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Boson-Fermion System

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1 Introduction and Main Results

As boson-fermion systems, we treat with the *generalized spin-boson model* proposed by Arai and the author in [AH1].

We consider mainly the following problems:

I We characterize the existence or absence of ground states of the generalized spin-boson model in terms of the ground state energy and correlation functions. It is one of the purposes to generalize Spohn's criterion by methods of functional analysis, and clarify the mathematical structure causing the existence or absence of the generalized model.

II We give expressions for the ground state energy of the standard spin-boson model with infrared cutoff, and *without infrared cutoff.*

III We investigate spectral properties of the Wigner-Weisskopf model.

Problem I is argued in [AHH, AH2], so see them. In this contribution, we consider Problems II and III. The proofs of all statements in this contribution appears in [mHi3].

The spin-boson model describes a two-level system coupled to a quantized Bose field. For the ground state energy of this model, we know several approximate expression by, for instance, [EG, Ts]. Recently the author gave an explicit one in the way of [mHi2, Theorems 1.3 and 1.4, the first equalities in Theorem 1.6 (i) and (ii)], still he proved it in the case with infrared cutoff. In this paper, we shall give new upper bounds for the ground state energy of the spin-boson model *without infrared cutoff*, and using it we shall express the ground state energy with a parameter in the way of [mHi2, (1.19) in Theorem 1.5, the second equalities in Theorem 1.6 (i) and (ii)], and argue how an effect by the spin appears in the ground state energy *without infrared cutoff.*

The Hamiltonian of the spin-boson model is given as follows:

We take a Hilbert space of bosons to be

$$\mathcal{F}_b := \mathcal{F}\left(L^2\left(\mathbf{R}^d\right)\right) \equiv \bigoplus_{n=0}^{\infty}\left[\otimes_s^n L^2\left(\mathbf{R}^d\right)\right] \tag{1.1}$$

($d \in \mathrm{N}$) the symmetric Fock space over $L^2(\mathbf{R}^d)$ ($\otimes_s^n\mathcal{K}$ denotes the n-fold symmetric tensor product of a Hilbert space $\mathcal{K}$, $\otimes_s^0\mathcal{K} \equiv \mathbf{C}$). In this paper, we set both of $\hbar$ and c one, i.e., $\hbar = c = 1$, where $\hbar$ is the Planck constant divided by 2π, and c the velocity of the light.

Let $\omega : \mathbf{R}^d \to [0,\infty)$ be Borel measurable such that $0 \le \omega(k) < \infty$ for all $k \in \mathbf{R}^d$ and $\omega(k) \neq 0$ for almost everywhere (a.e.) $k \in \mathbf{R}^d$ with respect to the d-dimensional Lebesgue measure. We here assume that

$$\inf_{k\in\mathbf{R}^d} \omega(k) = 0 \tag{1.2}$$

because we are interested in the case without infrared cutoff. Let $\hat{\omega}$ be the multiplication operator by the function ω, acting in $L^2(\mathbf{R}^\nu)$. We denote by $d\Gamma(\hat{\omega})$ the second quantization of $\hat{\omega}$ [RS2, §X.7] and set

$$H_b = d\Gamma(\hat{\omega}) = \int_{\mathbf{R}^d} dk\, \omega(k)a(k)^*a(k),$$

where $a(k)$ is the operator-valued distribution kernels of the smeared annihilation operator, so $a(k)^*$ is that of creation operator:

$$a(f) = \int_{\mathbf{R}^d} dk\, a(k)\overline{f(k)}, \qquad a(f)^* = \int_{\mathbf{R}^d} dk\, a(k)^* f(k) \tag{1.3}$$

for every $f \in L^2(\mathbf{R}^d)$ on $\mathcal{F}_b$. Let Ω_0 be the Fock vacuum in $\mathcal{F}_b$: $\Omega_0 := \{1, 0, 0, \cdots\} \in \mathcal{F}_b$.

The Segal field operator $\phi_S(f)$ $(f \in L^2(\mathbf{R}^d))$ is given by

$$\phi_S(f) := \frac{1}{\sqrt{2}}\left(a(f)^* + a(f)\right). \tag{1.4}$$

The inner product (resp. norm) of a Hilbert space $\mathcal{K}$ is denoted $(\cdot,\cdot)_{\mathcal{K}}$, complex linear in the second variable (resp. $\|\cdot\|_{\mathcal{K}}$). For each $s \in \mathbf{R}$, we define a Hilbert space

$$\mathcal{M}_s = \left\{ f : \mathbf{R}^d \to \mathbf{C}, \text{Borel measurable} \mid \omega^{s/2} f \in L^2(\mathbf{R}^\nu) \right\}$$

with inner product $(f,g)_s := (\omega^{s/2} f, \omega^{s/2} g)_{L^2(\mathbf{R}^\nu)}$ and norm $\|f\|_s := \|\omega^{s/2} f\|_{L^2(\mathbf{R}^d)}$, $f \in \mathcal{M}_s$.

We shall assume the following (A.1) to obtain upper bounds for $E_{\mathrm{SB}}(0)$:

(A.1) The function $\lambda(k)$ of $k \in \mathbf{R}^d$ satisfies that $\lambda \in \mathcal{M}_{-1} \cap \mathcal{M}_0$.

We call the following condition the *infrared singularity condition* (see [AH2])

$$\|\lambda\|_{-2} = \infty, \qquad \left(\text{i.e., } \lambda/\omega \notin L^2\left(\mathbf{R}^d\right)\right). \tag{1.5}$$

The Hamiltonian of the spin-boson model is defined by

$$H_{\mathrm{SB}} := \frac{\mu}{2}\sigma_3 \otimes I + I \otimes H_b + \sqrt{2}\alpha\sigma_1 \otimes \phi_S(\lambda) \tag{1.6}$$

acting in the Hilbert space $\mathcal{F} := \mathbf{C}^2 \otimes \mathcal{F}_b$, where $0 < \mu$ is a splitting energy which means the gap of the ground and first excited state energy of uncoupled chiral molecule to a radiation field, $\alpha \in \mathbf{R}$ a coupling constant, and σ_1, σ_3 the standard Pauli matrices,

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

For simplicity, we denote the decoupled free Hamiltonian $(\alpha = 0)$ by H_0:

$$H_0 \ := \ \frac{\mu}{2}\sigma_3 \otimes I + I \otimes H_b. \tag{1.7}$$

For the above H_{SB}, we temporally introduce an infrared cutoff $\nu > 0$ as the *infrared regularity condition*

$$\lambda/\omega_\nu \in L^2(\mathbf{R}^d), \qquad \nu > 0, \tag{1.8}$$

which raise the bottom of the frequency $\omega(k)$ of bosons (see [AH2]):

$$\omega_\nu(k) := \omega(k) + \nu, \qquad H_b(\nu) := d\Gamma(\widehat{\omega}_\nu), \nu > 0, \tag{1.9}$$

$$H_{\mathrm{SB}}(\nu) := \frac{\mu}{2}\sigma_3 \otimes I + I \otimes H_b(\nu) + \sqrt{2}\alpha\sigma_1 \otimes \phi_S(\lambda). \tag{1.10}$$

Of course, we shall remove 'ν' later by taking the limit $\nu \downarrow 0$ such as making the precision better.

For simplicity, we put $H_{\mathrm{SB}}(0) := H_{\mathrm{SB}}$.

For a linear operator T on a Hilbert space, we denote its domain by $D(T)$. It is well-known that $H_{\mathrm{SB}}(\nu)$ is self-adjoint on

$$D\left(H_{\mathrm{SB}}(\nu)\right) = D\left(I \otimes H_b(\nu)\right), \text{and bounded from below for all } \alpha \in \mathbf{R} \tag{1.11}$$

for every $\nu \geq 0$ by [AH1, Proposition 1.1(i)] since σ_1 is bounded now.

For a self-adjoint operator T bounded from below, we denote by $E_0(T)$ the infimum of the spectrum $\sigma(T)$ of T: $E_0(T) = \inf \sigma(T)$. In this paper, when T is a Hamiltonian, we call $E_0(T)$ the *ground state energy* of T *even if* T *has no ground state.*

For $H_{\text{SB}}(\nu)$ $(\nu \geq 0)$ we set $E_{\text{SB}}(\nu) := E_0(H_{\text{SB}}(\nu))$

It is well known that for $\nu > 0$

$$-\frac{\mu}{2} - \alpha^2 \left\| \frac{\lambda}{\sqrt{\omega_\nu}} \right\|_0^2 \leq E_{\text{SB}}(\nu) \leq -\frac{\mu}{2} e^{-2\alpha^2 \|\lambda/\omega_\nu\|_0^2} - \alpha^2 \left\| \frac{\lambda}{\sqrt{\omega_\nu}} \right\|_0^2 \tag{1.12}$$

by easy estimation and the variational principle [Ar, Theorem 2.4]. So we have for every $\nu > 0$

$$E_{\text{SB}}(\nu) = -\frac{\mu}{2} e^{-2\alpha^2 \|\lambda/\omega_\nu\|_0^2 G_\nu} - \alpha^2 \left\| \frac{\lambda}{\sqrt{\omega_\nu}} \right\|_0^2 \tag{1.13}$$

for some $G_\nu \in [0, 1]$. Under a condition we know a concrete expression of G_ν [mHi2, Theorems 1.5 and 1.6]. On the other hand, we can prove that

$$-\frac{\mu}{2} - \alpha^2 \left\| \frac{\lambda}{\sqrt{\omega}} \right\|_0^2 \leq \lim_{\nu \downarrow 0} E_{\text{SB}}(\nu) = E_{\text{SB}}(0) \leq -\alpha^2 \left\| \frac{\lambda}{\sqrt{\omega}} \right\|_0^2 \tag{1.14}$$

even under the infrared singularity condition (1.5) (see [AH2, Proposition 3.2(iii)]), and we have now

$$\lim_{\nu \downarrow 0} \left\| \frac{\lambda}{\omega_\nu} \right\|_0 = \infty, \qquad 0 \leq G_\nu \leq 1, \qquad \nu > 0. \tag{1.15}$$

Then, the problem of expressing the $E_{\text{SB}}(0)$ in the case without infrared cutoff is as follows: Although $\lim_{\nu \downarrow 0} \|\lambda/\omega_\nu\|_0^2 G_\nu$ is apparently infinite (except for the fortunate case $\lim_{\nu \downarrow 0} G_\nu = 0$) and the term of μ is seemingly removed under the limit $\nu \downarrow 0$, we cannot believe $E_{\text{SB}}(0) = -\alpha^2 \|\lambda/\sqrt{\omega}\|_0^2$. So, *how does the term of* μ *from the effect by the spin survive in* $E_{\text{SB}}(0)$? This is what the author would like to consider, so this work is the sequel to his in [mHi2].

Moreover, this work is also the first step for another scheme: Considering the result in [BFS], there is a possibility that the generalized spin-boson (GSB) model [AH1] has a ground state even under the infrared singularity condition. Actually, as we showed it in [AH2, §6.2], a model of a quantum harmonic oscillator coupled to a Bose field with the rotating wave approximation has a ground state, and the Wigner-Weisskopf model [WW] has also a ground state under certain conditions even if we assume the infrared singularity condition [AH2, §6.3].By our recent theory in [AH2], we know that if the right differential $E'_{\text{SB}}(0+)$ of $E_{\text{SB}}(\nu)$ at $\nu = 0$ is less than 1, then we have a ground state of H_{SB} in the standard state space $\mathcal{F}$. It may be worth pointing out, in passing, that Spohn discovered a critical criterion between the existence and absence of a ground state in $\mathcal{F}$ for the spin-boson model [Sp2, Sp3] by a method of the functional integration. Our goal of the scheme is to characterize the existence or absence of ground states of the GSB model in terms of the ground state energy and correlation functions [AHH, AH2] by methods of functional analysis.

The estimation (1.12) is not suitable to check whether $E'_{\text{SB}}(0+) < 1$ or not. Because (1.12) is obtained by regarding $H_{\text{SB}}(\nu)$ as the van Hove model $H_{\text{VH}}(\nu)$ perturbed by bounded operator:

$$U_0^* H_{\text{SB}}(\nu) U_0 = H_{\text{VH}}(\nu) - \frac{\mu}{2}\sigma_1, \tag{1.16}$$

where $H_{\text{VH}}(\nu) = I \otimes H_b(\nu) + \sqrt{2}\alpha\sigma_3 \otimes \phi_s(\lambda)$, $U_0 = (\sigma_0 - i\sigma_2) \otimes I/\sqrt{2}$. And, under the infrared singularity condition (1.5), the right differential of the ground state energy $E_{\text{VH}}(\nu) = -\alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2$ $(\nu \geq 0)$ of the Hamiltonian $H_{\text{VH}}(\nu)$ of the van Hove model is infinite [AH2, §6.1], i.e.,

$$E'_{\text{VH}}(0+) = \left\| \frac{\lambda}{\omega} \right\|_0^2 = \infty.$$

So, we need another estimation which is not influenced by the van Hove model.

We shall show in Theorem 1.1 that the term of μ influenced by the spin remains, moreover, the spin may make $\mu/2$ play a role such as the lower bound of frequency (a mass) of bosons.

In this paper, we give an answer for the first problem above by using the *variational principle.* To do it, we have to assume the following (A.2) in addition to (A.1):

Fix arbitrarily δ with

$$0 < \delta < 1/3. \tag{1.17}$$

(A.2) The splitting energy μ and the coupling constant α satisfy

$$4\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k)} < \mu, \tag{1.18}$$

$$\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\left(\omega(k) + \frac{\mu}{2}\right)^2} < \frac{1-3\delta}{\delta^2} =: \gamma_\delta. \tag{1.19}$$

Theorem 1.1 *(without infrared cutoff) Assume (A.1). For the Hamiltonian H_{SB} of the spin-boson model without infrared cutoff (i.e., even under the infrared singularity condition (1.5)), upper bounds and an equality are given as follows:*

(a) *(upper bound)*

(a-1) $E_{\mathrm{SB}}(0) \le \min\left\{ -\frac{\mu}{2}, \inf_{f \in D(\widehat{\omega})} \frac{2\alpha \Re(f, \lambda)_0 + (f, \omega f)_0}{1 + \|f\|_0^2} \right\},$

(a-2) $E_{\mathrm{SB}}(0) \le -\frac{\mu}{2} + \inf_{f \in D(\widehat{\omega})} \frac{2\alpha \Re(f, \lambda)_0 + (f, \omega f)_0 + \mu \|f\|_0^2}{1 + \|f\|_0^2}.$

(b) *(equality) Let $\mu\alpha \neq 0$. Then, there exists $c_{\mu,\alpha} > \delta$ such that*

$$E_{\mathrm{SB}}(0) = -\frac{\mu}{2} - c_{\mu,\alpha}\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k) + \frac{\mu}{2}}. \tag{1.20}$$

Moreover, assume (A.2) in addition to (A.1). Then,

$$-\frac{\mu}{2} - \alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k)} \le E_{\mathrm{SB}}(0) < -\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k)}, \tag{1.21}$$

and

$$\begin{aligned} \lim_{\nu \downarrow 0} \left\| \frac{\lambda}{\omega_\nu} \right\|_0^2 G_\nu &= -\frac{1}{2\alpha^2} \ln\left\{ 1 + \frac{2\alpha^2}{\mu}(c_{\mu,\alpha} - 1) \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k) + \frac{\mu}{2}} \right. \\ &\qquad \left. -\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k)\left(\omega(k) + \frac{\mu}{2}\right)} \right\} < \infty. \end{aligned} \tag{1.22}$$

Remark 1.1 *By the equality in Theorem 1.1 (b), we know that*

$$E_{\mathrm{SB}}(0) < E_0(H_0). \tag{1.23}$$

So, considering the diamagnetic inequality by Hiroshima [fHi1, Theorem 5.1], (1.23) means that there is a deifference between the spin-boson model and the Pauli-Fierz model as far as concerning the ground state energy though the spin-boson model is regarded as an approximation of the Pauli-Fierz model in physics.

To make comment on a lower bound, we have to assume the following (A.3) at present because of the reason coming Proposition 2.2:

(A.3) $\lambda^{(1)}$, $\lambda^{(1)}/\omega \in L^2(\mathbf{R}^d)$, where

$$\lambda^{(1)}(k) := \frac{\partial}{\partial |k|}\lambda(k) + \frac{(d-1)\lambda(k)}{2|k|}, \qquad k \in \mathbf{R}^d. \tag{1.24}$$

Remark 1.2 *Assuming (A.3) practically amounts to assuming the infrared regularity condition, namely not the infrared singularity condition:*

$$\lambda/\omega \in L^2(\mathbf{R}^d). \tag{1.25}$$

Proposition 1.2 *Let* $\omega(k) = |k|$. *Assume (A.1), (A.3), (1.18) and (1.25). Then, for all* $\alpha \in \mathbf{R}$ *with*

$$\alpha^2 < \frac{1}{12\|\lambda^{(1)}\|_0^2}, \tag{1.26}$$

(a) (lower bound)

$$E_{\text{SB}}(0) > -\frac{\mu}{2} - 2\alpha^2 \int_{\mathbf{R}^d} dk \, \frac{|\lambda(k)|^2}{\omega(k) + \frac{\mu}{2}} \tag{1.27}$$

(b) Assume (1.19) in addition. Then $c_{\mu,\alpha}$ *in Theorem 1.1(b) is given as*

$$c_{\mu,\alpha} \in (\delta\,,\, 2) \tag{1.28}$$

2 Wigner-Weisskopf Model

To prove Theorem 1.1 we use the properties of the Wigner-Weisskopf model [WW, HüS, AH2]. So, in this section, we shall describe fundamental properties of the Wigner-Weisskopf model.

We define a matrix c by

$$c := \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{2.1}$$

And let

$$H_b(0) := H_b, \qquad \omega_0(k) := \omega(k), \qquad k \in \mathbf{R}^d. \tag{2.2}$$

Then, for every $\mu_0 \in \mathbf{R} \setminus \{0\}$ and $\nu \geq 0$, we define Hamiltonian $H_\alpha(\mu_0\,;\,\nu)$ of the Wigner-Weisskopf model by

$$H_\alpha(\mu_0\,;\,\nu) := \mu_0 c^* c \otimes I + I \otimes H_b(\nu) + \alpha\left(c^* \otimes a(\lambda) + c \otimes a(\lambda)^*\right). \tag{2.3}$$

We call $H_\alpha(\mu_0\,;\,\nu)$ the *Wigner-Weisskopf Hamiltonian.* We may put for $\nu = 0$

$$H_\alpha(\mu_0) := H_\alpha(\mu_0\,;\,0). \tag{2.4}$$

Remark 2.1 *The Wigner-Weisskopf model is one of several examples of the generalized spin-boson model. We know it if we put* $B_1 \equiv (c^* + c)/\sqrt{2}$, $B_2 \equiv i(c^* - c)/\sqrt{2}$; $\lambda_1 \equiv \lambda$ *and* $\lambda_2 \equiv i\lambda$. *This model is very simple, but it has an unusual property contrary to our expectation (see Remark 2.4).*

It is easy to prove that $H_\alpha(\mu_0\,;\,\nu)$ is self-adjoint on

$$D\left(H_\alpha(\mu_0\,;\,\nu)\right) = D\left(I \otimes H_b(\nu)\right), \text{and bounded from below} \tag{2.5}$$

for every $\nu \geq 0$ by [AH1, Proposition 1.1(i)] since each B_j is bounded. As we did in [AH2, §6.2], we introduce a function $D^\alpha_{\mu_0,\nu}$ for $\mu_0 \in \mathbf{R} \setminus \{0\}$ and $\nu \geq 0$ by

$$D^\alpha_{\mu_0,\nu}(z) := -z + \mu_0 - \alpha^2 \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega_\nu(k) - z}, \tag{2.6}$$

defined for all $z \in \mathbf{C}$ such that $|\lambda(k)|^2/|z - \omega_\nu(k)|$ is Lebesgue integrable on $\mathbf{R}^d$.

Remark 2.2 *It is well-known that the Wigner-Weisskopf model is the simplified Lee model [Le, KaMu, We] and [Ta, §5.2], and the solution of $D^\alpha_{\mu_0,\nu}(z) = 0$ gives the renormalized mass for the Lee model.*

In particular, as we mentioned it in [AH2, §6.2], $D^\alpha_{\mu_0,0,\nu}(z)$ is defined in the cut plane $\mathbf{C}_\nu := \mathbf{C} \setminus [\nu\,, \infty)$, $\nu \geq 0$, and analytic there. It is easy to see that $D^\alpha_{\mu_0,\nu}(x)$ is monotone decreasing in $x < \nu$. Hence, the limit

$$d^\alpha_\nu(\mu_0) := \lim_{x \uparrow \nu} D^\alpha_{\mu_0,\nu}(x) = -\nu + \mu_0 - \alpha^2 \lim_{t \downarrow 0} \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega_\nu(k) - \nu + t} \tag{2.7}$$

exists. Actually, for a.e. $k \in \mathbf{R}^d$,

$$0 < \frac{|\lambda(k)|^2}{\omega_\nu(k) - \nu + t} < \frac{|\lambda(k)|^2}{\omega(k)}, \ t > 0 \ \text{ and } \ \lim_{t \downarrow 0} \frac{|\lambda(k)|^2}{\omega_\nu(k) - \nu + t} = \frac{|\lambda(k)|^2}{\omega(k)},$$

and we assumed $\lambda/\sqrt{\omega} \in L^2(\mathbf{R}^d)$ in (A.1), moreover set $\omega_\nu(k) := \omega(k) + \nu$ $(\nu > 0,\ k \in \mathbf{R}^d)$. So, by the Lebesgue dominated convergence theorem, we have

$$d^\alpha_\nu(\mu_0) = -\nu + \mu_0 - \alpha^2 \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega(k)}. \tag{2.8}$$

We may put for $\nu = 0$ $D^\alpha_{\mu_0}(z) := D^\alpha_{\mu_0,0}(z)$ and $d^\alpha(\mu_0) := d^\alpha_0(\mu_0)$.

The Wigner-Weisskopf model has a conservation low for a kind of the particle number in the following sense:

We define

$$N^\pm_P := \frac{1 \pm \sigma_3}{2} \otimes I + I \otimes N_b, \tag{2.9}$$

which appeared in [HüS, §6], where N_b is the boson number operator,

$$N_b := d\Gamma(1) = \sum_{\ell=0} \ell P^{(\ell)}. \tag{2.10}$$

Here (2.10) is the spectral resolution of N_b, and $P^{(\ell)}$ is the orthogonal projection onto ℓ-particle space in $\mathcal{F}_b$ for each $\ell \in \{0\} \cup \mathbf{N}$. The spectral resolution of $N^\pm_P$ is given as

$$N^\pm_P = \sum_{\ell=0} \ell P^\pm_\ell, \tag{2.11}$$

where

$$P^\pm_\ell = \begin{cases} \frac{1 \mp \sigma_3}{2} \otimes P^{(0)} & \text{if } \ell = 0, \\ \\ \frac{1 \pm \sigma_3}{2} \otimes P^{(\ell-1)} + \frac{1 \mp \sigma_3}{2} \otimes P^{(\ell)} & \text{if } \ell \in \mathbf{N}. \end{cases} \tag{2.12}$$

$H_\alpha(\mu_0; \nu)$ is reduced by $P^\pm_\ell \mathcal{F}$ for every $\alpha \in \mathbf{R}$ and each $\ell \in \{0\} \cup \mathbf{N}$. So, for every $\alpha \in \mathbf{R}$, $H_\alpha(\mu_0; \nu)$ is decomposed to the direct sum of $H_{\ell,\alpha}(\mu_0; \nu)$'s as

$$H_\alpha(\mu_0; \nu) = \bigoplus_{\ell=0}^{\infty} H_{\ell,\alpha}(\mu_0; \nu), \tag{2.13}$$

where $H_{\ell,\alpha}(\mu_0; \nu)$ is self-adjoint on the closed subspace $\mathcal{F}_\ell$ defined by

$$\mathcal{F}_\ell := P_\ell \mathcal{F} \tag{2.14}$$

for each $\ell \in \{0\} \cup \mathbf{N}$ and

$$\mathcal{F} = \bigoplus_{\ell=0}^{\infty} \mathcal{F}_\ell. \tag{2.15}$$

The proof of the above statement is that, for instance, we have only to extend [Ka, Problem 3.29] to its infinite version by repeating [Ka, Problem 3.29] with the closedness of $H_\alpha(\mu_0;\nu)$.

We call $\mathcal{F}_\ell$ the *ℓ sector.*

We define vector $\Omega^0 \in \mathcal{F}_0$ by

$$\Omega^0 \;:=\; \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \Omega_0. \tag{2.16}$$

For every $f \in D(\widehat{\omega})$, we define vector $\Omega^1(f) \in \mathcal{F}_1$ by

$$\Omega^1(f) := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \Omega_0 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes a(f)^*\Omega_0. \tag{2.17}$$

When a zero $E^\alpha_{\mu_0,\nu}$ of $D^\alpha_{\mu_0,\nu}(z)$ exists, we define a function by

$$g^\alpha_{\mu_0,\nu}(k) := -\alpha\frac{\lambda(k)}{\omega_\nu(k) - E^\alpha_{\mu_0,\nu}} \in D(\widehat{\omega}_\nu), \qquad k \in \mathbf{R}^d. \tag{2.18}$$

Especially, we may for $\nu = 0$ $g^\alpha_{\mu_0} := g^\alpha_{\mu_0,0}$ and $E^\alpha_{\mu_0} := E^\alpha_{\mu_0,0}$.

For a self-adjoint operator T, we denote the set of all essential spectra of T by $\sigma_{ess}(T)$, and pure point spectra by $\sigma_{pp}(T)$.

By the definition (2.3) of the Hamiltonian $H_\alpha(\mu_0;\nu)$, the free Hamiltonian of the Wigner-Weisskopf model is $H_0(\mu_0;\nu)$ for every $\mu_0 \in \mathbf{R}$ and $\nu \geq 0$. Then, it is clear that

$$\sigma_{pp}\left(H_0\left(\mu_0;\nu\right)\right) = \left\{0\,,\,\mu_0\right\}, \tag{2.19}$$

$$\sigma_{ess}\left(H_0\left(\mu_0;\nu\right)\right) = \left[\min\left\{0\,,\,\mu_0\right\}\,,\infty\right), \tag{2.20}$$

$$0 \text{ and } \mu_0 \text{ are simple,} \tag{2.21}$$

$$\text{the unique eigenvector of } 0 \text{ is } \Omega^0_+ \in \mathcal{F}_0, \tag{2.22}$$

$$\text{and}\quad \text{the unique eigenvector of } \mu_0 \text{ is } \Omega^1_+(0) \in \mathcal{F}_1. \tag{2.23}$$

The following theorem follows from [AH2, Proposition 6.13, Theorems 6.14 and 6.15]. We note here that the proof of [AH2, Theorem 6.15] had already proved part (c) below:

Theorem 2.1 **(a)** *Let $\nu, d^\alpha_\nu(\mu_0) \geq 0$. Then,*

$$0 \in \sigma_{pp}\left(H_\alpha\left(\mu_0\,;\,\nu\right)\right), \tag{2.24}$$

$$\sigma_{ess}\left(H_\alpha\left(\mu_0\,;\,\nu\right)\right) = \left[\nu\,,\,\infty\right). \tag{2.25}$$

In particular, 0 is the ground state energy of $H_\alpha\left(\mu_0\,;\,\nu\right)$ with its unique ground state Ω^0_+.

(b) *Let $d^\alpha_\nu(\mu_0) < 0 < \nu$ and $\alpha^2\|\lambda/\sqrt{\omega_\nu}\|^2_0 \leq \mu_0$. Then,*

$$\left\{0\,,\,E^\alpha_{\mu_0,\nu}\right\} \subset \sigma_{pp}\left(H_\alpha\left(\mu_0\,;\,\nu\right)\right), \tag{2.26}$$

$$\sigma_{ess}\left(H_\alpha\left(\mu_0\,;\,\nu\right)\right) = \left[\nu\,,\,\infty\right), \tag{2.27}$$

with $0 \leq E^\alpha_{\mu_0,\nu} < \nu$. In particular, 0 is the ground state energy of $H_\alpha(\mu_0\,;\,\nu)$. Moreover,

$$0 < E^\alpha_{\mu_0,\nu};\ 0 \textit{ is simple, and } \Omega^0_+ \textit{ is the unique ground state of } H_\alpha(\mu_0\,;\,\nu) \quad \textit{if } \alpha^2\|\lambda/\sqrt{\omega_\nu}\|^2_0 < \mu_0, \tag{2.28}$$

$$0 = E^\alpha_{\mu_0,\nu},\ \textit{and } \Omega^0_+ \textit{ and } \Omega^1_+(g^\alpha_{\mu_0,\nu}) \textit{ are the degenerate ground states of } H_\alpha(\mu_0\,;\,\nu) \quad \textit{if } \alpha^2\|\lambda/\sqrt{\omega_\nu}\|^2_0 = \mu_0. \tag{2.29}$$

(c) *Let $d_\nu^\alpha(\mu_0) < 0 < \nu$ and $\mu_0 < \alpha^2\|\lambda/\sqrt{\omega_\nu}\|_0^2$. Suppose that*

$$2\nu - \mu_0 > \alpha^2 \left(\left\| \frac{\lambda}{\sqrt{\omega_\nu}} \right\|_0^2 - M(\alpha, \mu_0, \omega_\nu) \right) + \frac{\|\lambda\|_0^2}{M(\alpha, \mu_0, \omega_\nu)}, \tag{2.30}$$

where

$$M(\alpha, \mu_0, \omega_\nu) := \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega_\nu(k) - \mu_0 + \alpha^2\|\lambda/\sqrt{\omega_\nu}\|_0^2}. \tag{2.31}$$

Then,

$$\{E_{\mu_0,\nu}^\alpha, 0\} \subset \sigma_{pp}(H_\alpha(\mu_0; \nu)), \tag{2.32}$$

$$\sigma_{ess}(H_\alpha(\mu_0; \nu)) = [E_{\mu_0,\nu}^\alpha + \nu, \infty), \tag{2.33}$$

with $E_{\mu_0,\nu}^\alpha < 0$. In particular, $E_{\mu_0,\nu}^\alpha$ is the ground state energy of $H_\alpha(\mu_0; \nu)$ with its ground state $\Omega_+^1(g_{\mu_0,\nu}^\alpha)$.

Remark 2.3 *We are also interested in the case for large absolute value of the coupling constant(i.e., $|\alpha| \gg 1$). Fix μ_0 and make $|\alpha|$ so large. Then, we have $d_\nu^\alpha(\mu_0) < 0$. Thus, we have to investigate the case for $d_\nu^\alpha(\mu_0) < 0$ to know the case for large $|\alpha|$. See Theorem 2.5 below.*

Remark 2.4 *In $[\nu, \infty)$, we can make a different eigenvalue from $E_{\mu_0,\nu}^\alpha$ and 0 by adding some conditions to $\omega(k)$ and $\lambda(k)$ as we mentioned it in [AH2, Remark 6.4]. Namely, as an effect of the scalar Bose field, a new eigenvalue appears in (ν, ∞).*

We note here that, if $d^\alpha(\mu_0) < 0$, then

$$\mu_0 < \alpha^2 \lim_{t \downarrow 0} \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega(k) + t} \le \alpha^2 \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega(k)} \tag{2.34}$$

since

$$\int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega(k) + t} < \int_{\mathbf{R}^d} dk \frac{|\lambda(k)|^2}{\omega(k)}$$

for all $t > 0$.

In Theorem 2.1(c) for the case $d^\alpha(\mu_0) < 0$, we cannot show the ground state energy of $H_\alpha(\mu_0)$ for the massless bosons, but we can determine the pure point spectra of $H_\alpha(\mu_0)$ completely for the massless bosons *under the condition (A.3)* by using [Sk, Theorem 3.1]:

Proposition 2.2 *Assume (A.1), (A.3) and (1.25). Let $\omega(k) = |k|$ and $d^\alpha(\mu_0) < 0$. Then,*

$$\sigma_{pp}(H_\alpha(\mu_0)) = \{E_{\mu_0}^\alpha, 0\}, \tag{2.35}$$

$$\sigma_{ess}(H_\alpha(\mu_0)) = [E_{\mu_0}^\alpha, \infty) \tag{2.36}$$

for all $\alpha \in \mathbf{R}$ with

$$\alpha^2 < \frac{1}{4\|\lambda^{(1)}\|_0^2}. \tag{2.37}$$

Especially, $E_{\mu_0}^\alpha$ is the simple ground state energy with its unique ground state $\Omega_+^1(g_{\mu_0}^\alpha)$, and 0 is the simple first excited state energy with its unique first excited state Ω_+^0.

In the following proposition, we employ the conjugate operator D_{HS} in [HüS, (2.9)]:

$$D_{\mathrm{HS}} := \frac{1}{2}\left(\frac{1}{|\nabla_k \omega_\nu|^2} \nabla_k \omega_\nu \cdot \nabla_k + \nabla_k \cdot \nabla_k \omega_\nu \frac{1}{|\nabla_k \omega_\nu|^2} \right). \tag{2.38}$$

Proposition 2.3 *Let $\omega(k) = |k|$ and $\nu > 0$. Assume*

$$\int_{\mathbf{R}^d} dk\, |\lambda(k)|^2 \delta\left(\omega_\nu(k) - \mu_0\right) > 0, \tag{2.39}$$

$$\int_{\mathbf{R}^d} dk\, |D_{\mathrm{HS}}\lambda(k)|^2 < \infty \quad and \quad \int_{\mathbf{R}^d} dk\, |D^2_{\mathrm{HS}}\lambda(k)|^2 < \infty, \tag{2.40}$$

and $d^\alpha_\nu(\mu_0) < 0$. Then,

(a)

$$\sigma_{pp}\left(H_\alpha(\mu_0;\nu)\right) = \left\{E^\alpha_{\mu_0,\nu}\,,\, 0\right\}, \tag{2.41}$$

$$\sigma_{ess}\left(H_\alpha(\mu_0;\nu)\right) = \left[\min\left\{E^\alpha_{\mu_0}\,,\,0\right\} + \nu\,,\, \infty\right) \tag{2.42}$$

for all $\alpha \in \mathbf{R}$ with

$$|\alpha| \|D_{\mathrm{HS}}\lambda\|_0 < 1. \tag{2.43}$$

(b) *If $\mu_0 > \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2$, then 0 is the simple ground state energy with its unique ground state Ω^0_+, and $E^\alpha_{\mu_0,\nu}$ is the simple first excited state energy with its unique first excited state $\Omega^1_+(g^\alpha_{\mu_0,\nu})$ for all $\alpha \in \mathbf{R}$ with (2.43).*

(c) *If $\mu_0 < \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2$, then $E^\alpha_{\mu_0,\nu}$ is the simple ground state energy with its unique ground state $\Omega^1_+(g^\alpha_{\mu_0,\nu})$, and 0 is the simple first excited state energy with its unique first excited state Ω^0_+ for all $\alpha \in \mathbf{R}$ with (2.43).*

(d) *Assume $\mu_0 > 0$ and $\sqrt{\mu_0}\|D_{\mathrm{HS}}\lambda\|_0 < \|\lambda/\sqrt{\omega_\nu}\|_0$, then $H_\alpha(\mu_0;\nu)$ has degenerate ground states for $\alpha_c = \sqrt{\mu_0}/\|\lambda/\sqrt{\omega_\nu}\|_0$ with ground state energy $0 = E^\alpha_{\mu_0,\nu}$, and ground states are given by Ω^0_+ and $\Omega^1_+(g^\alpha_{\mu_0,\nu})$.*

We define expectations, $\overline{n}_{grd}$ and $\overline{n}_{1st}$, of the number of (massive) photons at the ground and first excited states, respectively, as follows:

$$\overline{n}_{grd} := \left(\Psi_{grd}\,,\, I \otimes N_b \Psi_{grd}\right)_{\mathcal{F}}, \tag{2.44}$$

$$\overline{n}_{1st} := \left(\Psi_{1st}\,,\, I \otimes N_b \Psi_{1st}\right)_{\mathcal{F}}, \tag{2.45}$$

where Ψ_{grd} and Ψ_{1st} denote the ground state and first excited state of $H_\alpha(\mu_0;\nu)$, respectively.

By Proposition 2.3, we obtain the following corollary:

Corollary 2.4 *Let $\omega(k) = |k|$ and $\nu > 0$. Assume (2.39) and (2.40), and $d^\alpha_\nu(\mu_0) < 0$. Then, for all $\alpha \in \mathbf{R}$ with (2.43),*

(a)

$$\overline{n}_{grd} = \begin{cases} 0 & \text{if} \quad \mu_0 > \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2, \\ \|g^\alpha_{\mu_0,\nu}\|_0^2 & \text{if} \quad \mu_0 < \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2. \end{cases}$$

(b) *A reverse between $\overline{n}_{grd}$ and $\overline{n}_{1st}$ occurs as follows:*

$$\begin{cases} \overline{n}_{grd} < \overline{n}_{1st} & \text{if} \quad \mu_0 > \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2, \\ \overline{n}_{1st} < \overline{n}_{grd} & \text{if} \quad \mu_0 < \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2. \end{cases}$$

(A.4) The functions $\omega(k)$ is continuous with

$$\lim_{|k|\to\infty} \omega(k) = \infty, \tag{2.46}$$

and there exist constants $\gamma_\omega > 0$ and $C_\omega > 0$ such that

$$|\omega(k) - \omega(k')| \leq C_\omega |k - k'|^{\gamma_\omega} \left(1 + \omega(k) - \omega(k')\right), \qquad k, k' \in \mathbf{R}^d. \tag{2.47}$$

The $\lambda(k)$ is also continuous.

Theorem 2.5 *Let $\nu \geq 0$. Assume (A.1). Then,*

(a) *there exists $\alpha_{\mathrm{WW}}(\nu) > 0$ such that*

$$\left\{E^\alpha_{\mu_0,\nu}\,,\, 0\right\} \subset \sigma_{pp}\left(H_\alpha\left(\mu_0;\nu\right)\right) \tag{2.48}$$

$$\text{with } E_0\left(H_\alpha\left(\mu_0;\nu\right)\right) < \min\left\{E^\alpha_{\mu_0,\nu}\,,\, 0\right\}, \tag{2.49}$$

$$\sigma_{ess}\left(H_\alpha\left(\mu_0;\nu\right)\right) = \left[E_0\left(H_\alpha\left(\mu_0;\nu\right)\right) + \nu\,,\, \infty\right) \tag{2.50}$$

for every $\alpha \in \mathbf{R}$ with $|\alpha| > \alpha_{\mathrm{WW}}(\nu)$.

(b) *let $\nu > 0$ (massive bosons). Assume (A.4) in addition. Then, there exists a ground state $\Psi_{\mathrm{WW}} \in \mathcal{F}$ of $H_\alpha\left(\mu_0;\nu\right)$, namely*

$$H_\alpha\left(\mu_0;\nu\right)\Psi_{\mathrm{WW}} = E_0\left(H_\alpha\left(\mu_0;\nu\right)\right)\Psi_{\mathrm{WW}},$$

such that

$$\left\{E_0\left(H_\alpha\left(\mu_0;\nu\right)\right)\,,\, E^\alpha_{\mu_0,\nu}\,,\, 0\right\} \subset \sigma_{pp}\left(H_\alpha\left(\mu_0;\nu\right)\right), \tag{2.51}$$

with (2.49)

$$\Psi_{\mathrm{WW}} \notin \mathcal{F}_0 \cup \mathcal{F}_1 \tag{2.52}$$

for every $\alpha \in \mathbf{R}$ with $|\alpha| > \alpha_{\mathrm{WW}}(\nu)$.

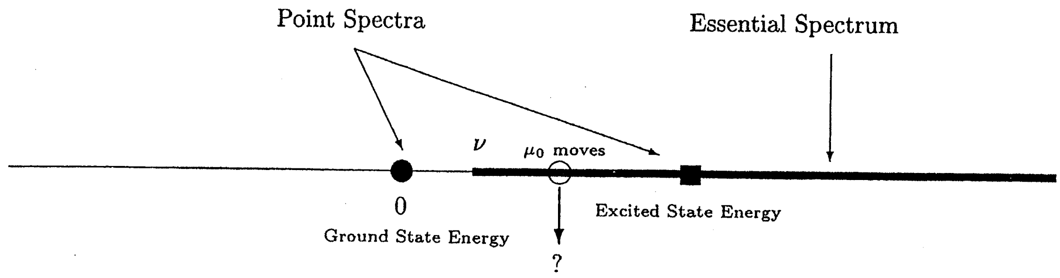
(c) *Let $\nu = 0$ (massless bosons). Assume (A.4), $\nabla\omega \in L^\infty(\mathbf{R}^d)$ and (1.25) in addition. Then, there exists a ground state $\Psi_{\mathrm{WW}} \in \mathcal{F}$ of $H_\alpha\left(\mu_0;\nu\right)$ such that (2.51), (2.49) and (2.52) hold for every $\alpha \in \mathbf{R}$ with $|\alpha| > \alpha_{\mathrm{WW}}(0)$.*

Remark 2.5 *When the case of massive bosons ($\nu > 0$), we can apply the regular perturbation theory to the Wigner-Weisskopf model for sufficiently small absolute value of the coupling constant $|\alpha|$, and then Theorem 1.1 says that we get either $E^\alpha_{\mu_0,\nu}$ or 0 as the ground state energy. Theorem 2.5 means that, for sufficiently large absolute value of the coupling constant, a non-perturbative ground state appears as an influence of the scalar Bose field with its ground state energy so low that we cannot conjecture it by the regular perturbation theory from sufficiently small absolute value of the coupling constant. For other models, the similar phenomenon were investigated by Hiroshima and Spohn So, Theorem 2.5 may make a statement on the existence of a superradiant ground state in physics (see, for instance, [Pr1, Pr2, En]) for the Wigner-Weisskopf model. Namely, we can say that, even for the Wigner-Weisskopf model which is simple and familiar in physics, we may be able to show a phenomena of superradiant ground state influenced by the scalar Bose field. [HiS].*

The following 図 1-3 show the spectra which we had found, so not all:

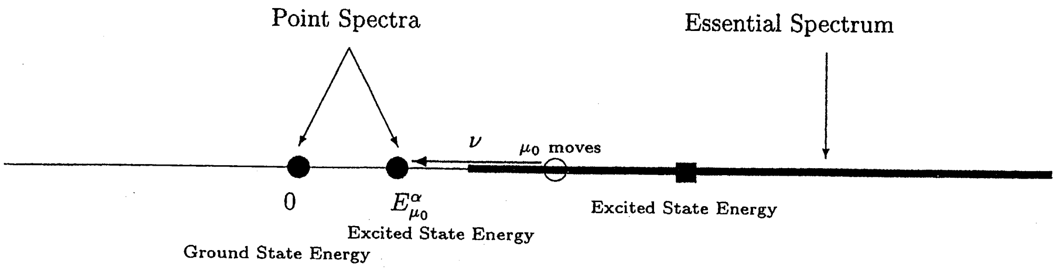
(I) For $|\alpha| < \alpha_{\mathrm{WW}}(\nu)$:

(I-a) Let $d_\nu^\alpha(\mu_0) \geq 0$. Then

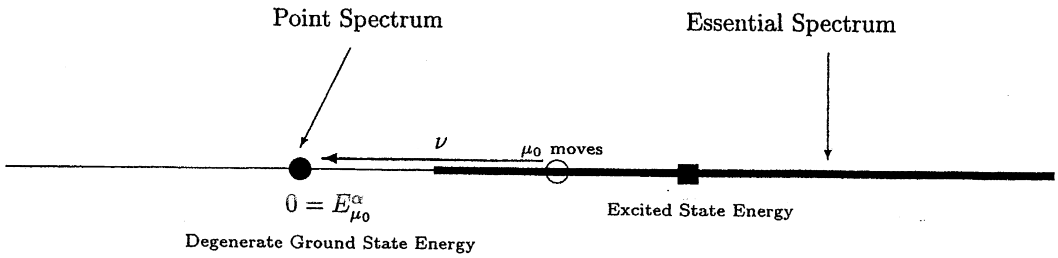


(I-b) Let $d_\nu^\alpha(\mu_0) < 0$.

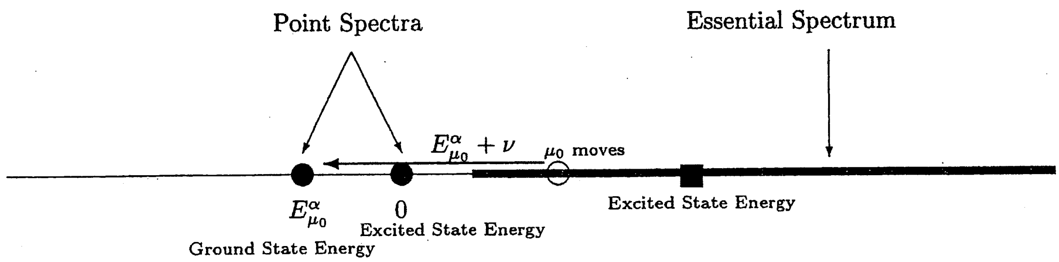
(I-b-1) If $\mu_0 > \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2$, then



(I-b-2) If $\mu_0 = \alpha^2 \|\lambda/\sqrt{\omega_\nu}\|_0^2$, then



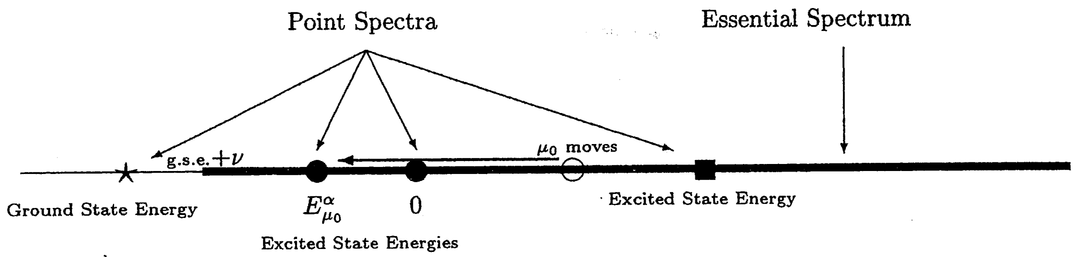
(I-b-3) If $\mu_0 < \alpha^2\|\lambda/\sqrt{\omega_\nu}\|_0^2$, and all other hypothese in Theorem 2.1(c) hold, then



Appearance or disappearance of ■ depends on the condition for λ by an effect of the scalar Bose field as non-purterbative eigenvalue.

図 1: Spectra We Had Found for WW Model (I) for $\nu > 0$

(II) For $|\alpha| > \alpha_{\rm WW}(\nu)$: If all hypotheses in Theorem 2.5 (b) hold, then

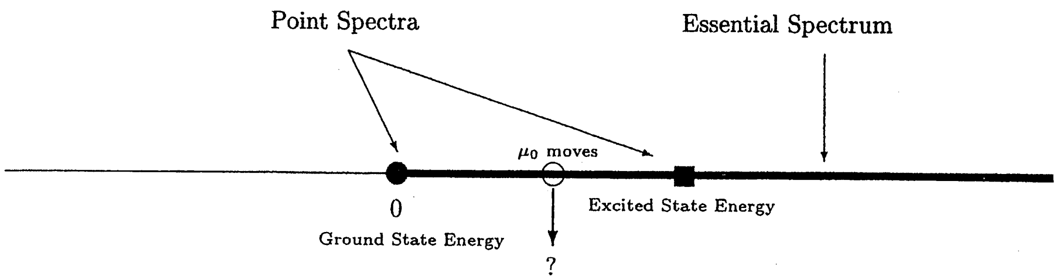


Appearance or disappearance of ■ depends on the condition for λ, and ★ appears by an effect of the scalar Bose field. Both of ★ and ■ are non-perturbative eigenvalues.

図 2: Spectra We Had Found for WW Model (II) for $\nu > 0$

(I) For $|\alpha| < \alpha_{\mathrm{WW}}(0)$:

(I-a) If $d^{\alpha}(\mu_0) \geq 0$, then



Appearance or disappearance of ■ depends on the condition for λ by an effect of the scalar Bose field as non-perturbative eigenvalue.

(I-b) If all hypotheses in Proposition 2.2 hold, then

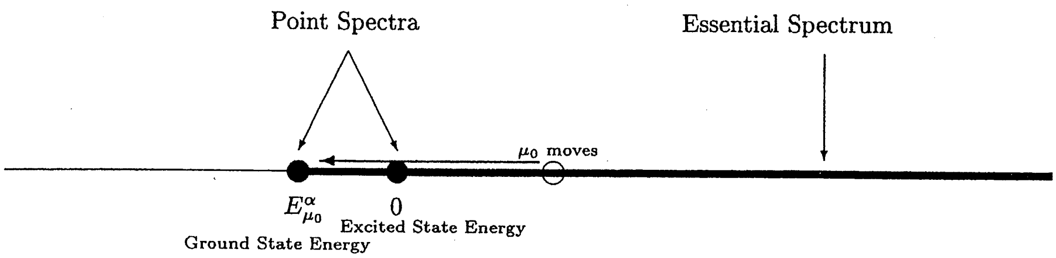
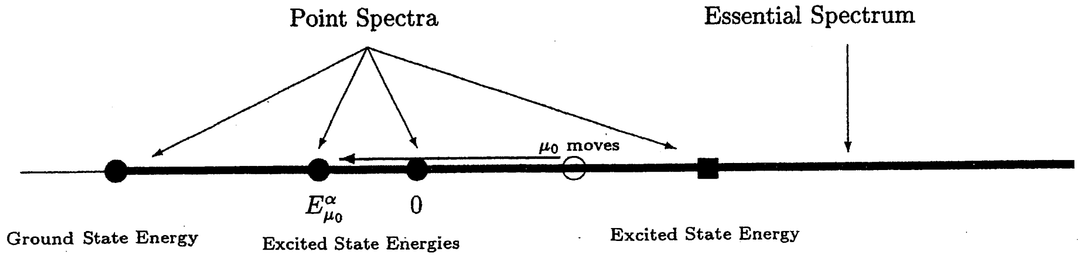


図 3: Spectra We Had Found for WW Model (I) for $\nu = 0$

(II) For $|\alpha| > \alpha_{\mathrm{WW}}(0)$: If all hypotheses in Theorem 2.5 (c) hold, then



Appearance or disappearance of ■ depends on the condition for λ, and ★ appears by an effect of the scalar Bose field. Both of ★ and ■ are non-perturbative eigenvalues.

図 4: Spectra We Had Found for WW Model (II) for $\nu = 0$

謝辞

H. Spohn proposed the problem of expressing $E_{\mathrm{SB}}(0)$ independently of the existence of its ground state to me when I held discussions on [mHi2] with him, though I assumed the existence in [mHi2]. So, this is the beginning of the problem I dealt with in this paper. I wish to thank him for giving me the beginning of the problem. I argued the problem about $E_{\mathrm{SB}}(0)$ given by the limit (1.14) of the explicit expression for $E_{\mathrm{SB}}(\nu)$ in [mHi2] with V. Bach and A. Elgart when I visited Technische Universität Berlin during September 8-10, '98. Then the above problem (1.13) - (1.15) on the survival of μ arose. I wish to thank them for arrangements of my visiting Technische Universität Berlin and the hospitality. I am indebted to A. Arai for useful discussions which proofs in this paper were based on. I thank H. Spohn and F. Hiroshima for their hospitality at Technische Universität München during April 15-22, '99, and discussing Spohn's unpublished results. I wish to express H. Spohn, R. A. Minlos, H. Ezawa, K. Watanabe, K. Yasue, M. Jibu and F. Hiroshima for valuable advice. I wish to thank J. Dereziński for discussing several aspects about the generalized spin-boson model at the summer school "Schrödinger Operators and Related Topics," Shonan Village Center, July 5-9, '99, and also C. Gérard for telling me how to get his recent result which broke through a wall in Theorem 2.5 (c). My research is supported by the Grant-In-Aid No.11740109 for Encouragement of Young Scientists from Japan Society for the Promotion of Science.

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