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Motives for Selecting a Strategy in a Non-cooperative Game

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ABSTRACT

There are many examples of non-cooperative games in which a player does not necessarily select the Nash equilibrium strategy in practice. In order to understand such a situation rationally, we propose a subjective game for each player which is constituted by taking his motive for selecting his strategy into consideration. In particular we give a rational model for explaining an actual behavior of each player in the prisoner's dilemma game.

1. INTRODUCTION

We consider the following two-person, non-cooperative, nonzero-sum, finite game:

$$G=[A,B]:\quad \mathrm{I}\quad \begin{array}{c|c} & \mathrm{II} \\ & \begin{array}{ccc}\beta_1 & \cdots & \beta_n\end{array} \\ \begin{array}{c}\alpha_1\\ \vdots \\ \alpha_m\end{array} & \begin{bmatrix} (a_{11},b_{11}) & \cdots & (a_{1n},b_{1n}) \\ \vdots & & \vdots \\ (a_{m1},b_{m1}) & \cdots & (a_{mn},b_{mn}) \end{bmatrix} \end{array} \tag{1}$$

where $A=[a_{ij}]$ and $B=[b_{ij}]$ are payoff matrixes for player I and II respectively. That is to say, player I (II) selects one of $m(n)$ pure strategies $\alpha_1,\cdots,\alpha_m$ $(\beta_1,\cdots,\beta_n)$ without knowing the choice of the opponent, and he obtains the payoff a_{ij} (b_{ij}) and the game is over. Let $x=\langle x_1,\cdots,x_m\rangle$ be a mixed strategy for player I where x_i is a probability that he selects the pure strategy $\alpha_i : x_i \geq 0 \quad (i = 1,\cdots,m)$ and $\sum_{i=1}^{m} x_i = 1$.

Similarly a mixed strategy for player Ⅱ is denoted by $y = \langle y_1,\cdots,y_n \rangle : y_j \geq 0$ $(j = 1,\cdots n)$ and $\sum_{j=1}^{n} y_j = 1$. When players I and Ⅱ select mixed strategies x and y respectively, the expected payoff for I is given by

$$M_1(x,y) = xAy = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_i y_j \tag{2}$$

and one for Ⅱ is given by

$$M_2(x,y) = xBy = \sum_{i=1}^{m} \sum_{j=1}^{n} b_{ij} x_i y_j \,. \tag{3}$$

For a non-cooperative game some equilibrium solutions have been proposed to explain the behaviors of players rationally. One is the Nash equilibrium point which is prposed by J.F.Nash[3].

Definition 1. The pair of mixed strategies for both players (x^*,y^*) is the Nash equilibrium point (NEP) if and only if it satisfies the following relations :

$$\begin{cases} M_1(x^*,y^*) = \max\limits_{x} M_1(x,y^*) & (4) \\ M_2(x^*,y^*) = \max\limits_{y} M_2(x^*,y). & (5) \end{cases}$$

The NEP is a point at which both players equilibrate each other by aiming at only maximizing his own expected payoff.
Another is the twisted equilibrium point which is proposed by R.A.Aumann[1].

Definition 2. The pair of mixed strategies for both players $(\tilde{x},\tilde{y})$ is the twisted equilibrium point (TEP) if and only if it satisfies the following relations :

$$\begin{cases} M_2(\tilde{x},\tilde{y}) = \min\limits_{x} M_2(x,\tilde{y}) & (6) \\ M_1(\tilde{x},\tilde{y}) = \min\limits_{y} M_1(\tilde{x},y). & (7) \end{cases}$$

The TEP is a point at which both players equilibrate each other by aiming at only minimizing the expected payoff of his opponent.
Moreover Aumann[1] defines an almost strictly competitive (a.s.c) game and shown that in an a.s.c. game the optimal strategies for both players exist.

By the way, can these equilibrium solution realy explain the actual behavior of each player rationally ? In connection with this, J.S.Minas et al.[2] reports the results of some actual experiments by non-cooperative games.

Experiment 1. We consider the non-cooperative game

$$G_1: \quad \mathrm{I} \quad \begin{array}{cc} & \mathrm{II} \\ & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (4,4) & (1,3) \\ (3,1) & (0,0) \end{bmatrix} \end{array} \tag{8}$$

which is a thirty-trial game using fifteen pairs of female subjects. This game G_1 has a unique NEP (α_1, β_1) with payoff vector (4,4), and besides, this point brings both players maximum payoff 4, that is , it satisfies the Pareto optimality. Then if each player aims at only maximizing his own expected payoff, he is sure to have no hesitation about selecting the pure strategy α_1. On the other hand, since a unique TEP of this game is (α_2, β_2), player I is sure to select the pure strategy α_2 if he aims at only minimizing the expected payoff of his opponent. However the result of this experiment was that fifty-three per cent of the individual selections were α_1 and the remainders were α_2. Note that the game G_1 is not an a.s.c. game since the NEP is not equal to the TEP.

Experiment 2. We consider the prisoner's dilemma game

$$G_2: \quad \mathrm{I} \quad \begin{array}{cc} & \mathrm{II} \\ & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{bmatrix} \end{array} \tag{9}$$

which is a fifty-trial game using thirty male subjects. This game has a unique NEP (α_2, β_2) which is also equal to the TEP. Then the game G_2 is an a.s.c. game . If player I aims at maximizing his own expected payoff or minimizing the expected payoff of his opponent, he is sure to have no hesitation about selecting the defection strategy α_2. But in this experiment thirty-six per cent of subjects select the cooperation strategy α_1 and the remainders select the defection strategy α_2.

From these experiments we can see the following facts:

(i) A player does not necessarily take the Nash equilibrium strategy (NES) only. Then the NES can not necessarily explain the behavior of a player in a non-cooperative game rationally .

(ii) A player does not necessarily select his strategy by only one motive. It seems that a player moves under the mixture of various motives. Though the payoff matrixes known to both players are A and B only, in the actual game a player seems to move under judgement by his subjective payoff matrix constituted from A and B.

Then in the next section we propose a subjective game for each player in a non-cooperative game.

2. SUBJECTIVE GAME

In the non-cooperative game G given by (1), we consider l motives $m_1, m_2, \cdots m_l$ under which each player selects his strategy. When player I and II select pure strategies α_i and β_j respectively, let f_{ij}^k (g_{ij}^k) be a subjective payoff for player I (II) with respect to the motive m_k. Then

$$f^k = [f_{ij}^k] \quad , \quad g^k = [g_{ij}^k] \tag{10}$$

are called by the subjective payoff matrixes for player I and II with respect to the motive m_k respectively $(k = 1, \cdots, l)$. Note that f^k and g^k are constituted by the public payoff matrixes A and B.

For example, the following motives can be considered :

(i) Motive m_1 : Maximizing my own expected payoff. In this case the subjective payoff matrixes are given by

$$f_{ij}^1 = a_{ij} \quad , \quad g_{ij}^1 = b_{ij} \quad (i = 1, \cdots, m \,;\, j = 1, \cdots, n). \tag{11}$$

In the case that both players move by the motive m_1 only, the equilibrium point is the NEP.

(ii) Motive m_2 : Minimizing the expected payoff of the opponent.

$$f_{ij}^2 = -b_{ij} \quad , \quad g_{ij}^2 = -a_{ij} \quad (i = 1, \cdots, m \,;\, j = 1, \cdots, n) \tag{12}$$

In the case that both players move by the motive m_2 only, the equilibrium point is the TEP.

(iii) Motive m_3 : Maximizing the difference in payoff between player I and II .

$$f_{ij}^3 = a_{ij} - b_{ij} \quad , \quad g_{ij}^3 = b_{ij} - a_{ij} \quad (i = 1, \cdots, m \,;\, j = 1, \cdots, n). \tag{13}$$

(iv) Motive m_4 : Maximizing my wining probability.

$$f_{ij}^4 = \mathrm{sgn}(a_{ij} - b_{ij})^+ \quad , \quad g_{ij}^4 = \mathrm{sgn}(b_{ij} - a_{ij})^+ \quad (i = 1,\cdots,m\,;\, j = 1,\cdots,n) \quad (14)$$

where $x^+ = \max(x,0)$ and

$$\mathrm{sgn}\ x = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0. \end{cases} \quad (15)$$

(v) Motive m_5 : Maximizing a probability of not losing.

$$f_{ij}^5 = 1 - \mathrm{sgn}(b_{ij} - a_{ij})^+ \quad , \quad g_{ij}^5 = 1 - \mathrm{sgn}(a_{ij} - b_{ij})^+ \quad (16)$$

$$(i = 1,\cdots,m\,;\, j = 1,\cdots,n).$$

(vi) Motive m_6 : Minimizing my regret.

$$f_{ij}^6 = -\left(\max_{r,s} a_{rs} - a_{ij}\right) \quad , \quad g_{ij}^6 = -\left(\max_{rs} b_{rs} - b_{ij}\right) \quad (17)$$

$$(i = 1,\cdots,m\,;\, j = 1,\cdots,n).$$

(vii) Motive m_7 : Maximizing the total social payoff (The sum of payoffs of both players).

$$f_{ij}^7 = g_{ij}^7 = a_{ij} + b_{ij} \qquad (i = 1,\cdots,m\,;\, j = 1,\cdots,n). \quad (18)$$

(viii) Motive m_8 : Maximizing the payoff of mutual prosperity. This motive can be considered only when there is a pair (i_0, j_0) satisfying

$$a_{i_0 j_0} + b_{i_0 j_0} = \max_{i,j}(a_{ij} + b_{ij}) \quad \text{and} \quad \left|a_{i_0 j_0} - b_{i_0 j_0}\right| = \min_{i,j}\left|a_{ij} - b_{ij}\right|. \quad (19)$$

In this case

$$f_{ij}^8 = g_{ij}^8 = \begin{cases} a_{i_0 j_0} + b_{i_0 j_0} & if\ (i,j) = (i_0, j_0) \\ 0 & otherwise \end{cases}. \quad (20)$$

Let $\lambda_k(\theta_k)$ be a weight which player I introduces for denoting a degree with which player I (II) regards the motive m_k as most important. That is, player I considers that player I (II) selects his strategy by the motive m_k with weight $\lambda_k(\theta_k)$. Similarly let $\xi_k(\eta_k)$ be a weight which player II introduces for denoting a degree with which player I (II) regards the motive m_k as most important.

We define four weight vectors as follows :

$$\lambda = \langle \lambda_1,\cdots,\lambda_l \rangle \ ; \ \lambda_k \geq 0\ for\ any\ k \ , \ \sum_{k=1}^{l} \lambda_k = 1 \quad (21)$$

$$\theta = \langle \theta_1, \cdots, \theta_l \rangle \;\; ; \;\; \theta_k \geq 0 \textit{ for any } k \;\; , \;\; \sum_{k=1}^{l} \theta_k = 1 \tag{22}$$

$$\xi = \langle \xi_1, \cdots, \xi_l \rangle \;\; ; \;\; \xi_k \geq 0 \textit{ for any } k \;\; , \;\; \sum_{k=1}^{l} \xi_k = 1 \tag{23}$$

$$\eta = \langle \eta_1, \cdots, \eta_l \rangle \;\; ; \;\; \eta_k \geq 0 \textit{ for any } k \;\; , \;\; \sum_{k=1}^{l} \eta_k = 1 \; . \tag{24}$$

Definition 3. When l motives $m_1, \cdots, m_l$ are considered in a non-cooperative game $G = [A, B]$, a subjective game $G^{\mathrm{I}} = [A^{\mathrm{I}}, B^{\mathrm{I}}]$ for player I is defined by

$$A^{\mathrm{I}} = \left[\sum_{k=1}^{l} \lambda_k f_{ij}^k \right] \text{ and } B^{\mathrm{I}} = \left[\sum_{k=1}^{l} \theta_k g_{ij}^k \right] . \tag{25}$$

Similarly a subjective game $G^{\mathrm{II}} = [A^{\mathrm{II}}, B^{\mathrm{II}}]$ for player II is defined by

$$A^{\mathrm{II}} = \left[\sum_{k=1}^{l} \xi_k f_{ij}^k \right] \text{ and } B^{\mathrm{II}} = \left[\sum_{k=1}^{l} \eta_k g_{ij}^k \right] . \tag{26}$$

Moreover we call the Nash equilibrium strategy for player I (II) in the game $G^{\mathrm{I}}(G^{\mathrm{II}})$ by the subjective equilibrium strategy (SES) for player I (II).

Each player dose not know the subject game for the opponent since he cannot know weight vectors for both players which the opponent guesses. Namely each player faces his own subjective game individually, and it is optimal for him to select a strategy, which is most desirable for his own subjective game. If we consider that the public payoff matrixes A and B are the final results after deliberating various motives, it seems that it is not necessary to propose a subjective game. But it is not so since it is impossible to know completely the final payoff matrix considering all motives of the opponent. Even if the public payoff matrixes A and B are final, in the actual game, except guessing there is no method of knowing the motive of the opponent for selecting a strategy. Really the results of many experiments dose not necessarily show that a player aims at only maximizing his own expected payoff and selects the Nash equilibrium strategy (NES). Then it is necessary to consider a subjective game, and if so, it is possible to explain behaviors of a player rationally.

When player I faces his subjective game G^{I}, he considers as followings : All speculations of both players are incorporated in the game G^{I} and therefore each player will move by motive m_1 only (namely, maximizing of his own subjective expected payoff). Then it is most desirable for each player to take his subjective equilibrium strategy (SES).

In the game G^{I}, when player I and II select mixed strategies x and y respectively, the expected payoff of each player is given by

$$M_1^{\mathrm{I}}(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{k=1}^{l}\lambda_k f_{ij}^k\right)x_i y_j \tag{27}$$

and

$$M_2^{\mathrm{I}}(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{k=1}^{l}\theta_k g_{ij}^k\right)x_i y_j\,. \tag{28}$$

Then the NEP $(x^{\mathrm{I}},y^{\mathrm{I}})$ for G^{I} is given by

$$\begin{cases} M_1^{\mathrm{I}}(x^{\mathrm{I}},y^{\mathrm{I}})=\max\limits_x M_1^{\mathrm{I}}(x,y^{\mathrm{I}}) & (29)\\ M_2^{\mathrm{II}}(x^{\mathrm{I}},y^{\mathrm{I}})=\max\limits_y M_2^{\mathrm{I}}(x^{\mathrm{I}},y). & (30)\end{cases}$$

The strategy x^{I} is the SES for player I .

Similarly the expected payoff of each player in the subjective game G^{II} for player II is given by

$$M_1^{\mathrm{II}}(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{k=1}^{l}\xi_k f_{ij}^k\right)x_i y_j \tag{31}$$

and

$$M_2^{\mathrm{II}}(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{k=1}^{l}\eta_k g_{ij}^k\right)x_i y_j\,. \tag{32}$$

Then the NEP $(x^{\mathrm{II}},y^{\mathrm{II}})$ for G^{II} is given by

$$\begin{cases} M_1^{\mathrm{II}}(x^{\mathrm{II}},y^{\mathrm{II}})=\max\limits_x M_1^{\mathrm{II}}(x,y^{\mathrm{II}}) & (33)\\ M_2^{\mathrm{II}}(x^{\mathrm{II}},y^{\mathrm{II}})=\max\limits_y M_2^{\mathrm{II}}(x^{\mathrm{II}},y). & (34)\end{cases}$$

The strategy y^{II} is the SES for player II .

When player I and II select the SESs x^{I} and x^{II} respectively, the expected payoffs which both players expect to receive are $M_1^{\mathrm{I}}(x^{\mathrm{I}},y^{\mathrm{I}})$ and $M_2^{\mathrm{II}}(x^{\mathrm{II}},y^{\mathrm{II}})$, but the expected payoffs which both players can receive really are $M_1(x^{\mathrm{I}},y^{\mathrm{II}})$ and $M_2(x^{\mathrm{I}},y^{\mathrm{II}})$.

Remark 1. (i) Since each player plays his own subjective game individually, there is no direct relation among the SESs x^{I} and y^{II} of both players.

(ii) If both players move by the motive m_1 (m_2) only and know it, the SES for each player is equal to the NES (TES) for the original game G. Specially if both players move by the motives m_1 and m_2 only in a zero-sum game, the SES is equal to the optimal strategy for the original game G.

3. NUMERICAL EXAMPLES

In this section we consider two numerical examples.

Example 1. We consider a non-cooperative game as follows :

$$G_3: \quad \mathrm{I} \quad \begin{array}{c} \\ \alpha_1 \\ \alpha_2 \end{array} \begin{array}{c} \mathrm{II} \\ \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{bmatrix} (10,2) & (1,9) \\ (0,10) & (8,1) \end{bmatrix} \end{array}. \tag{35}$$

This game G_3 has a unique NEP $(\langle 9/16, 7/16\rangle, \langle 7/17, 10/17\rangle)$ and a unique TEP $(\langle 8/17, 9/17\rangle, \langle 1/2, 1/2\rangle)$. Then this game is not an a.s.c. game. We consider the following two motives only:

m_1 : maximization of my own expected payoff

m_2 : minimization of the expected payoff of the opponent.

Then

$$f^1 = \begin{bmatrix} 10 & 1 \\ 0 & 8 \end{bmatrix}, \qquad g^1 = \begin{bmatrix} 2 & 9 \\ 10 & 1 \end{bmatrix} \tag{36}$$

$$f^2 = \begin{bmatrix} -2 & -9 \\ -10 & -1 \end{bmatrix}, \qquad g^2 = \begin{bmatrix} -10 & -1 \\ 0 & -8 \end{bmatrix}. \tag{37}$$

We put

$$\begin{array}{lcl} \lambda_1 = \lambda, \ \lambda_2 = 1-\lambda & ; & \theta_1 = \theta, \ \theta_2 = 1-\theta \\ \xi_1 = \xi, \ \xi_2 = 1-\xi & ; & \eta_1 = \eta, \ \eta_2 = 1-\eta. \end{array} \tag{38}$$

Then the subjective game $G^{\mathrm{I}} = [A^{\mathrm{I}}, B^{\mathrm{II}}]$ for player I is given by

$$A^{\mathrm{I}} = \begin{bmatrix} 12\lambda - 2 & 10\lambda - 9 \\ 10\lambda - 10 & 9\lambda - 1 \end{bmatrix}, \qquad B^{\mathrm{I}} = \begin{bmatrix} 12\theta - 10 & 10\theta - 1 \\ 10\theta & 9\theta - 8 \end{bmatrix}. \tag{39}$$

The game G^{I} has a unique NEP

$$\left(\left\langle \frac{8+\theta}{17-\theta}, \frac{9-2\theta}{17-\theta} \right\rangle, \left\langle \frac{8-\lambda}{16+\lambda}, \frac{8+2\lambda}{16+\lambda} \right\rangle \right), \tag{40}$$

and then the SES for player I is

$$x^{\mathrm{I}} = \left\langle \frac{8+\theta}{17-\theta}, \frac{9-2\theta}{17-\theta} \right\rangle. \tag{41}$$

On the other hand, the subjective game $G^{\mathrm{II}} = \left[A^{\mathrm{II}}, B^{\mathrm{II}}\right]$ for player II is given by

$$A^{\mathrm{II}} = \begin{bmatrix} 12\xi - 2 & 10\xi - 9 \\ 10\xi - 10 & 9\xi - 1 \end{bmatrix}, \quad B^{\mathrm{II}} = \begin{bmatrix} 12\eta - 10 & 10\eta - 1 \\ 10\eta & 9\eta - 8 \end{bmatrix} \tag{42}$$

which can be given by replacing λ and θ with ξ and η respectively in G^{I}.

Then the SES for player II is

$$y^{\mathrm{II}} = \left\langle \frac{8-\xi}{16+\xi}, \frac{8+2\xi}{16+\xi} \right\rangle. \tag{43}$$

The characteristics of the SES for each player are as follows:

(i)The SES for each player depends on not only the weight of his own motive but also one of the opponent's motive.

(ii)If the opponent attaches greater importance to the motive m_1 (m_2), then each player must select a similar strategy to the NES (TES) for the original game G_3, namely, he must also attach greater importance to the same motive.

Example 2. We consider the game G_1 in Experiment 1 in Section 1. As motives of selecting strategies, we consider the following two motives:

m_1 : maximization of my own expected payoff

m_5 : maximization of a probability of not losing.

Then

$$f^1 = \begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}, \quad g^1 = \begin{bmatrix} 4 & 3 \\ 1 & 0 \end{bmatrix} \tag{44}$$

$$f^5 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad g^5 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}. \tag{45}$$

We put

$$\begin{aligned} \lambda_1 = \lambda,\ \lambda_5 = 1-\lambda \quad &: \quad \theta_1 = \theta,\ \theta_5 = 1-\theta \\ \xi_1 = \xi,\ \xi_5 = 1-\xi \quad &: \quad \eta_1 = \eta,\ \eta_5 = 1-\eta. \end{aligned} \tag{46}$$

The subjective game $G^{\mathrm{I}} = \left[A^{\mathrm{I}}, B^{\mathrm{I}}\right]$ for player I is given by

$$A^{\mathrm{I}} = \begin{bmatrix} 3\lambda+1 & \lambda \\ 2\lambda+1 & 1-\lambda \end{bmatrix}, \quad B^{\mathrm{I}} = \begin{bmatrix} 3\theta+1 & 2\theta+1 \\ \theta & 1-\theta \end{bmatrix}. \tag{47}$$

The NEP of the game G^{I} is as follows: If $\lambda < 1/2$ and $\theta < 1/2$, then the NEPs are (α_1, β_1), (α_2, β_2) and

$$(\tilde{x}, \tilde{y}) = \left(\left\langle \frac{1-2\theta}{1-\theta}, \frac{\theta}{1-\theta} \right\rangle, \left\langle \frac{1-2\lambda}{1-\lambda}, \frac{\lambda}{1-\lambda} \right\rangle \right). \tag{48}$$

Otherwise the NEP is (α_1, β_1). Therefore the SES x^{I} for player I is as follows:

$$x^{\mathrm{I}}=\begin{cases}\alpha_1,\alpha_2,\tilde{x} & \text{if } \lambda<1/2,\ \theta<1/2 \\ \alpha_1 & \text{otherwise.}\end{cases} \tag{49}$$

Since the game G_1 is symmetric, the SES y^{II} for player II can be obtained by replacing λ, θ with ξ and η respectively in one of player I. Then y^{II} is given as follows:

$$y^{\mathrm{II}}=\begin{cases}\alpha_1,\alpha_2,\hat{y} & \text{if } \xi<1/2,\ \eta<1/2 \\ \alpha_1 & \text{otherwise.}\end{cases} \tag{50}$$

where $\hat{y}=\langle(1-2\xi)/(1-\xi),\xi/(1-\xi)\rangle$.

If player I selects the mixed strategy $\tilde{x}$ in the case that $\lambda<1/2$ and $\theta<1/2$. his SES x^{I} has the following characteristics:

(i) If player I attaches greater importance to the motive m_1 ($\lambda>1/2$), then he must select the pure strategy α_1 (the NES for the original game G_1) beyond a doubt.

(ii) If the opponent (II) attaches greater importance to the motive m_1 ($\theta>1/2$), then player I must also select α_1 since the opponent is very likely to select β_1.

(iii) If the opponent is noncommittal ($\theta<1/2$ and θ is near to $1/2$), then player I must select α_2 (which is not the NES) with a higher probability.

We suppose that player I considers that the distribution of the weights λ and θ are given by $\delta(\lambda)$ and $\mu(\theta)$ respectively. Then the probability that the SES x^{I} selects the strategy α_2 is given by

$$P\{x^{\mathrm{I}}\ \textit{selects}\ \alpha_2\}=\int_0^{1/2}\delta(\lambda)d\lambda\int_0^{1/2}\frac{\theta}{1-\theta}\mu(\theta)d\theta. \tag{51}$$

If we assume

$$\delta(x)=\mu(x)=\begin{cases}7.46x & 0\le x\le 0.5 \\ 55.53-103.6x & 0.5\le x\le 0.536 \\ 0 & 0.536\le x\le 1\end{cases} \tag{52}$$

then

$$P\{x^{\mathrm{I}}\ \textit{selects}\ \alpha_2\}=\int_0^{1/2}7.46\lambda d\lambda\int_0^{1/2}\frac{\theta}{1-\theta}\times 7.46\theta d\theta \fallingdotseq 0.473$$

which is nearly equal to the result (0.47) of the experiment by Minas et.al. Then we can consider that if a person faces the game G_1, he seems to consider that the distributions the behaviors of players rationally.

4. THE PRISONER'S DILEMMA GAME

In this section we consider the general prisoner's dilemma game as follows:

$$G_2: \qquad \begin{array}{cc} & \text{II} \\ & \begin{array}{cc} C & D \end{array} \\ \text{I} \; \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} (b,b) & (d,a) \\ (a,d) & (c,c) \end{bmatrix} \end{array} \tag{53}$$

where $a > b > c > d$ and $2b > a + d$. The symbols C and D denote a cooperation strategy and a defection strategy respectively. Since the game G_2 has a unique NEP (D,D) which is a unique TEP also, this game is an a.s.c. game and both players have the same optimal strategy D (defection). But it has been reported that in the real game each player has not necessarily take the optimal strategy D. We shall propose a model explaining such a move of a player rationally.

We consider the following two motives:

m_1 :maximization of my own expected payoff.

m_8 :maximization of the payoff of mutual prosperity.

Then

$$f^1 = \begin{bmatrix} b & d \\ a & c \end{bmatrix}, \quad g^1 = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$
$$f^8 = g^8 = \begin{bmatrix} 2b & 0 \\ 0 & 0 \end{bmatrix}. \tag{54}$$

We put

$$\lambda_1 = \lambda, \lambda_8 = 1 - \lambda \; ; \; \theta_1 = \theta, \theta_8 = 1 - \theta$$
$$\xi_1 = \xi, \xi_8 = 1 - \xi \; : \; \eta_1 = \eta, \eta_8 = 1 - \eta. \tag{55}$$

The subjective game $G^{\mathrm{I}}=[A^{\mathrm{I}}\ ,\ B^{\mathrm{I}}]$ for player I is given by

$$A^{\mathrm{I}}=\begin{bmatrix}(2-\lambda)b & \lambda d\\ \lambda a & \lambda c\end{bmatrix}\ ,\ B^{\mathrm{I}}=\begin{bmatrix}(2-\theta)b & \theta a\\ \theta d & \theta c\end{bmatrix}. \tag{56}$$

The SES $\boldsymbol{x}^{\mathrm{I}}$ for player I is as follows:

$$x^{\mathrm{I}}=\begin{cases}\alpha_1,\alpha_2,\langle\hat{x}(\theta)\ ,\ 1-\hat{x}(\theta)\rangle & \text{if}\ \ \lambda<\dfrac{2b}{a+b}\ \ \text{and}\ \ \theta<\dfrac{2b}{a+b}\\ \alpha_2 & \text{otherwise}\end{cases} \tag{57}$$

where

$$\hat{x}(\theta)=(c-d)\theta/\{2b-(a+b-c+d)\theta\}. \tag{58}$$

By the symmetry of this game the SES y^{II} for player II can be obtained by replacing λ , θ with ξ and η respectively, that is,

$$y^{\mathrm{II}}=\begin{cases}\alpha_1,\alpha_2,\langle\hat{y}(\xi)\ ,\ 1-\hat{y}(\xi)\rangle & \text{if}\ \ \xi<\dfrac{2b}{a+b}\ \ \text{and}\ \ \eta<\dfrac{2b}{a+b}\\ \alpha_2 & \text{otherwise}\end{cases} \tag{59}$$

where

$$\hat{y}(\xi)=(c-d)\xi/\{2b-(a+b-c+d)\xi\}. \tag{60}$$

When player I selects the mixed strategy $\langle\hat{x}(\theta)\ ,\ 1-\hat{x}(\theta)\rangle$ in the case that $\lambda<2b/(a+b)$ and $\theta<2b/(a+b)$, in Figure 1 we show the probability $p(\theta)$ that the SES $\boldsymbol{x}^{\mathrm{I}}$ selects α_1 (which is not the NES for the original game G_2).(Fig.2)

$$p(\theta)=\begin{cases}\hat{x}(\theta) & \text{if}\ \ 0\le\theta\le\dfrac{2b}{a+b}\\ 0 & \text{if}\ \ \dfrac{2b}{a+b}\le\theta\le 1\end{cases} \tag{61}$$

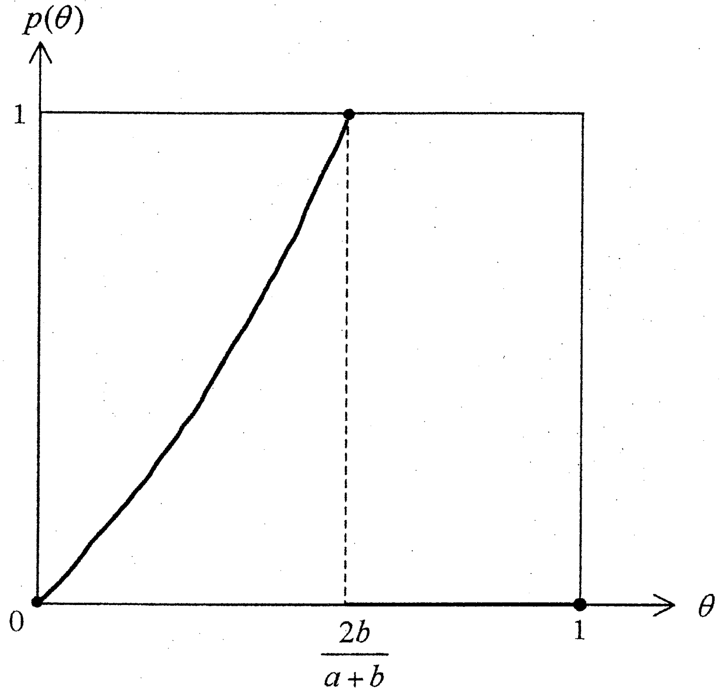

Fig.1 The probability $p(\theta)$ that SES for player I selects the cooperation strategy α_1.

The SES x^{I} for player I has the following characteristics:

(i) If the opponent(II) is egoistic $(\theta > 2b/(a+b))$, then player I must select the defection strategy α_2 since player II also seems to take the defection strategy β_2.

(ii) If the opponent has a larger intention to mutual prosperity $(\theta < 2b/(a+b))$, then the larger the degree of the intention becomes, the higher the probability for player I of selecting the defection strategy α_2 becomes, and as a result he can take the expected payoff by outwitting.

(iii) If the opponent is noncommittal $(\theta < 2b/(a+b)$ and θ is near to $2b/(a+b))$, then player I must select the cooperation strategy α_1 with a higher probability to show his sincerity.

We consider the prisoner's dilemma game in Experiment 2 in Section 1 which is the case that $a=5, b=3, c=1$ and $d=0$ in (53). In this case the probability that the SES x^{I} for player I selects the cooperation strategy α_1 is given by

$$p(\theta)=\begin{cases}\dfrac{\theta}{6-7\theta} & \text{if} \quad 0\le\theta\le\dfrac{3}{4}\\[2ex] 0 & \text{if} \quad \dfrac{3}{4}\le\theta\le 1.\end{cases} \tag{62}$$

We divide the interval $[0,1]$ into the following three subintervals:

$$I_1=\left[0\ ,\ \frac{2}{3}\right],\ I_2=\left[\frac{2}{3}\ ,\ \frac{3}{4}\right],\ \text{and}\ I_3=\left[\frac{3}{4}\ ,\ 1\right]. \tag{63}$$

For any $\theta\in I_1$, $p(\theta)\le 1/2$ and then the subinterval I_1 is called by the relative defection region in which the probability of defection is higher than the probability of cooperation. For any $\theta\in I_2$, $p(\theta)\ge 1/2$ and then I_2 is called by the relative cooperation region in which the probability of cooperation is higher. For any $\theta\in I_3$, $p(\theta)=0$ and then I_3 is called by the perfect defection region in which player I selects the defection strategy with probability one.

We think that for many players the egoistic tendency, if anything, is slightly larger than the tendency to mutual prosperity and that there is no person with perfect egoism. Then we suppose that the probability density function $\mu(\theta)$ of the weight θ is given by

$$\mu(\theta)=\begin{cases}\dfrac{4}{3}(8\theta-3) & \text{if} \quad \dfrac{3}{8}\le\theta\le\dfrac{3}{4}\\[2ex] 4(7-8\theta) & \text{if} \quad \dfrac{3}{4}\le\theta\le\dfrac{7}{8}\\[2ex] 0 & \text{otherwise}.\end{cases} \tag{64}$$

In addition, we assume that the weight λ distributes only the interval [0,3/4].
In this case the probability that the SES x^{I} selects the cooperation strategy α_1 is

$$P\{x^{\mathrm{I}}\ \ select\ \ \alpha_1\}=\int_{3/8}^{3/4}\frac{\theta}{6-7\theta}\cdot\frac{4}{3}(8\theta-3)d\theta\ \fallingdotseq\ 0.350 \tag{65}$$

which is nearly equal to the result (36 per cent) of Minas et al. [2]. Then we can consider that if a person faces this game, he thinks the weight of egoistic tendency of the opponent is a random variable with the above probability density function(64). Thus we can explain a behavior of a player in the prisoner's dilemma game rationally by introducing a subjective game for each player. (It cannot be explained by the NES or the TES only.)

Next we try another approach to the prisoner's dilemma game G_2 given by (9). We consider the following two motions:

m_1 :maximization of my own expected payoff.

m_7 :maximization of the total social payoff.

Then

$$f^1 = A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}, \quad g^1 = B = \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix} \tag{66}$$

$$f^7 = g^7 = \begin{bmatrix} 6 & 5 \\ 5 & 2 \end{bmatrix}. \tag{67}$$

We put

$$\begin{aligned} &\lambda_1 = \lambda, \lambda_7 = 1-\lambda \ ; \ \theta_1 = \theta, \theta_7 = 1-\theta \\ &\xi_1 = \xi, \xi_7 = 1-\xi \ ; \ \eta_1 = \eta, \eta_7 = 1-\eta \end{aligned} \tag{68}$$

The subjective game $G^{\mathrm{I}} = [A^{\mathrm{I}}, B^{\mathrm{I}}]$ for player I is given by

$$A^{\mathrm{I}} = \begin{bmatrix} 6-3\lambda & 5-5\lambda \\ 5 & 2-\lambda \end{bmatrix}, \quad B^{\mathrm{I}} = \begin{bmatrix} 6-3\theta & 5 \\ 5-5\theta & 2-\theta \end{bmatrix}. \tag{69}$$

The SES x^{I} for player I is as follows: For three region regions R_1, R_2 and R_3 in Figure 2

$$x^{\mathrm{I}} = \begin{cases} \alpha_1 & \text{if} \quad (\lambda,\theta) \in R_1 \\ \alpha_2 & \text{if} \quad (\lambda,\theta) \in R_2 \\ \alpha_1, \alpha_2 \ \textit{and} \ \langle \bar{x}, \ 1-\bar{x} \rangle & \text{if} \quad (\lambda,\theta) \in R_3 \end{cases} \tag{70}$$

where $\bar{x}$ is the probability that player I selects the cooperation strategy α_1 and is given by $\bar{x} = (3-4\theta)/(2-\theta)$. (Fig.2)

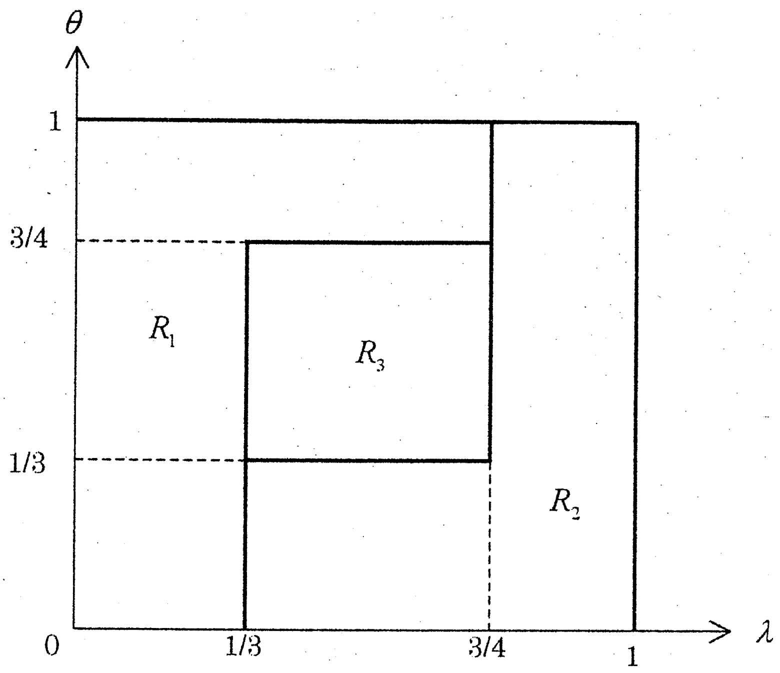

Fig.2 The decision regions of the SES for player I

By the symmetry of this game the SES x^{II} for player II can be taken by replacing λ, θ with ξ and η respectively.

Namely

$$x^{\mathrm{II}} = \begin{cases} \alpha_1 & \text{if} \quad (\xi, \eta) \in R_1 \\ \alpha_2 & \text{if} \quad (\xi, \eta) \in R_2 \\ \alpha_1, \alpha_2 \ \ \textit{and} \ \ \langle \bar{y}, \ 1-\bar{y} \rangle & \text{if} \quad (\xi, \eta) \in R_3 \end{cases} \tag{71}$$

where $\bar{y} = (3-4\xi)/(2-\xi)$.

The characteristics of the SES are as follows:

(i) If player I attaches greater importance to the motives m_1 (m_7) then he must select the defection strategy α_2 (the cooperation strategy α_1).

(ii) When player I attaches the nearly equal weight to the motives m_1 and m_7 $(1/3 < \lambda < 3/4)$, if the opponent attaches greater importance to the motive m_1 (m_7), then player I selects the strategy α_1 (α_2) and is beaten on purpose(beats his opponent).

(iii) If both players attach the nearly equal weight to the motives m_1 and m_7, then player I must select one of three strategies α_1, α_2 and $\langle \bar{x}, 1-\bar{x} \rangle$.

In this model, since one of motives is to maximize the total social payoff, a player becomes happy even if the payoff of the opponent increases, and therefore the SES may indicate to be beaten on purpose. But it is unusual to assume such a motive in the prisoner's dilemma game.

5. DISCUSSION

In this paper we have introduced a subjective game for each player in a non-cooperative game and tried to explain behaviors of players rationally. That is to say, by taking various motives into account we have constructed a subjective payoff matrix and let the NES of the subjective game for each player be a rational selection for him. Moreover we have proposed some models to explain the results of experiments rationally.

We can extend our results to the case of n persons.

REFERENCES

1. R. J. Aumann (1961), Almost Strictly Competitive Games, J. Soc. Indust. Appl. Math., Vol.9, pp.544-550.

2. J. S. Minas, A. Scodel, D. Marlowe and H. Rawson (1960), Some Descriptive Aspects of Two-Person Non-Zero-Sum Games II, J. of Conflict Resolution, Vol.4, pp.193-197.

3. J. F. Nash (1951), Non-Cooperative Games, Annals of Mathematics, Vol.54, pp.286-295.