker\partial_m / \mathrm{Im}\partial_{m-1}$ is defined for $m \geq 1$, where $\mathcal{F}_0 := A$ and $(\partial_0 a)(g) := ga - a$ for all $a \in A$. An element in $\ker\partial_m$, (in $\mathrm{Im}\partial_{m-1}$) is called m-cocycle, (m-coboundary), respectively. Now let us apply the above general theory to our situation. That is, we take A as the equivalence class of all measurable S^1-valued function to modulo μ, and define the action of G on A such that $(gf)(x) := f(g^{-1}x)$ for all $g \in G$ and for all $f \in A$. Then it is easily checked that 1-cocycle (1-coboundary) is just the same with 1-cocycle (1-coboundary) in the cohomological sense.

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2. 1-COCYCLES DERIVED FROM COMMUTATION RELATION IN QUANTUM MECHANICS

First we shall consider 1-cocycles on the algebraic dual space X^a of an infinite dimensional real linear space X, which come from the representation of commutation relation in quantum mechanics. So we consider X^a as the basic space and take a linear subspace X' of X^a as a transformation group G such that for any $x \in X$ there exists $x' \in X'$ such that $< x, x' > \neq 0$, where $< \cdot, \cdot >$ is a natural duality bracket for X and X^a. The action of X' on X^a is defined by $x^a \longmapsto x' + x^a$. Now let us consider unitary representations $(U, \mathcal{H}), (V, \mathcal{H})$ of X and X' respectively which satisfy,

(1) $U(x)$ is continuous on any finite dimensional subspace of X,

(2) U is cyclic, and

(3) $U(x)V(x') = \exp(\sqrt{-1} < x, x' >)V(x')U(x)$ for all $x \in X$ and for all $x' \in X'$.

Then the following theorems hold which are already well known.

Theorem 2.1. *There exist some probability measure μ on the cylindrical σ-algebra $\mathfrak{B}$ on X^a and 1-cocycle θ on $X^a \times X'$ such that the representations $(U, \mathcal{H})$ and $(V, \mathcal{H})$ are realized as follows,*

$$U(x) : f(x^a) \in \mathrm{L}^2_\mu(X^a) \longmapsto \exp(\sqrt{-1} < x, x^a >)f(x^a) \in \mathrm{L}^2_\mu(X^a), \tag{2.1}$$

$$V(x') : f(x^a) \in \mathrm{L}^2_\mu(X^a) \longmapsto \theta(x^a, x')\sqrt{\frac{d\mu_{x'}}{d\mu}}(x^a)f(x^a - x') \in \mathrm{L}^2_\mu(X^a). \tag{2.2}$$

Theorem 2.2. (1) *For the pair of representations (U_i, V_i) $(i = 1, 2)$ which are defined by* (2.1) *and* (2.2), *(U_1, V_1) are equivalent to (U_2, V_2) if and only if the corresponding μ_1 and μ_2 are equivalent as measures and the corresponding θ_1 and θ_2 are 1-cohomologus. i.e, there exists some 1-coboundary ϕ such that $\theta_1 = \phi \cdot \theta_2$.*

(2) *In order that the representation (U, V) is irreducible, it is necessary and sufficient that μ is X'-ergodic. i.e., $\mu(A) = 0$ or 1, provided that $\mu(A \ominus (A - x')) = 0$ for all $x' \in X'$.*

From the above theorems, we see that the pair of representations (U, V) is characterized by two factors, that is, measure and 1-cocycle. So we shall look them quickly.

In the finite dimensional case, the problem is so simple. Namely, every translationally quasi-invariant measure is equivalent to the Lebesgue measure and every 1-cocycle is a 1-coboundary. While in the infinite dimensional case the situation is quite complicated. First of all there exist quasi-invarianr measures much enough to nonclassify them. Secondly, even if a quasi-invariant measure is specified, there exist also many 1-cocycles which are essentially different from each other. Therefore it seems to be meaningless to try to classfy 1-cocycles for a general μ. However it seems to be meaningful and important to consider 1-cocycles θ for Gaussian measures g picked up among many quasi-invariant measures. Here the Gaussian measure g on the algebraic dual H^a of a Hilbert space H is defined by,

$$\int_{H^a} \exp(\sqrt{-1} < x, x^a >) g(dx^a) = \exp(-\frac{1}{2}\|x\|_H^2). \tag{2.3}$$

It is well known that g is H^*-quasi-invariant, where H^* is the topological dual space of H. So the problem becomes as follows.

(P) What kinds of 1-cocycles θ on $H^a \times H^*$ for the Gaussian measure g do there exist ? Especially, it is a matter worthy to be considered when θ is continuous with the norm topology on H^*.

The following theorem is a modest result along this line.

Theorem 2.3. *For any $s \in \mathbf{R}$ consider a continuous 1-cocycle*

$$\theta_s(x^a, x^*) := \left(\frac{dg_{x^*}}{dg}(x^a)\right)^{\sqrt{-1}s}.$$

(1) *Then the canonical representations $(R_s, \mathrm{L}_g^2(H^a))$ defined by*

$$R_s(x^*) : f(x^a) \in \mathrm{L}_g^2(H^a) \longmapsto \theta_s(x^a, x^*)\sqrt{\frac{dg_{x^*}}{dg}}(x^a) f(x^a - x^*) \in \mathrm{L}_g^2(H^a) \tag{2.4}$$

give mutually inequivalent representations for all different s's.

(2) *Let g_s be the image measure of g by a homothety, $x^a \longmapsto (1+4s^2)^{-\frac{1}{2}} x^a$. Then $(R_s, \mathrm{L}_g^2(H^a))$ is equivalent to $(R_0, \mathrm{L}_{g_s}^2(H^a))$, where the last representation is defined by,*

$$R_0(x^*) : f(x^a) \in \mathrm{L}_{g_s}^2(H^a) \longmapsto \sqrt{\frac{d(g_s)_{x^*}}{dg_s}}(x^a) f(x^a - x^*) \in \mathrm{L}_{g_s}^2(H^a). \tag{2.5}$$

(3) *There exists another family of representations $(R_{\zeta_c}, \mathrm{L}_g^2(H^a))$ $(c \in \mathbf{R})$ with the property that $(R_{\zeta_c}, \mathrm{L}_g^2(H^a))$ are inequivalent to $(R_s, \mathrm{L}_g^2(H^a))$ for all $c, s \in \mathbf{R}$. Moreover $(R_{\zeta_c}, \mathrm{L}_g^2(H^a))$ are mutually inequivalent.*

The definition of ζ_c is as follows. For any $h \in H^*$ we take a unique $W_h \in \mathrm{Cl}\{< x, x^a > |x \in H\} (\subset \mathrm{L}_g^2(H^a))$ such that

$$< x, h > = \int_{H^a} < x, x^a > W_h(x^a) g(dx^a).$$

Put

$$\zeta_c(x^a, h) := \exp\{\sqrt{-1}c \sum_{n=1}^{\infty} (W_{h_n}^3(x - \varphi) - 3W_{h_n}(x - \varphi) - W_{h_n}^3(x) + 3W_{h_n}(x))\}.$$

For the further and detailed informations see [13].

3. 1-COCYCLES FOR ROTATIONALLY INVARIANT MEASURES

In this section we set up the following situation.
Let H be a real separable Hilbert space ($\dim(H) < \infty$ or $= \infty$), $\mathfrak{B}$ be a cylindrical σ-algebra on H^a, $O(H)$ be a rotation group ($O(H)$=SO(n), *if* $\dim(H) = n < \infty$), and μ be an $O(H)$-quasi-invariant probability measure. Now consider a continuous 1-cocycle θ defined on $H^a \times O(H)$ which satisfies the following conditions.

(1) For any fixed $U \in O(H)$, $\theta(x^a, U)$ is a S^1-valued $\mathfrak{B}$-measurable function.

(2) For any $U_1, U_2 \in O(H)$,

$$\theta(x^a, U_1)\theta({}^tU_1 x^a, U_2) = \theta(x^a, U_1U_2) \quad \text{for} \quad \mu-\text{a.e.}x^a.$$

(3) $\theta(x^a, U) \longrightarrow 1$ in μ, if $U \longrightarrow \mathrm{Id}$ in the strong operator topology.

Such 1-cocycles arises in the representations of the semi-direct product of H and $O(H)$. That is, let $(V, \mathcal{H})$ and $(T, \mathcal{H})$ be unitary representations of H and $O(H)$, respectively which satisfy,

(1) V is cyclic,

(2) V is continuous on any finite dimensional subspace of H and T is continuous with the strong operator topology, and

(3) for all $h \in H$ and for all $U \in O(H)$,

$$T(U)V(h) = V(Uh)T(U).$$

Then there exist an $O(H)$-quasi-invariant probability measure μ on $(H^a, \mathfrak{B})$ and a continuous 1-cocycle θ for μ such that $(V, \mathcal{H})$ and $(T, \mathcal{H})$ are realized as follows.

$$V(h) : f(x^a) \in \mathrm{L}^2_\mu(H^a) \longmapsto \exp(\sqrt{-1} < h, x^a >) f(x^a) \in \mathrm{L}^2_\mu(H^a). \tag{3.1}$$

$$T(U) : f(x^a) \in \mathrm{L}^2_\mu(H^a) \longrightarrow \theta(x^a, U)\sqrt{\frac{d\mu_U}{d\mu}}(x^a) f({}^tUx^a) \in \mathrm{L}^2_\mu(H^a). \tag{3.2}$$

Moreover similar results with Theorem 2.2 also holds. We have only to change the ergodic part to "$O(H)$-ergodic". Thus the pair of representations (V, T) is also controled by the same two factors. However the situation is quite different from the previoue one. First for the measure the following results are already known.

Theorem 3.1. (1) *For any rotationally quasi-invariant probability measure μ, there exists a rotationally invariant probability measure ν such that $\mu \simeq \nu$.*

(2) *ν is represented as a superposition of probability measures $\{g_c\}_{c\in[0,\infty)}$, where g_c is the uniform measure on the sphere of radious c centered at the origin, if* $\dim(H) < \infty$, *and g_c is the centered Gaussian measure with variance c^2, if* $\dim(H) = \infty$.

For the proof, see [12] and [17]. Second the structure of 1-cocycles is very simple as is shown in the following theorem.

Theorem 3.2. *Assume that* $\dim(H) \neq 3$. *Then any continuous 1-cocycle θ for μ is a 1-coboundary. That is, there exists S^1-valued $\mathfrak{B}$-measurable function ϕ on H^a such that for each fixed $U \in O(H)$,*

$$\theta(x^a, U) = \frac{\phi({}^tUx^a)}{\phi(x^a)}$$

for μ-a.e.x^a.

For the proof see [14]. From these theorems, we see that the pair of representation (V,T) is equivalent to the following one,

$$(3.3)\qquad V_\nu(h) : f(x^a) \in \mathrm{L}^2_\nu(H^a) \longrightarrow \exp(\sqrt{-1} < h, x^a >) f(x^a) \in \mathrm{L}^2_\nu(H^a),$$

$$(3.4)\qquad T_\nu(U) : f(x^a) \in \mathrm{L}^2_\nu(H^a) \longrightarrow f({}^tUx^a) \in \mathrm{L}^2_\nu(H^a),$$

and the equivalence (irreducibility) of the pair (V,T) defined by (3.3) and(3.4) are reduced to the equivalence of the corresponding rotationally invariant (ergodic) probability measure ν, respectively. Further single representation of $O(H)$,

$$R_\theta(U) : f(x^a) \in \mathrm{L}^2_\mu(H^a) \longmapsto \theta(x^a, U)\sqrt{\frac{d\mu_U}{d\mu}}(x^a) f({}^tUx^a) \in \mathrm{L}^2_\mu(H^a)$$

is equivalent to the representation defined by (3.4), and the properties for the decomposition are derived from the decomposition of

$$\nu = \int_{[0,\infty)} g_c m(dc),$$

where m is a Borel probability measure on $[0,\infty)$ and from the result for the irreducible decomposition, $\mathrm{L}^2_g(H^a) = \sum \oplus \mathcal{H}_n$, using multiple Wiener integrals $\mathcal{H}_n (n = 0,1,2,\cdots)$ for the Gaussian measure $g = g_1$. Namely,

Theorem 3.3. *Assume that* $\dim(H) = \infty$. *Then* $(T_\nu, \mathrm{L}^2_\nu(H^a))$ *is completely reducible, and as its irreducible components,*

(1) $(R_g, \mathcal{H}_n)$ $(n = 1,2,\cdots)$ *appears* $\dim(\mathrm{L}^2_m)$*- times in it and*

(2) $(R_g, \mathcal{H}_0)$ *appears* $\dim(\mathrm{L}^2_m) + 1$*-times or* $\dim(\mathrm{L}^2_m)$*-times, according to* $m(0) > 0$ *or* $m(0) = 0$.

N.B. Here we give a counter example for Theorem 3.2, when $\dim(H) = 3$. Let $e := e_3 = {}^t(0,0,1)$, $\mathcal{M} : U \in SO(3) \longmapsto Ue \in S^2$ and $\mathcal{N}$ be a Borel cross section of $\mathcal{M}$. Then for any $x \in S^2$ and for any $U \in SO(3)$ there exists $\tau \in \mathbf{R}$ such that

$$U^{-1}\mathcal{N}(x) = \mathcal{N}(U^{-1}x)\begin{pmatrix} \cos\tau & -\sin\tau & 0 \\ \sin\tau & \cos\tau & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Put

$$\theta(x,U) := \exp(\sqrt{-1}\tau).$$

Then θ is a continuous 1-cocycle for the uniform measure on S^2. However it is not a 1-coboundary. For the detailed informations in this section, see [14].

4. 1-COCYCLES ON THE GROUP OF DIFFEOMORPHISMS

Let $M = M^d$ be a paracompact C^∞-manifold and $\mathrm{Diff}_0(M)$ be the set of all diffeomorphisms g with compact supports. This section is a study of 1-cocycle θ on $M \times \mathrm{Diff}_0(M)$. So let μ be a σ-finite smooth measure on M which is locally equivalent to the Lebesgue measure on $\mathbf{R}^d$, and take a canonical representation U_θ of $\mathrm{Diff}_0(M)$ such that

$$(4.1)\qquad U_\theta(g) : f(P) \in \mathrm{L}^2_\mu(M) \longmapsto \theta(P,g)\sqrt{\frac{d\mu_g}{d\mu}}(P) f(g^{-1}(P)) \in \mathrm{L}^2_\mu(M),$$

where θ is a continuous 1-cocycle. Here the topology τ on $\mathrm{Diff}_0(M)$ is the inductive limit topology of τ_K on $\mathrm{Diff}(K)$, $K \uparrow X$, where $\mathrm{Diff}(K) := \{g \in \mathrm{Diff}_0(M) \mid \mathrm{supp} g \subset K\}$ is equipped with the topology τ_K of uniform convergence with every derivative on the compact set K. Exactly speaking, the continuity of θ is as follows. $\theta(P, g_n) \longrightarrow 0$ in μ if there exists some compact set K such that supp$g_n \subseteq K$ $(n = 1, \cdots)$ and $g_n \longrightarrow \mathrm{Id}$ in Diff(K). τ is never a group topology unless M is compact. (See [16] in this issue). These 1-cocycles often appears in the representation theory of Diff$_0(M)$. In this report we will give some characterization of 1-cocycles which have much stronger continuous property than the original one. We anew give the definition of our present 1-cocycle θ.

Definition 4.1. *A S^1-valued function θ on $M \times$ Diff$_0(M)$ is said to be continuous 1-cocycle, if and only if the following conditions are satisfied.*

(1) *For any $g_1, g_2 \in$ Diff$_0(M)$,*

$$\theta(P, g_1)\theta(g_1^{-1}(P), g_2) = \theta(P, g_1 g_2).$$

(2) *For each fixed $P \in M$, $\theta(P, g)$ is a continuous function of $g \in$ Diff$_0(M)$ with respect to τ.*

The analysis of continuous 1-cocycles is based on the following theorems.

Theorem 4.1. (Campbell – Hausdorff formula)

Let $X, Y \in \Gamma_0(M)$ and $\{\mathrm{Exp}(tX)\}_{t\in\mathbf{R}}, \{\mathrm{Exp}(tY)\}_{t\in\mathbf{R}}$ be 1-parameter subgroups of diffeomorphisms generated by X, Y, respectively. Then as n tends to $+\infty$,

(1) $\{\mathrm{Exp}(\frac{tX}{n}) \circ \mathrm{Exp}(\frac{tY}{n})\}^n$ *converges to* $\mathrm{Exp}(t(X+Y))$, *and*

(2) $\{\mathrm{Exp}(-\frac{tX}{\sqrt{n}}) \circ \mathrm{Exp}(-\frac{tY}{\sqrt{n}}) \circ \mathrm{Exp}(\frac{tX}{\sqrt{n}}) \circ \mathrm{Exp}(\frac{tY}{\sqrt{n}})\}^n$ *converges to* $\mathrm{Exp}(-t^2[X, Y])$

in τ_K unifomly on every compact interval of t, respectively, where K is any compact set containig supp$X$ *and* supp$Y$.

Theorem 4.2. *The group $\tilde{G}$ generated by* Exp(X), *where X runs through all C^∞-vector fields with compact supports, forms a dense subset of* Diff$_0^*(M)$ *which is the connected component of* Id *in* Diff$_0(M)$.

Using these theorems we restrict θ to the subgroup $\tilde{G}$ and analize it locally. Then, but many lemmas are needed, the following results are obtained which is expected by T.Hirai in the case of $M = \mathbf{R}^d$.

Theorem 4.3. *Assume that M is simply connected. Then any continuous 1-cocycle θ has the following canonical form,*

$$\theta(P, g) = \frac{c(g^{-1}(P))}{c(P)} \left(\frac{d\mu_g}{d\mu}(P)\right)^{\sqrt{-1}s} \eta(g)$$

, *where c is a S^1-valued continuous function on M, s is a real number and η is a unitary character on* Diff$_0(M)$. (*Actually, η is a trivial character on* Diff$_0^*(M)$, *so it is a function on the discrete group* Diff$_0(M)/$Diff$_0^*(M)$.) *S and η are uniquely determined for θ, while c is determined up to constant factors.*

Corollary 4.4. *If M is a compact Lie group, then the same holds for any continuous 1-cocycle θ.*

If M is not simply connected, then it is possible to exists a new 1-cocycle. For example in the case $M = \mathbf{R} \times S^1$, we have a following result. Let $g \in$ Diff$_0^*(\mathbf{R} \times S^1)$ and take a continuous path $\{g_t\}_{0\leq t\leq 1}$ connecting Id and g. Then for each fixed $p = (u, z) \in \mathbf{R} \times S^1$, the second component of $g_t^{-1}(u, z)$ has a continuous angular function $\theta(t, u, z)$.

The value $\varphi(u,z) := \theta(1,u,z) - \theta(0,u,z)$ only depends on (g,u,z) and does not depend on a particular choice of $\{g_t\}_{0\le t\le 1}$. So for any $\Omega \in [0,1)$ put

$$\zeta_\Omega((u,z),g) := \exp(\sqrt{-1}\Omega\varphi(u,z)).$$

Then ζ_Ω is a continuous 1-cocycle on $\mathrm{Diff}_0^*(\mathbf{R}\times S^1)$ and it is extended to the whole group in an essential unique way. We denote it again by ζ_Ω.

Theorem 4.5. *If $M = \mathbf{R}\times S^1$, the general form of continuous 1-cocycles is as follows,*

$$\theta(P,g) = \frac{c(g^{-1}(P))}{c(P)}\left(\frac{d\mu_g}{d\mu}(P)\right)^{\sqrt{-1}s}\zeta_\Omega(P,g)\eta(g).$$

Any ζ_Ω $(0<\Omega<1)$ is never 1-cohomologus with any 1-cocycles appeared in Theorem4.3. *s,Ω and η are uniquely determined from θ and c is determined up to constant factors.*

Lastly, we shall list the following results with canonical representations defined by (4.1).

Theorem 4.6. *Assume that M is connected. Then*
(1) *The representation $(U_\theta, \mathrm{L}^2_\mu(M))$ is irreducible for all continuous 1-cocycle θ.*
(2) *$(U_{\theta_1}, \mathrm{L}^2_\mu(M))$ is equivalent to $(U_{\theta_2}, \mathrm{L}^2_\mu(M))$, if and only if θ_1 and θ_2 are 1-cohomologus.*

For detailed informations in this section, see [15].

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